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SUPERCONFORMAL INVARIANT QFT IN TWO-DIMENSIONAL SPACE-TIME

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Суперконформно-инвариантная КТП в двухмерном пространстве-времени

Даны представления суперконформной группы в двухмерном пространстве-времени. Найдены инвариантные двух- и трехточечные функции. Получен локальный свободный лагранжиан, из которого следуют уравнения для струны со спином.

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Superconformal Invariant QFT in Two-Dimensional Space-Time

The representations of the superconformal group in two-dimensional space-time are given. The invariant two- and three-point functions are constructed and is obtained the free local Lagrangian, from which the equations of a spinning string are implicit.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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The superconformal transformations in two-dimensional space were introduced in the dual string model [1]. The corresponding transformations in 4-dimensional Minkowsky space were considered in paper [2]. The representations of the last group and corresponding invariant two- and three-point functions for the fields transformed by some of these representations were found in papers [3-5].

In the present paper we consider the representations of the superconformal group in two-dimensional space-time. The corresponding Lie algebra is more simple than the Lie algebra of superconformal group in 4-dimensional space-time. For example, it is closed with γ_5 transformations excluded. Consequently the representations of this group are labeled only by one dimension - the scale one. In view of the dual string models [6,7] it is interesting to consider the QFT invariant with respect to $SO_s(2,2)$ transformations (superconformal group in two-dimensional space-time). As in the case of conventional conformal invariant QFT the two- and three-point functions are determined from the invariance condition up to the normalization constants. But there exist some more strong restrictions. The two-point function for scalar superfields is nonvanishing for any scale dimensions. For tensor fields it is not the case - the corresponding two-point function is nonvanishing only for coinciding points and consequently the scale dimension is fixed. Such a theory can describe the "quarks" which do not propagate. In the case of OPE we will have only one parameter expansion.

In section 4 the invariant action is considered from which free equations are obtained of a spinning string [6,7]. It is found that in the framework of superconformal invariant QFT there are no self-interactions.

2. TWO-DIMENSIONAL SUPERCONFORMAL ALGEBRA

Consider the superconformal algebra in two-dimensional space-time given by:

$$[M_{\mu\nu}, S_\alpha] = (\sigma_{\mu\nu})_\alpha^{\ \beta} S_\beta\,; \quad [M_{\mu\nu}, T_\alpha] = (\sigma_{\mu\nu})_\alpha^{\ \beta} T_\beta\,,$$

$$[P_\mu, S_\alpha] = 0, \qquad [P_\mu, T_\alpha] = i(\gamma_\mu)_\alpha^{\ \beta} S_\beta\,,$$

$$[D, S_\alpha] = -\frac{i}{2} S_\alpha\,, \qquad [D, T_\alpha] = \frac{i}{2} T_\alpha\,,$$

$$[K_\mu, S_\alpha] = i(\gamma_\mu)_\alpha^{\ \beta} T_\beta\,, \quad [K_\mu, T_\alpha] = 0,$$

$$\{S_\alpha, S_\beta\} = -2(\gamma^\mu C)_{\alpha\beta} P_\mu\,,$$

$$\{T_\alpha, T_\beta\} = -2(\gamma^\mu C)_{\alpha\beta} K_\mu\,,$$

$$\{S_\alpha, T_\beta\} = -2i((\sigma^{\mu\nu} C)_{\alpha\beta} M_{\mu\nu} + iC_{\alpha\beta} D) \qquad (2.1)$$

where

$$\sigma_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu]\,, \quad \gamma_5 = \gamma_0 \gamma_1$$

and $C = -C^T = \gamma_1$ is the charge conjugation operator. The CR between $M_{\mu\nu}$, P_μ, D and K_μ are given elsewhere [8].

Consider the scalar superfield

$$\Psi(x;\Theta) = A(x) + \Psi^\alpha(x)\Theta + \frac{1}{2}\Theta C\Theta B(x) \qquad (\alpha = 1,2) \qquad (2.2)$$

which transforms by some (in generally, reducible) representations of superconformal algebra. Here Θ is an anticommuting Majorana spinor, $A(x)$ and $B(x)$ are scalar fields and $\Psi(x)$ is a spinor field. For tensor supermultiplet $A(x)$. $B(x)$ and $\Psi(x)$ are tensors of a corresponding rank.

