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Об импульсном распределении нуклонов в ядрах

Строится модель "когерентных флуктуаций" ядерной плотности (альтернативная к теории Бракнера) с целью теоретически строгого объяснения в импульсном распределении нуклонов существования больших компонент, ответственных за рождение высокоэнергетических частиц в инклюзивных реакциях. Полученная в рамках этой модели формула для импульсного распределения не содержит свободных параметров, и компоненты с большими импульсами возникают естественным образом.

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On the Nucleon Momentum Distribution in Nuclei

The nucleon momentum distribution in nuclei has been obtained within the framework of a "coherent flucton" model. The present theoretical calculations agree well with experimental data on particle backward production in inclusive reactions.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. The production of particles in inclusive reactions povides an interesting possibility of studying the nucleon momentum in atomic nuclei. The authors of Refs. [1,2] consider the production of particles $(N, \pi \ldots)$ in scattering processes of fast particles on nuclei in the forbidden energy range for the corresponding elementary events such as $(p + N \to \pi + X)$. These inclusive reactions are pictured as a process of interaction of the projectile particle with an ensemble of nucleons of the target. The kinetic energy of the nucleons in such a correlated formation (the so-called flucton [1-3]) is several times larger than the average value of the nucleon kinetic energy in the nucleus. In such a way in nucleon momentum distribution there appear high momentum components that are responsible for the particle inclusive production. In Refs. [4-6] the dependence of the production cross section on the energy of the detected particles is qualitatively described on the basis of a phenomenological single-particle momentum distribution. In spite of the success of the above approaches there still remains the problem of the strict theoretical explanation of the existence of high components in the momentum distribution.

In this paper we propose a model of the coherent nuclear density fluctuations, the former being an alternative to Brueckner theory. One can determine the single-nucleon momentum distribution, which agrees with the results of the phenomenological analysis.

2. Nuclear matter theory studies the "asymptotic" state of a system with mass number $A \to \infty$ in the volume $\Omega_x \equiv \frac{4}{3}\pi x^3 \to \infty$. The nuclear density $\rho(x) = \frac{A}{\Omega_x} = \frac{3A}{4\pi x^3}$ is finite within a sphere of radius x. The energy of such a homogeneous "piece" of nuclear matter is:

$$\mathcal{E}(x) = A\mathcal{E}_0(\rho(x)), \qquad (1)$$

where $\mathcal{E}_0(\rho(x))$ is the energy per nucleon. The latter can be expanded in powers of the Fermi-momentum, corresponding to density $\rho(x)$ /7/.

Let us assume that the properties of finite Fermi systems depend only on the energy density and the local nuclear density distribution $\rho(r)$. This assumption has alredy been used in statistical theory of nuclei /7/. At finite values of x (that holds the nucleon number A fixed at a given value of the density) the quantity has the meaning of the energy of an abstract nuclear object, which should be treated as a virtual state of the real nucleus. This virtual state will be referred to as a coherent flucton in what follows. The calculation of the energy of the system requires the determination of the probability for existence of this object (or the wave function) and the kinetic energy connected with the collective momentum, conjugate to the variable x. Keeping in mind the definition of the density:

$$\rho(r) = \langle f(x) | \hat{\rho}_x(r) | f(x) \rangle \qquad (2)$$

one can obtain the flucton wave function $f(x)$.

The density distribution is obviously determined by:

$$\hat{\rho}_x(r) = \frac{3A}{4\pi x^3}\hat{\theta}(x-r), \qquad (3)$$

where $\hat{\theta}(x-r)$ is the well-known θ-function

$$\hat{\theta}(x-r) = \begin{cases} 1, & r < x \\ 0, & r > x \end{cases}.$$

Equations (2) and (3) define the wave function

$$f(x) = \left(-\frac{4\pi x^3}{3A}\frac{d\rho(r)}{dr}\Big|_{r=x}\right)^{1/2}. \qquad (4)$$

The ground state energy of the nucleus is determined by the following density functional:

$$E = \langle f(x) | \hat{T}_x + \mathcal{E}(x) | f(x) \rangle. \qquad (5)$$

The collective kinetic energy operator $\hat{T}_x$ that corresponds to the coherent motion of the nucleus is written as:

$$\hat{T}_x = -\frac{h^2}{2m_{ef}(x)}\frac{d^2}{dx^2}, \qquad (6)$$

where m_{ef} is the effective mass depending on the mass number A. In general, m_{ef} is a function of the collective variable x. Eq. (5) corresponds to a dynamic differential equation of the Schrödinger-type, describing zero "breathing" vibrations of the system:

$$\left[-\frac{h^2}{2m_{ef}}\frac{d^2}{dx^2} + \mathcal{E}(x)\right] f(x) = Ef(x). \qquad (7)$$

The wave function $f(x)$ is normalized by:

$$\int_0^\infty |f(x)|^2 dx = 1. \qquad (8)$$

The numerical analysis of eq. (5) has shown that m_{ef} is essentially independent of x and is also a simple function of the mass number A :

$$m_{ef} \approx \frac{4M_N}{A} \quad . \qquad (9)$$

That dependence of the effective mass on A is most probably due to the adding contributions of single nucleon kinetic energies to the collective kinetic energy.

