ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
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ИССЛЕДОВАНИЙ
ДУБНА

3/IV-78

E2 - 11297

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1978

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Submitted to ТМФ

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Кварки в квантованном пространстве

В работе рассматриваются следствия для кинематики кварков, основанной на предположении о квантовании пространства. Показано, что существует только две упорядоченные в пространстве структуры из двух частиц или из трех частиц. На этой основе дана геометрическая интерпретация цвета.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

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Quarks in Quantized Space

Consequences are considered for the quark kinematics based on the quantized space. It is shown that there exist only two space-ordered structures composed of two or three particles. A geometrical interpretation of colour is given, too.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1978

1. ON SPACE QUANTIZATION

A general statement of the problem is as follows: The usual (c -number) coordinates of points x_1, x_2, x_3, x_4 which form a differentiable manifold $\mathfrak{M}_4(x)$ (with a certain metrics) are changed by linear operators $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$, $\hat{x}_4$, in general, noncommuting with each other. Then, the question immediately arises concerning the numerical ("measurable") coordinates of a point event and the ordering of events in this operational space, $\mathfrak{M}_4(\hat{x})$.

The only reasonable answer to that question is to admit a mapping of such an operational space on a space of eigenvalues of $\hat{x}$ or of functions of $f(\hat{x})$ which form a complete set of variables. This set should be sufficient for ordering points in the four-dimensional Pseudo-Euclidean space.

Along this line, we postulate the space $\mathcal{H}(\Phi)$ of eigenfunctions Φ of the complete set at each point of space $\mathfrak{M}_4(x)$.

Further, we will consider three examples of the operational space, and apply the latter of them to the quark theory.

Example I. Let $\hat{x}_\mu = i \frac{\partial}{\partial p_\mu}$, p_μ is the conjugate momentum. Points p form the Minkowski flat space $\mathfrak{M}_4(p)$. Operators $\hat{x}_\mu$ commute with each other and possess common eigenfunctions, $\Phi = e^{i p_\mu x'_\mu}$, where x'_μ are eigenvalues of operator $\hat{x}_\mu$. This example is trivial: the space $\mathfrak{M}_4(\hat{x})$ corresponds to the numerical (c -number) space $\mathfrak{M}_4(x')$.

Example II. Let the momentum space $\mathfrak{M}_4(p)$ be of constant curvature. The operators $\hat{x}_\mu$ are then considered as displacement operators in this curved space.

This possibility was pointed out by Snyder /1/ many years ago and was thoroughly studied by Kadyshevsky et al. /2/.

The space $\mathfrak{M}_4(p)$ was taken to be the De Sitter space, the space with constant negative curvature. Its geometry is equivalent to the geometry of the hyperboloid in a flat five-dimensional space

$$p_1^2 + p_2^2 + p_3^2 + p_4^2 + p_5^2 = -1/a^2 \tag{1}$$

with $p_4 = ip_0$, $p_5 = iq_0$ (p_0, q_0 real). Operators $\hat{x}_\mu$ ($\mu = 1,2,3,4$) are now noncommuting

$$[\hat{x}_\mu, \hat{x}_\nu] = ia^2 \hat{\mathfrak{M}}_{\mu\nu}, \tag{2}$$

where $\mathfrak{M}_{\mu\nu}$ is the rotation operator, a is an elementary length defining the curvature of momentum space.

As follows from the commutator (2), the eigenvalues of operators $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$, $\hat{x}_4$ cannot form the complete set of four variables. As shown in refs. /2,3/, this set can be constructed in terms of the eigenvalues of the Casimir operator of the De Sitter group SO(2,3):

$$\hat{s}^2 = \mathfrak{M}_{k\ell} \mathfrak{M}^{k\ell} \tag{3}$$

with k, $\ell = 1,2,3,4,5$. This invariant operator is taken as an interval operator $\hat{s}^2$. As $a \to 0$ it changes to usual operator of interval in the flat space $\mathfrak{M}_4(p)$: $\hat{s}^2 = -\sum_1^4 \frac{\partial^2}{\partial p_\mu^2}$.

It turns out that any possible eigenvalue of $\hat{s}^2$ is compatible with a certain value of a 4-vector N directed along the interval s .

The "unit" vector $N(N_1, N_2, N_3, N_4)$ is subjected to the usual condition: $N^2 = -1$, or $+1$ for time-like and space-like interval, respectively. The value $N^2 = 0$ (light cone) is excluded.