The field $\Psi(x;\Theta)$ has the following transformation properties with respect to the infinitesimal superconformal transformations

$$[M_{\mu\nu}, \Psi(x;\Theta)] = \{i(x_\mu\partial_\nu - x_\nu\partial_\mu) + \Theta\sigma_{\mu\nu}\frac{\partial}{\partial\theta} + \Sigma_{\mu\nu}\}\Psi(x,\Theta), \qquad (2.3)$$

$$[P_\mu, \Psi(x;\Theta)] = i\partial_\mu \Psi(x;\Theta),$$

$$[D, \Psi(x;\Theta)] = \{-i(x^\nu\partial_\nu + \frac{1}{2}\Theta\frac{\partial}{\partial\Theta}) + \Delta\}\Psi(x;\Theta),$$

$$[K_\mu, \Psi(x;\Theta)] = \{i[2x_\mu x^\nu\partial_\nu - x^2\partial_\mu - 2ix^\nu(g_{\mu\nu}\Delta + \Sigma_{\mu\nu})] + i\Theta\gamma_\mu\hat{x}\frac{\partial}{\partial\Theta} + k_\mu\}\Psi(x;\Theta),$$

$$[S_\alpha, \Psi(x;\Theta)] = [\frac{\partial}{\partial\Theta\alpha} - (\Theta\hat{P}C)_\alpha]\Psi(x;\Theta).$$

$$[T_\alpha, \Psi(x;\Theta)] = \{2i(\Theta\sigma^{\mu\nu}C)_\alpha M_{\mu\nu} + 2i\Theta_\alpha D -$$

$$- i(\Theta\sigma^{\mu\nu}C)_\alpha \Theta\sigma_{\mu\nu}\frac{\partial}{\partial\Theta} + \frac{1}{2}\Theta_\alpha\Theta\frac{\partial}{\partial\Theta} + t_\alpha\} \times$$

$$\times \Psi(x;\Theta),$$

where $\Sigma_{\mu\nu}$, Δ, k_μ and t_α are generators of stability subgroup, i.e., the subgroup which leaves $x=\Theta=0$. We restrict ourselves to the case $k_\mu = t_\alpha = 0$. For scalar supermultiplet (2,2) we have $\Sigma_{\mu\nu} = 0$ and $\Delta = -i\alpha$.

3. INVARIANT TWO AND THREE-POINT FUNCTIONS

a) Two-Point Function

The invariant (in infinitesimal form) two-point function for tensor multiplet is given as a solution of the following system of Eqs.

$$(X_1 + X_2)F(x_1,\Theta_1,\xi_1; x_2,\Theta_2,\xi_2) = 0, \qquad (3.1)$$

where X_a ($a = 1,2$) are the generators of the superconformal group acting on the field Ψ_a according to (2.3). The two-component vector variables ξ replace the tensor indices /9/.

To solve eq. (3.9) we use the method given in paper /9/. It is convenient to consider first the following Eqs.

$$(D_1 + D_2)F = 0,$$
$$(K^1_\mu + K^2_\mu)F = 0. \qquad (3.2)$$

The relativistic invariant solution of (3.2) has the following form

$$F = (x^2_{12})^{-d}(2\xi_1\xi_2 - \frac{(\xi_1 x_{12})(\xi_2 x_{12})}{x^2_{12}})^n f(\frac{\Theta_1 C\hat{x}_{12}\Theta_2}{x^2_{12}}) \qquad (3.3)$$

where $x_{12} = x_1 - x_2$, $d = d_1 = d_2$, $n = n_1 = n_2$ is the rank of tensor superfield and f is an arbitrary function. This function can be determined from the following eq.

$$(S^1_\alpha + S^2_\alpha)F = 0. \qquad (3.4)$$

The nonvanishing solution of this equation with an arbitrary scale dimension exists only if $n = 0$. In this case it is given by

$$f = N_d \exp(2id\frac{\Theta_1 C\hat{x}_{12}\Theta_2}{x^2_{12}}), \qquad (3.5)$$

where N_d is a normalization constant. Substituting (3.5) into (3.3) we have

$$F = N_d(x^2_{12})^{-d}\exp(2id\frac{\Theta_1 C\hat{x}_{12}\Theta_2}{x^2_{12}}). \qquad (3.6)$$

It can be checked that the equation

$$(T^1_\alpha + T^2_\alpha)F = 0$$

is also satisfied. This follows from the C.R. (2.1).

