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A THEORETICAL INVESTIGATION OF THE BOUND STATES AND RESONANCES IN THE NΔ SYSTEM

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Теоретическое рассмотрение связанных состояний и резонансов в системе $N\Delta$

На основе решения задачи Гильберта-Шмидта для уравнений Фаддеева показано, что в системе $N\Delta$-изобара отсутствуют связанные и резонансные состояния.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Belyaev V.B., Möller K., Simonov Yu.A.

E2 - 11306

A Theoretical Investigation of the Bound States and Resonances in the $N\Delta$ System

By solving the Hilbert-Schmidt problem for the Faddeev equations it is shown, that under the condition of π-meson exchange between N and Δ_{33} in the system $N\Delta_{33}$ bound and resonance states are absent.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

The problem of the bound states and resonances in the NΔ system is very puzzling both from the theoretical and the experimental point of view.

The experimental situation up to 1969 was summarized in [1]. Since then some new data appeared [2]. One can argue in the NΔ final and intermediate states peaks are present, but it is not proved experimentally whether one can associate these peaks with bound states and resonances of the NΔ system. The theory could help in this respect, but it only complicates the situation. Namely, the poles in the NΔ amplitude should show up as poles in the πd scattering amplitude and consequently as loops in the πd Argand plots. The latter were found theoretically [3] but an argument was given in [4] that the Argand loops do not necessarily imply the poles, but might be due to the logarithmic singularities. The latter fact is known as a pseudoresonance phenomenon [5]. The experimental phase-shift analysis in the πd system is absent at present, but even if it would show the existence of Argand loops, we could not deduce from this fact the experimental evidence for the existence of NΔ bound states and resonances.

Theoretically, three different approaches were tested for the NΔ system. In the potential approach one considers NΔ as a two-body system with the specific exchange potential of oscillating character due to decay pion exchange, Fig. 1*. In the first investigation of this kind [7] the exchange potential was found to be weak and therefore no bound states and resonances were predicted (note the overall wrong sign in the definition of the potential in [7] which changes the notion of attraction and repulsion in different spin-isospin states). The subsequent paper of Arenhövel [8] contained additionally the potential due to the non-decay pion exchange, Fig. 2, which is usually called a direct pion (or boson) exchange. In ref. [8] a bound state ** of NΔ was

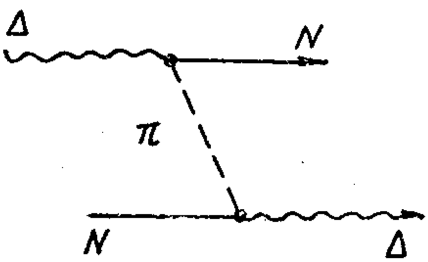

Fig. 1

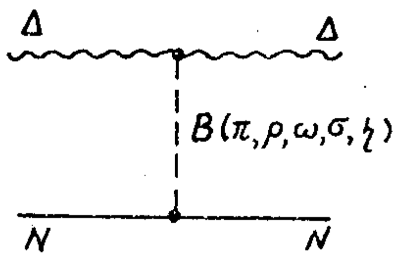

Fig. 2

predicted with the spin-isospin $J^P, T = 2^+, 2$. A more extensive paper [9] contained also the ρ and π exchange in the $2^+, 2$ state and again predicted a bound $2^+, 2$ state.

In the second approach the unitarity and analyticity for the properly defined resonance-particle amplitude result in the N/D equations with the dynamical input corresponding to the diagrams of Figs. 1 and 2 treated relativistically in the momentum

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*A detailed review on the NΔ potential approach can be found in [6].

** Note that this is not a bound state in the usual sense, since it is unstable against NNπ decay.

space[10,11]. The calculations for the $N\Delta$ system were performed by A.B.Badalyan and M.I.Polikarpov [11] with a negative answer to the existence of $N\Delta$ bound states, with $\ell=0$ (only pion exchange in Figs. 1 and 2 was taken into account.