The eigenvectors $\Phi(p)$ of $\hat{s}^2$ are different for $N^2 = -1$ and, $N^2 = +1$:

$$\Phi(p) = \langle L, N | p \rangle, \text{ or } \Phi(p) = \langle \Lambda, N | p \rangle, \tag{4}$$

where L and Λ spesify two series of eigenvalues of operator $\hat{s}^2$ *

$$s^2 = -L(L+3), \quad L = -1, 0, 1, 2, 3, \ldots, N^2 = -1; \tag{5}$$

$$s^2 = (\tfrac{3}{2})^2 + \Lambda(\Lambda+1), \quad 0 < \Lambda < \infty, \quad N^2 = +1. \tag{5'}$$

Therefore, points of the space $\mathfrak{M}_4(x)$ can be defined by four numbers, s^2 and N , and by field Φ which may be called the geometrical field.

As the eigenfunction of interval (4) depends on vector N , the intervals for different N possess different eigenfunctions and hence are incompatible with each other (i.e., belong to different complete sets). Therefore, each point of that space can be crossed only by one (though arbitrary) straight line N with a discrete or continuous measure of lenght, (5) or (5').

Example III. Consider now in detail another possibility indicated in refs. /4,5/.

Unlike the previous variant (example II) where the flat space $\mathfrak{M}_4(p)$ was replaced by the curved space $S_4(p)$, this possibility is based on the quantum generalization of the Finsler space, $F_4(x)$ /6/.

* For the explicit form of functions (4), see ref. /3/. There exist also another, "spherical" complete set.

2. GENERALIZATION OF THE FINSLER SPACE

In contrast to the Riemannian space, the Finsler space is anisotropic. In this geometry, the element of length (interval) ds is a first-order form of the coordinate differentials $dx(dx_1, dx_2, dx_3, dx_4)$:

$$ds = L(dx, x) \tag{6}$$

and depends on the direction of dx. The Minkowski four-dimensional space is a particular case of the homogeneous Finsler space because in the $\mathfrak{M}_4(x)$ the space-like and time-like directions are distinguished. Indeed, the length element in ds can be represented in the form characteristic of the Finsler geometry

$$ds = N_\mu dx^\mu \tag{7}$$

where the vector N_μ is a zero-order form in dx. This form is different for space-like, time-like directions and light cone, having three possible values, $N^2 = \pm 1, 0$.

The quantum generalization of the Finsler space consists in the change of coordinate differentials dx^μ in (7) by the finite operators

$$\Delta\hat{x}_\mu = a\gamma^\mu \tag{8}$$

with γ^μ being the Dirac matrices and a a certain length.

Then the forms (7) and (8) produce the operator of interval as follows:

$$\Delta\hat{s} = N_\mu \Delta\hat{x}^\mu \tag{9}$$

for $N^2 = 1$, and

$$\Delta\hat{s} = \frac{1}{i} N_\mu \Delta\hat{x}^\mu \tag{9'}$$

for $N^2 = -1$ and $N^2 = 0$*. From (8) it follows that

$$[\Delta\hat{x}_\mu, \Delta\hat{x}_\nu] = 2ia^2 \hat{I}_{\mu\nu}, \tag{10}$$

where $\hat{I}_{\mu\nu}$ is the four-dimensional spin operator. This commutator is adequate to the commutator (2). The space determined by formulae (8), (9), (9′) will be called $\Gamma_4(\hat{x})$ -space.

According to (10), the eigenvalues of operators $\Delta\hat{x}_1$, $\Delta\hat{x}_2$, $\Delta\hat{x}_3$, $\Delta\hat{x}_4$ do not form the complete set. Again, this set can be built out of the eigenvalues of the interval $\Delta\hat{s}$ and unit vector N. By solving the equation for eigenfunctions Φ_λ and eigenvalues of operator $\hat{\sigma}(N) = \frac{1}{a}\Delta\hat{s}(N)$:

$$\hat{\sigma}(N) = \Phi_\lambda = \lambda\Phi_\lambda \tag{11}$$

it is not difficult to find the eigenvalue λ

$$\lambda = \pm\sqrt{N^2} \qquad \text{for} \quad N^2 > 0, \tag{12}$$

$$\lambda = \pm\sqrt{-N^2} \qquad \text{for} \quad N^2 < 0. \tag{12'}$$

Therefore, the eigenvalues $\Delta s = \pm a$, or 0. As to the geometrical field Φ, it appears that it cannot be interpreted as a probability since for the tachyon states (12) the invariant $\bar{\Phi}_\lambda \Phi_\lambda = 0$.

As follows from (9) and (10) the interval operators $\Delta s(N')$ and $\Delta s(N'')$ for two nonparallel directions N' and N'' do not commute:

$$[\Delta\hat{s}(N'), \Delta\hat{s}(N'')] = a^2 \gamma^\mu \gamma^\nu (N' \times N'')_{\mu\nu} \tag{13}$$

* This definition of interval differs from that given in ref. /5/ but coincidences with the earlier definition of ref. /4/.

(symbol × represents the vector product). Hence, each point of the quantized space $\Gamma_4(\hat{x})$ can be crossed only by one (though arbitrary) straight line.