Taking into account the following identity

$$(-\Theta_1 C\hat{\partial}\Theta_2)^k(x^2)^{-d} = (2d(\frac{\Theta_1 C\hat{x}\Theta_2}{x^2})^k(x^2)^{-d}, \ (k=1,2) \qquad (3.7)$$

eq. (3.6) can be written down in the following, more convenient from

$$F = N_d\exp(-i\Theta_1 C\hat{\partial}\Theta_2)(x^2_{12})^{-d}. \qquad (3.8)$$

Equations (3.1), (3.2) and (3.4) have also the second solution given by

$$F = N\delta(x_1 - x_2)\delta(\Theta_1 - \Theta_1)(\xi_1\xi_2)^n \qquad (3.9)$$

if $d_1 + d_2 = 1 - n$.

b) Three-Point Function

Taking into account the results for the two-point function, we analyse only the scalar superfield. The invariant three-point function satisfies the following system of

$$(X_1 + X_2 + X_3) G(x_j, \Theta_j) = 0, \quad (j = 1, 2, 3). \qquad (3.10)$$

The relativistic invariant solutions of eqs.

$$(D_1 + D_2 + D_3) G = 0,$$
$$(K^1_\mu + K^2_\mu + K^3_\mu) G = 0, \qquad (3.11)$$

have the following form

$$G = (x^2_{12})^{\frac{d_3 - d_1 - d_2}{2}} (x^2_{23})^{\frac{d_1 - d_2 - d_3}{2}} (x^2_{13})^{\frac{d_2 - d_1 - d_3}{2}}$$
$$\times g\left(\frac{\Theta_1 C \hat{x}_{12} \Theta_2}{x^2_{12}}, \frac{\Theta_1 C \hat{x}_{13} \Theta_3}{x^2_{13}}, \frac{\Theta_2 C \hat{x}_{23} \Theta_3}{x^2_{23}}\right). \qquad (3.12)$$

Here g is an arbitrary function of

$$\lambda_{jk} = -\lambda_{kj} = \frac{\Theta_j C \hat{x}_{jk} \Theta_k}{\lambda^2_{jk}} \qquad (j, k = 1, 2, 3).$$

The function g can be determined from the following eq.

$$(S^1_\alpha + S^2_\alpha + S^3_\alpha) G = 0. \qquad (3.13)$$

The solution of (3.13) is given by

$$g = \kappa \exp\{ i (d_1 + d_2 - d_3) \lambda_{12} + i (d_1 + d_3 - d_2) \lambda_{13} +$$
$$+ i (d_1 + d_3 - d_2) \lambda_{23} \}, \qquad (3.14)$$

where κ is an arbitrary constant. The invariance with respect to generators T_α follows from the CR (2.1).

In the case when

$$d_1 + d_2 + d_3 = 2$$

there exists the second solution of (3.10) in the following form

$$G' = \kappa_1 \delta(x_{12}) \delta(x_{23}) \delta(\Theta_1 - \Theta_2) \delta(\Theta_2 - \Theta_3). \qquad (3.15)$$

The four-point function and any other higher functions cannot be determined without dynamical assumptions, as in the case of conformal invariant field theory [8].

4. SUPERCONFORMAL INVARIANT FIELD EQUATIONS

Consider the following (in general, nonlocal) action

$$A = \int d^2x_1 d^2x_2 d^2\Theta_1 d^2\Theta_2 \Psi(x_1, \Theta_1) \times$$
$$\times S^{-1}(x_1, \Theta_1; x_2, \Theta_2) \Psi(x_2, \Theta_2), \qquad (4.1)$$

where S^{-1} is the inverse operator which can be determined from equation.

$$\int d^2y \, d^2\theta \, S(x_1, \Theta_1; y, \theta) S^{-1}(y, \theta; x_2, \Theta_2) = \delta(x_1 - x_2) \delta(\Theta_1 - \Theta_2). \qquad (4.2)$$

From (3.8) and (4.2) it follows that

$$S^{-1} \approx \exp\{-i \Theta_1 C \hat{\partial} \Theta_2\} (x^2_{12} - i\epsilon)^{-\tilde{d}}, \qquad (4.3)$$

where $\tilde{d} = 1 - d$. It can be proved that the fields with scale dimensions d and $\tilde{d} = 1 - d$ are transformed according to the equivalent representations of superconformal group.

In the case when $d = 0$, from (4.3) it follows that (4.1) is local. Then we have

$$A = \int d^2x\, d^2\Theta_1\, d^2\Theta_2 \Psi(x,\Theta_1) e^{-i\Theta_1 c\hat{\partial}\Theta_2} \Psi(x;\Theta_2). \qquad (4.4)$$

From this action, we derive the following free equation of motion

$$\Box A(x) = 0, \qquad (4.5)$$

$$\gamma^\mu \partial_\mu \Psi(x) = 0,$$

$$B(x) = 0 \ . \qquad (4.6)$$

Equations (4.5) coincide with equations for spinning considered elsewhere [6, 7]. Because the scale dimension of $\Psi(x, \Theta)$ in (4.4) is zero, there exists no local self-interaction term with dimensionless coupling constant.

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