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ON INTERACTION WITH EXTERNAL FIELDS. II

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О взаимодействии с внешними полями. II

Для взаимодействия квантованного заряженного поля с любыми зависящими от времени внешними полями с помощью S-матрицы в N-упорядоченной форме исследовано условие унитарности S-матрицы и характеризуются генерируемые распределения по числу конечных частиц. В этих терминах обсуждается проблема связи спина со статистикой.

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On Interaction with External Fields. II

For interactions of any quantized charged field with arbitrary external classical fields the S-matrix unitarity condition is analysed, using the S-matrix in N-ordered form. Particle number distributions, produced, are characterized. In these terms the problem on connection between spin and statistics is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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I. INTRODUCTION

Theory of interactions of quantized fields with external classical fields is one of important models both from experimental and theoretical point of view /1-23/.

In Sec. 2 we analyse the unitarity of S-matrix, describing an interaction of any charged quantized field with arbitrary external classical fields. The unitarity condition is expressed in terms of its four sectors: one-particle, one-antiparticle, vacuum-pair and pair-vacuum (like the S-matrix itself). The latter are identities for the one-particle propagators. They follow, in fact, from more simple Schwinger relations/5/, which in turn are direct consequences of the equation of motion (e.g., Dirac equation).

In Sec. 3 a generating function is defined for probabilities of transitions into states with given numbers of particles and antiparticles.

In Sec. 4 the problem of connection between spin and statistics, as is stated by Feynman/1,2/, is discussed, using the above identities. Important arguments for this connection were given by Feynman (in the framework of quantum electrodynamics with arbitrary external electromagnetic field) long ago. Recently the problem of connection between spin and statistics in the same theory has been treated, using the unitarity condition, by Nikishov et al./10-12,18/, but they used terms different from the Feynman ones.

2. UNITARITY CONDITION

We are interested in what way quantities C, G (or S^A_+) (for the definition see ref./24/) are combined to give the identity for S^+S. To this end we decompose the product $:e^{C^+}::e^{C}:$ into N-products. A straightforward combinatorial analisys (see Appendix I.B *)) leads us to

$$:e^{C^+}::e^{C}:= e^{\overline{C^+C}+\frac{1}{2}\overline{\overline{C^+C\,C^+C}}+\cdots}$$

$$\cdot :e^{C^+ + C + \overline{C^+C} + \overline{C\,C^+} + \overline{C\,C^+C} + \overline{C^+C\,C^+} + \overline{C^+C\,C^+C} + \overline{C\,C^+C\,C^+} + \cdots}: \qquad (1)$$

To avoid entangling of the lines of pairing we alternate C and C^+, although all C^+'s stand, in fact, to the left of all C's , and this defines correct Dyson pairings.

Formula (1) is general in the sense that it holds for the product of any two N-ordered exponentials with independent C and C^+ bilinear in charged field (i.e., $C = \hat{\bar{\psi}} A \hat{\psi}$, $C^+ = \hat{\bar{\psi}} B \hat{\psi}$, where A and B are any c-number integral kernels). In our case C and C^+ are not independent. For $:e^{C^+}::e^{C}:$ we expect to obtain a constant multiple of the identity. Therefore C and C^+ are connected by the relation

$$C^+ + C + \overline{C^+C} + \overline{C\,C^+} + \overline{C\,C^+C} + \overline{C^+C\,C^+} + \overline{C^+C\,C^+C} + \overline{C\,C^+C\,C^+} + \cdots = 0 . \qquad (2)$$

In order to inspect all infinite series in a closed form we consider the case of the spinor field (for β-field analogously). Then coherent state expectation values of $:e^{-i\hat{\bar{\psi}}\bar{I}\hat{\psi}}::e^{i\hat{\bar{\psi}}I\hat{\psi}}:$ may be represented/5/ as

$$\langle\psi|:e^{-i\hat{\bar{\psi}}\bar{I}\hat{\psi}}::e^{i\hat{\bar{\psi}}I\hat{\psi}}:|\psi\rangle =$$