A similar investigation for the negative parity states was performed by J.Tjon and one of the present authors (Yu.S.) which resulted in a very narrow peak in the $J^P=0^-$ state; the effect, however, could arise from the narrow-width approximation in the $N\Delta$ phase-space factor used by these authors.

A third approach deals with the Faddeev equation. If the two-body t-matrix is replaced by the separable Breit-Wigner form with off-shell formfactors, the Faddeev equation reduces to a one-dimensional Lippman-Schwinger-type equation. In paper [12] the Faddeev equations were solved for the πNN system. A possibility of the formation of bound and resonant states in such systems has been studied. The πN interaction was described by a separable t-matrix with parameters fitted to the P_{33} phase shifts. The influence of the NN interaction has been neglected. In the non-relativistic version of the calculations two resonances with quantum numbers $J^P, T=2^-, 2$ were found. In the relativistic version, however, no definite conclusion could be made. One can summarize the theoretical results as puzzling and contradictive. First of all, the potential approach is not very reliable because it treats the exchange interaction of Fig. 1 in the configuration space as a local potential. In reality it is a rough approximation since 1) the exchange diagram of Fig. 1 results in a nonlocal and energy dependent interaction, 2) this potential oscillates in the configuration space with an amplitude decreasing only as an inverse power of r and special care should be taken to treat such an ill-behaved potential, 3) the effects of the width of Δ are not taken properly into account in the potential approach. But even if the calculations performed within the potential model are not quite correct, certainly they will be correct in order of magnitude and from that we know that the interaction in the NΔ system is strong and is most attractive in the $J^P, T = 2^+, 2$ state /8/. From the N/D approach we can infer a negative result on the existence of bound states due to pion exchange only. This result does not contradict the predictions of the Faddeev equation approach /12/. In principle these approaches seem to be rather consistent in their treatment of the NΔ system. One should notice, however, that it is not known whether the approximation applied in paper/12/to solve the Faddeev equations above the NNπ threshold is justified. Therefore it was necessary to reanalyze the situation in the NΔ system and to search again for bound states and resonances in the framework of the Faddeev equation. Such an analysis is done on the basis of a newly developed numerical procedure for solving the Faddeev equations at positive energies/13/and a compute code for the calculation of the eigenvalues of the Faddeev-kernel described in paper /14/.

2. METHOD

We begin with the πN two-body t-matrix, chosen in the following form:

$$t(q,q';\sigma) = \frac{8\pi\sqrt{\sigma}}{p(\sigma)} \sin\delta \cdot e^{i\delta} \cdot F(q^2) \cdot F(q'^2) = \frac{8\pi\Gamma\sigma F(q^2)F(q'^2)}{p(\sigma)[(M_R - i\frac{\Gamma}{2})^2 - \sigma]} \tag{1}$$

with the off-shell form factors $F(q^2)$

$$F(q^2) = \frac{q_o^2 + \gamma^2}{q^2 + \gamma^2}, \quad q_o = p(\sigma = M^2), \tag{2}$$

$$p(\sigma) = \sqrt{\frac{[\sigma - (m+M)^2][\sigma - (M-m)^2]}{4\sigma}}, \Gamma = \Gamma_o\left(\frac{p(\sigma)}{q_o}\right)^{2\lambda+1}. \tag{3}$$

The orbital momentum of the resonance λ is equal to 1, M is the nucleon mass and m is that of a pion. The radius of the form factor γ is chose to be 2m. In what follows the NN interactions are neglected, since according to the calculations /15/ their contributions will not be very important in comparison with the πN interaction in the resonance region. We shall consider the $N\Delta$ system as a nonrelativistic one, while the exchanged pion is treated everywhere relativistically. Denoting the excitation energy by $\epsilon = M_R - M - m \cong 160$ MeV we now introduce the following dimensionless variables

$$y = \frac{k^2}{2\mu\epsilon}, \quad y' = \frac{k'^2}{2\mu\epsilon}, \quad \mu = \frac{MM_R}{M+M_R}, \quad y_o = \frac{E}{\epsilon}, \tag{4}$$

where k and k′ are the relative momenta of the $N\Delta$ system in the initial and final states which are off-shell momenta, y_o is connected with the kinetic energy E of the relative motion of Δ and N. On the energy shell we have $y = y' = y_o$.