Thus we are sure that there has been proposed a new nuclear model based upon the properties of the asymptotic state of nuclear matter and the realistic nuclear density distribution.

3. The offered theory must be applied to the description of those phenomena and properties of nuclei which are essentially dependent on the general geometric and dynamic characteristics of nuclei. In this paper we consider only the determination of the nucleon momentum distribution in nuclei.

The normalized to A distribution of the particles with respect to their momentum k inside a given coherent flucton must be determined by:

$$n_x(k) = n_0(x)\,\Theta(k^F(x) - k), \qquad (10)$$

where $n_0(x)$ is the volume of a flucton of radius x, and $k^F(x)$ is the flucton Fermi-momentum

$$k^F(x) = \left(\frac{3\pi^2}{2}\rho(x)\right)^{1/3} = \left(\frac{9}{8}\pi A\right)^{1/3} \cdot \frac{1}{x}, \qquad (11)$$

$$\rho(x) = 3A/4\pi x^3 \quad .$$

The resulting distribution n(k) of the nuclear ground state is:

$$n(k) = \int_0^\infty |f(x)|^2 n_x(k)\,dx. \qquad (12)$$

It is obvious that

$$4\int n(k)\frac{d^3k}{(2\pi)^3} = A$$

n(k) can be expressed as a function of the density $\rho(r)$ from eq. (4):

$$n(k) = \left(\frac{4\pi}{3}\right)^2 \frac{1}{A}\rho\left(\frac{a}{k}\right)\left(\frac{a}{k}\right)^6 \left\{ -1 + 6\int_0^\infty \frac{dy}{(1+y)^7} \frac{\rho\left(\frac{a}{k(1+y)}\right)}{\rho\left(\frac{a}{k}\right)} \right\}, \qquad (13)$$

where $a = 1{,}52\,A^{1/3}$, and y is a dimensionless integration variable. For the case of the Fermi-density distribution

$$\rho(r) = \frac{\rho_0(R,b)}{1 + \exp\left(\frac{r-R}{b}\right)},$$

where the R and b parameters determine the nuclear radius and surface thickness, eq. (13) leads to a power asymptotics of the momentum distribution n(k), when $k \to \infty$

$$n(k) \sim \exp\left(-\frac{R}{b}\right)\left(\frac{4\pi}{3}\right)^2 \frac{1}{A}\left(\frac{a}{k}\right)^6, \qquad (14)$$

when $k \to 0$

$$n(k) \sim \left(\frac{4\pi}{3}\right)^2 \frac{1}{A}\left\{ -\left(\frac{a}{k}\right)^6 \rho\left(\frac{a}{k}\right) + \rho_0(R,b)6b^6\gamma\left(6, \frac{a}{kb}\right)\right\}. \qquad (15)$$

$\gamma(a,z)$ in the above expression is the non-complete gamma-function.

$$\gamma(a,z) = \int_0^z e^{-t}\, t^{a-1}\, dt \, .$$

According to eq. (15) $n(k)$ is finite at $k = 0$. The momentum distributions $n(k)$ for ^{12}C and ^{181}Ta are calculated according to eq. (12) and are given in Figs. 1, 2. In the numerical calculations $n(k)$ has