$$=\langle\psi|e^{-i(\hat{\bar{\psi}}^{(-)}+\bar{\psi}^{(+)})\bar{I}(\hat{\psi}^{(-)}+\psi^{(+)})}\, e^{i(\hat{\bar{\psi}}^{(+)}+\bar{\psi}^{(-)})I(\hat{\psi}^{(+)}+\psi^{(-)})}|\psi\rangle =$$

$$=\langle 0|\, e^{-i(\hat{\bar{\psi}}^{(-)}+\bar{\psi})\bar{I}(\hat{\psi}^{(-)}+\psi)}\, e^{i(\hat{\bar{\psi}}^{(+)}+\bar{\psi})I(\hat{\psi}^{(+)}+\psi)}|0\rangle = \langle 0|F|0\rangle , \qquad (3)$$

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*) We add the Roman numeral I for reference to Sections, Appendices and formula of ref./24/, e.g., Sec. I.3, eq. (I.7.b).

where F is Schwinger's notation for the product of exponentials of the preceding expression. To find $\langle 0|F|0\rangle$ we construct functional derivative equations for it (like Schwinger did). However, in differentiating, we take into account that $\psi(x)$ and $\bar\psi(x)$ in $|\psi\rangle$ satisfy free equations (see /25/, p. 18 for definition $|\psi\rangle$), unlike Schwinger, who has implied $\psi(x)$ and $\bar\psi(x)$ to be arbitrary spinors. Therefore we differentiate with respect to spinors $\bar\eta(x)$ and $\eta(x)$, which are in fact arbitrary (see /25/, p. 18). Then we obtain the equations

$$\frac{\delta}{\delta\bar\eta}\langle 0|F|0\rangle=-i\mathcal{S}\bar{I}\langle 0|(\hat\psi^{(-)}+\psi)F|0\rangle+i\mathcal{S}I\langle 0|F(\hat\psi^{(+)}+\psi)|0\rangle=$$
$$=i\mathcal{S}(I-\bar{I})\psi\langle 0|F|0\rangle-i\mathcal{S}\bar{I}\langle 0|\hat\psi^{(-)}F|0\rangle+i\mathcal{S}I\langle 0|F\hat\psi^{(+)}|0\rangle \tag{4}$$

$$\frac{\delta}{\delta\eta}\langle 0|F|0\rangle=i\langle 0|(\hat{\bar\psi}^{(-)}+\bar\psi)F|0\rangle\bar{I}\mathcal{S}-i\langle 0|F(\hat{\bar\psi}^{(+)}+\bar\psi)|0\rangle I\mathcal{S}=$$
$$=-i\langle 0|F|0\rangle\bar\psi(I-\bar{I})\mathcal{S}+i\langle 0|\hat{\bar\psi}^{(-)}F|0\rangle\bar{I}\mathcal{S}-i\langle 0|F\hat{\bar\psi}^{(+)}|0\rangle I\mathcal{S} \tag{5}$$

with additional "factors" $\mathcal{S}=\{\hat\psi,\hat{\bar\psi}\}$. If one takes into account the Schwingers relations (ref. /5/, eq. (107)), connecting I and $\bar{I}$, then the right-hand sides of eq. (4) and (5) vanish. This immediately leads to $\mathcal{S}^{+}\mathcal{S}=1$ /5/. However, we wish, unlike Schwinger, to obtain $\langle 0|F|0\rangle$ without using the connection of I and $\bar{I}$. To this end we express quantities $\langle 0|\hat\psi^{(-)}F|0\rangle$, $\langle 0|F\hat\psi^{(+)}|0\rangle$, $\langle 0|\hat{\bar\psi}^{(-)}F|0\rangle$ and $\langle 0|F\hat{\bar\psi}^{(+)}|0\rangle$, having entered into eqs. (4) and (5), in terms of $\langle 0|F|0\rangle$. So, for $\langle 0|\hat\psi^{(-)}F|0\rangle$, commuting $\hat\psi^{(-)}$ and F, and then F and $\hat\psi^{(+)}$, we obtain

$$\langle 0|\hat\psi^{(-)}F|0\rangle=i\mathcal{S}^{(-)}I\langle 0|F(\hat\psi^{(+)}+\psi)|0\rangle=$$
$$=i\mathcal{S}^{(-)}I(1+i\mathcal{S}^{(+)}\bar{I})\psi\langle 0|F|0\rangle-\mathcal{S}^{(-)}I\mathcal{S}^{(+)}\bar{I}\langle 0|\hat\psi^{(-)}F|0\rangle. \tag{6}$$