We use the separable form (1) in the Faddeev equations in the framework of the LS coupling, and we denote by ℓ the relative orbital momentum of $N\Delta$ and by S the total spin of $N\Delta$, J being the total momentum of the ΔN system.

Then the following equation is obtained:

$$M^J_{\ell S, \ell'' S''}(y, y''; y_o) = B^J_{\ell S, \ell'' S''}(y, y''; y_o) +$$

$$+ \sum_{\ell' S'} \int_0^\infty \frac{\sqrt{y'}\, dy' (\frac{\Gamma}{\Gamma_o} \cdot \frac{q_o}{p})}{\pi (y' - y_o - \frac{i\Gamma}{2\epsilon})} B^J_{\ell S, \ell' S'}(y, y'; y_o) \cdot M^J_{\ell' S', \ell'' S''}(y', y''; y_o). \tag{5}$$

The eq. (5) is represented graphically in Fig. 3, where the rectangular box is for the Born term (it also enters into the kernel of the equation), the circle is for the $N\Delta$ amplitude $M^J_{\ell S, \ell' S'}$, Δ is denoted by the wavy line and N, by the straight line; the $N\Delta$

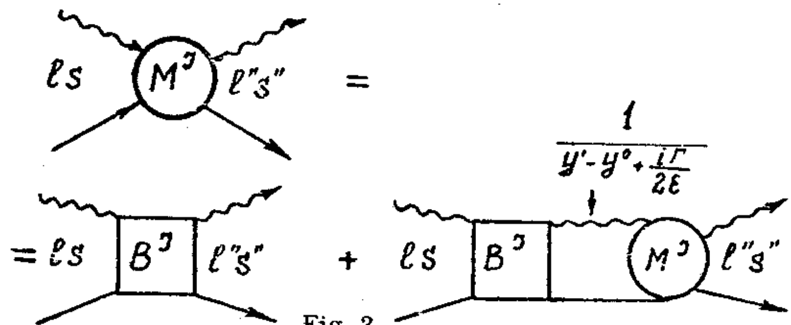

Fig. 3

free Green function is given by $(y'-y_0-i\frac{\Gamma}{2\epsilon})^{-1}$. One can see, that in the narrow width limit $\Gamma \to 0$ we obtain the usual two-body Green function, nevertheless the potential term B^J corresponds to an energy dependent and nonlocal interaction as was stated above.

The p and q_0 in (5) can be expressed through y' and y_0:

$$p=\sqrt{\frac{M\epsilon(1+y_0-y')[2m+\epsilon(1+y_0-y')]}{M+m}}, \quad q_0=p(y'=y_0)\,. \qquad (6)$$

Now the dynamical input B^J was considered as the diagram Fig. 4, with the form factors $F(q)$, $F(q')$ at both vertices and with the proper LS-projection, namely:

$$B^J_{\ell s,\,\ell' s'}(y,y';y_0)=-\frac{1}{\pi}\int_{-1}^{1}\frac{\xi_{\ell s,\ell' s'}(y,y';y_0;x)F(q^2)F(q'^2)dx}{x-u(y,y';y_0)}. \qquad (7)$$