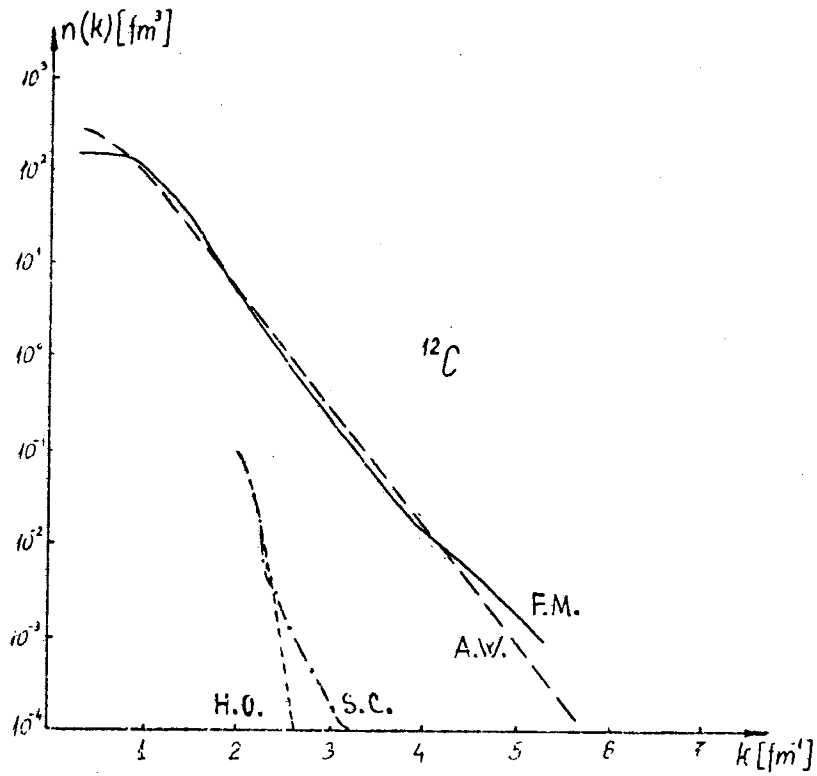

Fig. 1. Nuclear single-nucleon momentum distributions for ^{12}C. A.W. is the theoretical parametrization of Amado and Woloshyn, Ref [4]; F.M. is the result of the present calculation; H.O. is the nonrelativistic harmonic oscillator result with oscillator parameter equal to 1.65 fm, Ref. [6]; S.C. is the theoretical result from the Dirac equation for a self-consistent model, Refs. [6,9].

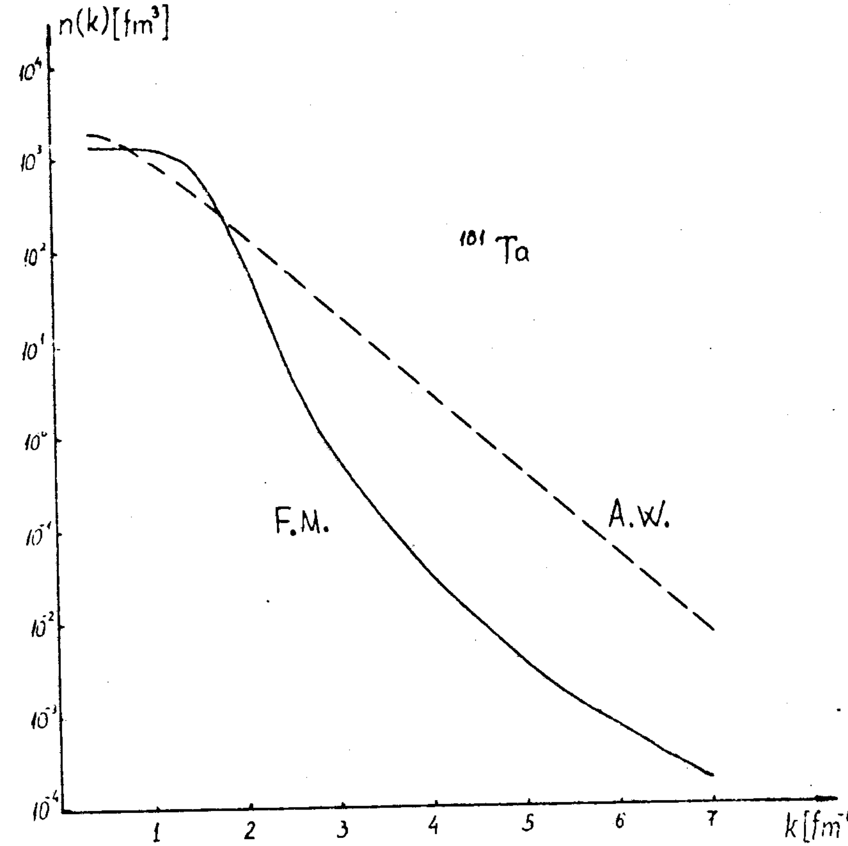

Fig. 2. Nuclear single-nucleon momentum distributions for ^{181}Ta. F.M. is the result of the present calculation; A.W. is the parametrization of Amado and Woloshyn, Ref. [4].

been normalized by the following condition:

$$2 \int \frac{d^3k}{(2\pi)^3} \frac{M_N}{E(k)} n(k) = A \qquad (16)$$

which takes into account the relativistic correction to the phase space volume. M_N is the nucleon mass and $E(k)=(k^2+M_N)^{1/2}$. Also given are the respective phenomenological distributions [4], that are successfully used in the analysis of the experimental data on inclusive reaction [8].

In Fig. 1 for the case of ^{12}C we also give the curves taken from Ref. [6] curves, which correspond to the distributions given by conventional models. It becomes clear that values of $n(k)$ obtained in the harmonic oscillator model for momentum $k > 2\,fm^{-1}$ are a few orders of magnitude less than the results of the present flucton model. The distribution $n(k)$ for ^{12}C corresponds to an average nucleon kinetic energy determined by the distribution integral

$$\frac{1}{A}\frac{1}{\pi^2}\int_0^\infty k^2 dk \frac{M_N}{E(k)}[E(k)-M_N]\, n(k)$$

and is equal to 33.6 MeV.

There is a certain disagreement between the present calculations and the results of Ref. [4]. It should be noted, however, that the experimental cross section lies about an order of magnitude lower than the approximation of Amado and Woloshyn.

We would like to point out, in conclusion, that the obtained formula about the momentum distibution $n(k)$ does not contain any free parameters, in case that $f(x)$ is related to the local density distribution $\rho(r)$, which is determined by electron scattering experiments.

The necessary high momentum components of the distribution $n(k)$ appear quite naturally within the framework of the present flucton model in contrast to other approaches.

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