Next, we would like to comment on the choice of sign for the interval. Since $\lambda = \pm 1$, we meet with an ambiguity of the same type as in the Minkowski geometry in which $ds = \pm\sqrt{\sum_1^4 dx_i^2}$. We will choose that sign in accordance with the concept of time τ and distance ℓ. For the time-like interval $\hat{s} = \hat{\tau}$, $N^2 = -1$, at each point, the rule

$$\lambda = \pm 1, \quad \Phi_\lambda \equiv \Phi_\pm(\pm N) \tag{14}$$

gives two values of τ, i.e., $\tau = \pm a$, whereas for the space-like interval $\hat{s} = \hat{\ell}$, $N^2 = +1$:

$$\lambda = +1, \quad \Phi_\lambda = \Phi_+(\pm N) \tag{14'}$$

only one sign is admitted, i.e., $\ell = a$.

With this choice, at each point in the space-like direction there can be only one ray (N), while in the time-like direction two rays (± N). Thereby the ordering of events is determined in the space $\Gamma_4(\hat{x})$. It is realized in the same way as in the Minkowski space with the help of interval s and unit vector N. The important difference is that only one line (for $N^2 = -1$) and only one ray (for $N^2 = +1$) are admitted at each point. The eigenvalue of interval for $N^2 = -1$ coincides with time τ in the reference frame, where $N = (1, 0, 0, 0)$, and for $N^2 = +1$ with length ℓ in the frame where $N = (0, \vec{N})$. As to the interval $\Delta s = 0$, $N^2 = 0$, it defines neither length nor time because at $\Delta s = 0$ operators $\hat{x}_4$ and $\hat{x}_k$ ($k = 1,2,3$) do not commute with $\Delta\hat{s}$ in any reference frame. Therefore the seat of points separated by the light cone $N^2 = 0$ is undetermined.

In the vicinity of every point A of space $\Gamma_4(\hat{x})$, one can indicate a neighbouring point B, for $N^2 = +1$, defined by the eigenvector $\Phi_\lambda(A, N)$ of operator $\Delta\hat{s}(N)$ given at point A. This definite geometrical state can be achieved through two equivalent methods due to the chiral degeneracy of field Φ_λ. From point A one cannot draw two or more lines connecting A with B, C,... (Fig. 1a). To any point A, in its vicinity, one can extend arbitrary number of lines from outside, e.g., from neighbouring points C,C′,C″,... (Fig. 1b).

However, distances between these points, CC′, C′C″, etc., will be indefinite. A vicinity of point A of this type is nonordered. In the vicinity of any point B, neighbouring with A, one can indicate a new point C defined by the eigenvector $\Phi_B(\lambda, N')$ of operator $\Delta\hat{s}(N')$ at point B, and so on. In this way, there arises the curve ABCD..., consisting of space-like or time-like intervals whose length is multiple to Figs. 1c and 1d.

The ordered vicinity of any point A consists, at the most, of three points (Fig. 1d).

This maximum ordering is possible only when the contour ABC can be closed, however, it is not always the case as intervals in $\Gamma_4(\hat{x})$-geometry are discrete. The fourth point can no longer possess definite positron with respect to all the three ordered points, because this would give rise to two conflicting eigenfunctions Φ_λ. For instance, definite values of interval BC, by relation (13). do not allow definite values for BD (cf. Fig. 1c). For the same reason, lines cannot intersect in this space.

Thus, in $\Gamma_4(\hat{x})$-geometry only two or three points can be ordered relative each other.

The space $\Gamma_4(\hat{x})$ can be embedded into the space $\mathfrak{M}_4(x)$ like any lattice is only a part of continuum.

It looks like, relative coordinates of points in $\mathfrak{M}_4(x)$ are subjected to some constraints.