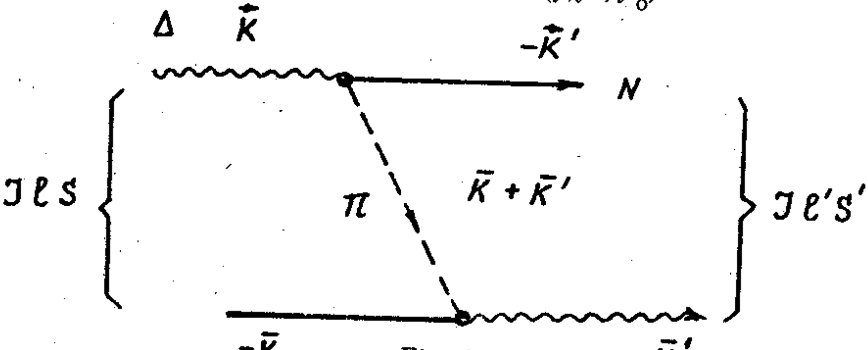

Fig. 4

Here x is the cosine of the angle between $\vec{k}$ and $\vec{k}'$ (see Fig. 4) and u is its value at the pole of the pion propagator:

$$u(y,y';y_0) = \frac{(m+\epsilon)y_0 \dfrac{M+M_R}{M_R^2} + \dfrac{(2m+\epsilon)(M+M_R)}{2M_R^2} - y - y'}{2\alpha\sqrt{yy'}} \qquad (8)$$

and in the half-off-shell limit it simplifies to:

$$u(y,y';y) = \frac{\beta - \alpha y - y'/\alpha}{2\sqrt{yy'}}, \qquad (9)$$

where

$$\beta = \frac{2m+\epsilon}{2M\,M_R}(M+M_R); \quad \alpha = \frac{M}{M_R}. \qquad (10)$$

It is easy to see from Fig. 4 that q^2 and q'^2 in (7) can be expressed through k, k' and $x = \cos(\vec{k}\vec{k}')$:

$$q^2 = \alpha^2 k^2 + 2\alpha k k' x + k'^2 = 2\mu\epsilon(y\alpha^2 + 2\alpha x\sqrt{yy'} + y'), \qquad (11)$$

$$q'^2 = k^2 + 2\alpha k k' x + \alpha^2 k'^2 = 2\mu\epsilon(y + 2\alpha x\sqrt{yy'} + \alpha^2 y'). \qquad (12)$$

Finally, we need the expressions for $\xi_{\ell S,\ell' S'}(y,y';y_0,x)$. They are computed for the diagram of Fig. 4 using the LS-coupling and the usual spin-isospin recoupling coefficients τ^T. In the Appendix they are given for the cases $\ell = 0,1$ and $J^P = 1^+, 2^+, 0^-$ and all $T (T = 1,2)$. We keep only the lowest value of ℓ for a given J^P and neglect the admixture of higher values of ℓ. Thus equations (5) reduce to one-channel equations for the cases $J^P = 1^+, 2^+$ and $J^P = 0^-$, while for $J^P = 1^-, 2^-$ even with this assumption we obtain two-channel equations. In the case $J = 0^-$ there is no admixture of higher ℓ whatever, but in the cases $J^P = 1^+, 2^+$ the value $\ell = 2$ is possible together with $\ell = 0$. We neglect $\ell = 2$ in those cases, because we are interested in the near-threshold effects, where $\ell = 2$ is suppressed by the centrifugal barrier. So we are left with one-channel equations for the cases $J^P = 1^+, 2^+, 0^-$ and $T = 1,2$. Instead of equations (5) the corresponding homogeneous equations with the kernel multiplied by $1/\lambda_n(E)$ have been solved. The two largest eigenvalues were found, which have both real and imagibary part. We have used the fact, that ξ (as is seen from the Appendix) is the same (up to a constant factor) in all the $J^P = 1^+, 2^+$ and $T = 1,2$ states. Consequently the quantities B^J are also the same and one needs to solve only one homogeneous equation for all $J^P = 1^+, 2^+$ states, different eigenvalues can be expressed through a single one. The same is true for $J^P = 0^-$ and different isospin states. So one is left with only two different equations, one for all $J^P = 1^+, 2^+$ states and the other for all $J^P = 0^-$ states. These equations were solved both below and above threshold using the newly proposed computational procedure /14/. By a simple transformation of the variables the eigenvalue equation corresponding to Eq. (5) can be written in the form

$$\lambda_n \phi_n(x) = \int^{\infty} dx' K(x,x';y_o) \phi_n(x') \tag{13}$$

with the kernel

$$K(x,x';y_o) = \frac{f_1(x')}{x'^2 - x_o^2 - i\Gamma_o w(x')} \int_{-1}^{1} dy \frac{f_2(x,x',y;y_o)}{y - u(x,x';y_o)} . \tag{14}$$