Hence,

$$\langle 0|\hat\psi^{(-)}F|0\rangle=(1+\mathcal{S}^{(-)}I\mathcal{S}^{(+)}\bar{I})^{-1}\,i\mathcal{S}^{(-)}I(1+i\mathcal{S}^{(+)}\bar{I})\psi\langle 0|F|0\rangle \tag{7}$$

$$\langle 0|F\hat\psi^{(+)}|0\rangle=(1+\mathcal{S}^{(+)}\bar{I}\mathcal{S}^{(-)}I)^{-1}\,i\mathcal{S}^{(+)}\bar{I}(1+i\mathcal{S}^{(-)}I)\psi\langle 0|F|0\rangle \tag{8}$$

$$\langle 0|\hat{\bar\psi}^{(-)}F|0\rangle=-i\langle 0|F|0\rangle\bar\psi(1-i\bar{I}\mathcal{S}^{(-)})I\mathcal{S}^{(+)}(1+\bar{I}\mathcal{S}^{(-)}I\mathcal{S}^{(+)})^{-1} \tag{9}$$

$$\langle 0|F\,\hat{\bar{\psi}}^{(+)}|0\rangle = -i\langle 0|F|0\rangle\,\bar{\psi}\left(1-iI\mathcal{S}^{(+)}\right)\bar{I}\mathcal{S}^{(-)}\left(1+I\mathcal{S}^{(+)}\bar{I}\mathcal{S}^{(-)}\right)^{-1} \tag{10}$$

and eqs. (4) and (5) take the form

$$\frac{\delta}{\delta\bar{\eta}}\langle 0|F|0\rangle = i\mathcal{S}K\psi\langle 0|F|0\rangle \tag{11}$$

$$\frac{\delta}{\delta\eta}\langle 0|F|0\rangle = -i\bar{\psi}K\mathcal{S}\langle 0|F|0\rangle\,. \tag{12}$$

There are many possible equivalent representations for integral kernel K (all without using the connection between I and $\bar{I}$):

$$K = I-\bar{I}-i\bar{I}\frac{1}{1+\mathcal{S}^{(-)}I\mathcal{S}^{(+)}\bar{I}}\mathcal{S}^{(-)}I\left(1+i\mathcal{S}^{(+)}\bar{I}\right)+iI\frac{1}{1+\mathcal{S}^{(+)}\bar{I}\mathcal{S}^{(-)}I}\mathcal{S}^{(+)}\bar{I}\left(1+i\mathcal{S}^{(-)}I\right)= \tag{13.a}$$

$$=I-\bar{I}+i\left(1-i\bar{I}\mathcal{S}^{(-)}\right)I\mathcal{S}^{(+)}\frac{1}{1+\bar{I}\mathcal{S}^{(-)}I\mathcal{S}^{(+)}}\bar{I}-i\left(1-iI\mathcal{S}^{(+)}\right)\bar{I}\mathcal{S}^{(-)}\frac{1}{1+I\mathcal{S}^{(+)}\bar{I}\mathcal{S}^{(-)}}I= \tag{13.b}$$

$$=I\frac{1}{1+\mathcal{S}^{(+)}\bar{I}\mathcal{S}^{(-)}I}\left(1+i\mathcal{S}^{(+)}\bar{I}\right)-\bar{I}\frac{1}{1+\mathcal{S}^{(-)}I\mathcal{S}^{(+)}\bar{I}}\left(1+i\mathcal{S}^{(-)}I\right)= \tag{13.c}$$

$$=\left(1-i\bar{I}\mathcal{S}^{(-)}\right)\frac{1}{1+I\mathcal{S}^{(+)}\bar{I}\mathcal{S}^{(-)}}I-\left(1-iI\mathcal{S}^{(+)}\right)\frac{1}{1+\bar{I}\mathcal{S}^{(-)}I\mathcal{S}^{(+)}}\bar{I}= \tag{13.d}$$