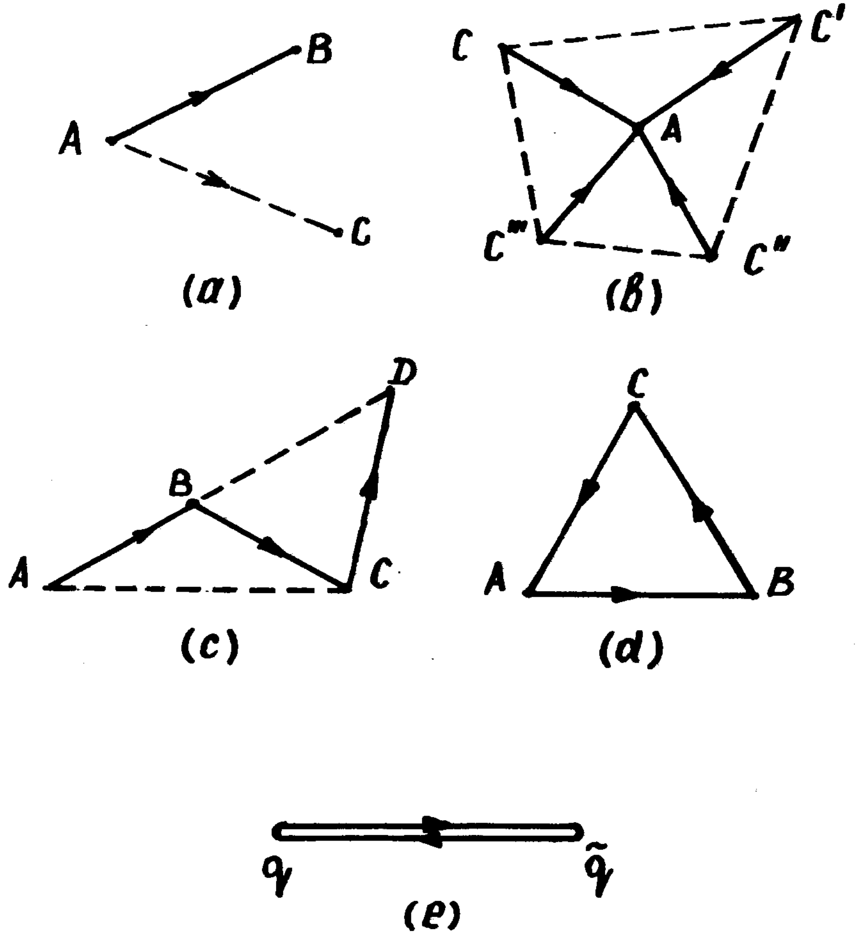

Fig. 1. Vicinity of points in the space $\Gamma_4(\hat{x})$. Solid lines represent definite intervals, dashed ones, indefinite intervals.

3. KINEMATICS OF QUARKS

The kinematics of quarks we consider below is based on the following idea: the gluon field $\mathcal{G}$ which couples quarks q (or antiquarks $\tilde{q}$) is defined in the space $\Gamma_4(\hat{x})$, i.e.,

$$\mathcal{G} = \mathcal{G}(\hat{x}) \equiv \mathcal{G}(N, \hat{\sigma}(N)).$$

From the properties of $\Gamma_4(\hat{x})$ -geometry we may conclude that each quark has only one, directed to a neighbour line N along which the one-dimensional interaction can propagate. In other words, quarks possess the directed valence, and what is more, they are monovalent.

Space-like configurations which are completely ordered and, consequently, possess definite interactions between quarks, consist of quark pairs or of triads only.

It is just this conclusion that follows from the above assumption on the gluon field.

Our second conclusion is due to the discreteness of space $\Gamma_4(\hat{x})$. Because of this property, the space-like intervals cannot be smaller than the element of length a. Therefore, quarks cannot be at the same point that removes the well-known difficulty of quark statistics: different positions of quarks can be treated as their different colour. In other words, the difference in colour of quarks can be identified with difference in their position.

A reasonable picture of quark interactions follows from the merger of the idea of directed one-dimensional valences with the assumption that quark structures are formed only by closed valences.

For the quark traids, this assumption follows straightforward from the geometrical properties of the point vicinity in the quantized space.

As to the pair of quark-antiquark, $(q, \tilde{q})$ it is of the form shown in Fig. 1e, according to that assumption (the saturation of valences). As fields Φ_q and $\Phi_{\tilde{q}}$ have two signs of chirality, the total number of methods for possible ordering of the pair

$(q,\tilde{q})$ equals four. Thereby, the ratio: $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ is quadrupled, which under the threshold of production of charmed quarks is equal to 8/3.

Other structures with a larger amount of quarks cannot be ordered with respect to all pair interactions.

The simplest motion in the quark triad is defined by the constancy of eigenvectors $\Phi_\lambda(A)$, $\Phi_\lambda(B)$, $\Phi_\lambda(C)$. Under this condition the triangle stays similar to itself. Note that the smallest triangle have sides equal to a.

The one-dimensional character of interaction admits the linear growth of the binding energy with increasing distance between quarks $L = \ell a$. This grows takes place, probably, to a certain limit. Indeed, if the binding energy of quarks reaches the value $2mc^2$ (m is the quark mass), the states with $\pm m$ cannot be separated at all (the Klein paradox). The simplest interpretation of this difficulty is to suppose the bond breaking at a distance $L \cong \frac{2mc^2}{gB}$, where g is the interaction constant and B is the strength of the gluon field. And just in that breaking the pair $(q,\tilde{q})$ is created.

In conclusion note that an analogous theory may be developed on the basis of the curved space $S_4(p)$ with discrete space-like intervals.

The author thanks Drs. B.Barbashov, A.Efremov and R.Mir-Kasimov for useful discussions.

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Received by Publishing Department
on January 31, 1978.