Here $f_1(x')$, $f_2(x,x',y;y_o)$ and $w(x')$ are non-singular functions. The quantities x_o and u are defined as

$$x = \frac{y}{\left| y + \frac{m + \epsilon/2}{m + \epsilon} \right|}, \tag{15}$$

$$u(x,x';y_o) = \frac{e_3 + a(x^2 + x'^2)}{xx'} \tag{16}$$

with

$$a = -\frac{M_R}{2M}; \; / \; e_3 = \frac{1}{2\mu}\,\mathrm{sign}[y_o + \frac{m+\epsilon/2}{m+\epsilon}]\,. \tag{17}$$

The kernel contains logarithmic singularities x'_i at the points:

$$x'_{(1,2,3,4)} = \pm\frac{x}{2a} \pm \sqrt{-\frac{e_3}{a} - x^2(1-\frac{1}{4a^2})} \tag{18}$$

and poles x'_p defined by the equation

$$x'^2_p - x^2_o - i\Gamma_o\, w(x'_p) = 0\,. \tag{19}$$

Thus we have reduced our problem to the same equations as given in paper [14] (apart from slight differences which are not essential for the method of solutions). As has been shown in [14] a kernel of the type discussed can be splitted into four terms containing the singularities in such a form that they can be integrated over analytically. To achieve this all non-singular functions occuring in the splitted kernel are approximated by Lagrange interpolating polynomials.

3. RESULTS

The results for the two largest eigenvalues are given in Tables 1,2. We have quoted there only the eigenvalues $\lambda(J^P, T=1^+, 2)$ and $\lambda(J^P, T=0^-, 2)$. Other eigenvalues are obtained from these using the equations:

$$\lambda(2^+,2) = -3\lambda(1^+,2);\; \lambda(2^+,1) = \lambda(1^+,2);\; \lambda(1^+,1) = -\frac{1}{3}\lambda(1^+,2) \tag{20}$$

$$\lambda(0^-,1) = -\frac{1}{3}\lambda(0^-,2). \tag{21}$$

We see from Tables 1 and 2 that the eigenvalues are never close to unity, even if the real part of λ

Table 1

$\lambda(J^P, T = 1^+, 2)$

y_o	$Re\lambda_1$	$Im\lambda_1$	$Re\lambda_2$	$Im\lambda_2$
0.7	2.17	-1.19	-0.248	0.268
0.2	1.13	-1.71	-0.252	0.276
0	0.380	-1.67	-0.210	0.289
-0.2	-0.290	-1.09	-0.216	0.262
-0.3	-0.386	-0.703	-0.214	0.232
-0.4	-0.346	-0.390	-0.218	0.333
-0.5	-0.243	-0.179	-0.214	0.343
-0.6	-0.130	-0.055	-0.230	0.340
-0.8	-0.071	-0.047	-0.227	0.300
-1.0	0.072	$6 \cdot 10^{-3}$	-0.243	0.356

is near to one, the imaginary part is quite sizeable (of an order of 1); hence, there are no resonances and bound states in the energy region considered here ($E \leq 240$ MeV).

In addition, we considered the direct pion contribution to B^J, that is we have taken B^J to be equal to the sum of the diagrams, Figs. 1 and 2. The latter contribution is again described by eq. (7) with ξ_d for $J^P = 1^+, 2^+$ and $T = 1,2$ given by:

$$\xi_d(y, y'; y_o) = -\pi\sqrt{\frac{\mu}{8\epsilon}} \cdot \frac{V_o(J,T)}{\sqrt{yy'}}, \tag{22}$$

Table 2

$\lambda(J^P, T = 0^-, 2)$

y_o	$Re\lambda_1$	$Im\lambda_1$	$Re\lambda_2$	$Im\lambda_2$
1.5	0.204	-0.129		
1.0	0.204	-0.125		
0.5	0.167	-0.182	0.132	-0.0477
0.2	0.109	-0.224	0.0535	-0.066
-0.1	-0.034	-0.187	0.011	-0.056
-0.4	-0.058	-0.04	-0.037	0.04
-0.7	-0.050	0.30	-0.009	0.039

Table 3

$\lambda(J^P, T = 1^+, 2)$ with direct pion exchange

y_o	$Re\lambda_1$	$Im\lambda_1$	$Re\lambda_2$	$Im\lambda_2$
0.7	2.11	-1.05	0.13	0.33
0.2	1.17	-1.61	-0.25	0.27

where

$$V_o(2,2) = \frac{3f_{NN\pi} \cdot f_{\Delta\Delta\pi}}{4\pi} \cong 0.06\,, \tag{23}$$

$$V_o(1,2) = V_o(2,1) = -\frac{5f_{NN\pi} \cdot f_{\Delta\Delta\pi}}{4\pi} \cong -0.1\,. \tag{24}$$

We have used the values for the coupling constants, in accordance with [9], namely

$$\frac{f^2_{NN\pi}}{4\pi} = 0.08\,; \quad \frac{f^2_{\Delta\Delta\pi}}{4\pi} = 0.005\,. \tag{25}$$

The resulting figures for the eigenvalues λ_1 and λ_2 in the state $J^P = 1^+$, $T=2$ are given in Table 3.

A glance at the figures in Tables 1-3 shows that the pion exchange mechanism due to the diagrams of Figs. 1 and 2 is not operative in producing bound states and resonances in the $N\Delta$ system. Specifically, the direct pion exchange, Fig. 2, is not substantial as compared to the stronger decay pion exchange mechanism, Fig. 1.

Our results agree with the calculations of A.M.Badalyan and M.I.Polikarpov [11], where also no bound states and resonances appear due to the pion exchange mechanism in the N/D equations. At the same time our results disagree with the results of Arenhövel [8], who obtained a bound state in the $J^P, T = 2^+, 2$ state. In ref. [8] two-channel equations were considered, in contrast with our one-channel case with $\ell=0$, however, it is not very like that this fact is responsible for the discrepancy between the result of ref. [8] and ours. However, we know from OBEP models for NN interaction that the one pion exchange is also rather unimportant for $\ell = 0,1$, whereas ω and σ exchanges contribute mostly to the short range repulsive and medium range attractive NN interactions. The same conclusion can be drawn from the potential approach formulas in [9].

So, we believe that the ω, ρ and σ contribute essentially to the NΔ interaction and might bring about the bound states and the resonances in the NΔ system. These exchanges can be taken into account easily in our equations and will be treated in a next publication.

APPENDIX

The expressions for $\xi(y, y'; y_o; x)$

$\ell = 0$, $J^P = 1^+, 2^+$; any T

$$\xi = -\frac{\omega^{TJ}}{8}\zeta[x(1+\alpha^2) + \alpha(\sqrt{\frac{y}{y'}} + \sqrt{\frac{y'}{y}})],$$

$\ell = 1$, $J^P = 0^-$; any T

$$\xi = \frac{\zeta\tau^T}{4}[\frac{3}{4}x^2 + \frac{1}{4} + \alpha^2 + \alpha x(\sqrt{\frac{y}{y'}} + \sqrt{\frac{y'}{y}})].$$

Here $\alpha = \dfrac{M}{M_R}$,

$$\zeta = 4\pi\sqrt{2}\frac{\Gamma_o}{2\epsilon}\;\frac{M_R^4}{M(M+M_R)^{3/2}\;(\epsilon + 2m)^{3/2}},$$

$$\omega^{TJ} = \tau^T \cdot \tau^J,\quad \tau^j = \begin{cases} -1/3, & j = 1 \\ 1, & j = 2 \end{cases}.$$

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