$$=I-\bar{I}-i\bar{I}\mathcal{S}^{(-)}I+i\left(1-i\bar{I}\mathcal{S}^{(-)}\right)I\mathcal{S}^{(+)}\frac{1}{1+\bar{I}\mathcal{S}^{(-)}I\mathcal{S}^{(+)}}\bar{I}\left(1+i\mathcal{S}^{(-)}I\right)= \tag{13.e}$$

$$=I-\left(1-iI\mathcal{S}^{(+)}\right)\bar{I}\frac{1}{1+\mathcal{S}^{(-)}I\mathcal{S}^{(+)}\bar{I}}\left(1+i\mathcal{S}^{(-)}I\right)= \tag{13.f}$$

$$=-\bar{I}+\left(1-i\bar{I}\mathcal{S}^{(-)}\right)\frac{1}{1+\mathcal{S}^{(+)}\bar{I}\mathcal{S}^{(-)}I}\left(1+i\mathcal{S}^{(+)}\bar{I}\right)\,. \tag{13.g}$$

Integrating eqs. (11) and (12) , we obtain

$$\langle 0|F|0\rangle = c\;e^{i\bar{\psi}K\psi}\,, \tag{14}$$

where the integration constant is defined by $c=\langle 0|F|0\rangle_{\bar{\eta}=\eta=0}$ and hence is the vacuum expectation value

$$c = \langle 0|:e^{-i\hat{\bar{\psi}}\bar{I}\hat{\psi}}::e^{i\hat{\bar{\psi}}I\hat{\psi}}:|0\rangle = e^{D}\,. \tag{15}$$

According to the theorem : coherent state expectation values of an operator $\langle\psi|\hat{Q}|\psi\rangle = Q(\bar\psi,\psi)$ determine uniquely the N-ordered form of the operator $\hat{Q}$ itself, we obtain finally

$$:e^{-i\hat{\bar\psi}\bar{I}\hat\psi}::e^{i\hat{\bar\psi}I\hat\psi}: = e^{D}:e^{i\hat{\bar\psi}K\hat\psi}: \qquad (16)$$

which is in fact the same as (1).

Again eq. (16) is general, i.e., it is valid for arbitrary I and $\bar{I}$. If we take into account the Schwinger relation between I and $\bar{I}$

$$I-\bar{I} = i\bar{I}(S^{(-)}-S^{(+)})I = iI(S^{(-)}-S^{(+)})\bar{I} \qquad (17)$$

then we can obtain

$$K = 0 \qquad (18)$$

what guarantees unitarity of the S-matrix.

Matrix elements of $:e^{C^+}::e^{C}:$ between states with definite number of quanta reduce to

$$\langle j_1\cdots j_k\bar{j}_1\cdots\bar{j}_\ell|:e^{C^+}::e^{C}:|i_1\cdots i_m\bar{i}_1\cdots\bar{i}_n\rangle =$$

$$= e^{D}\langle j_1\cdots j_k\bar{j}_1\cdots\bar{j}_\ell|:e^{i\hat{\bar\psi}K\hat\psi+\mathcal{N}}e^{-\mathcal{N}}:|i_1\cdots i_m\bar{i}_1\cdots\bar{i}_n\rangle =$$

$$= e^{D}\delta_{k-\ell-m+n}\frac{1}{(k+n)!}\langle j_1\cdots j_k\bar{j}_1\cdots\bar{j}_\ell|:(i\hat{\bar\psi}K\hat\psi+\mathcal{N})^{k+n}:|i_1\cdots i_m\bar{i}_1\cdots\bar{i}_n\rangle =$$

$$= e^{D}\delta_{km}\delta_{\ell n}\frac{1}{(k+n)!}\langle j_1\cdots j_k\bar{j}_1\cdots\bar{j}_\ell|:(i\hat{\bar\psi}K\hat\psi+\mathcal{N})^{k+n}:|i_1\cdots i_m\bar{i}_1\cdots\bar{i}_n\rangle \qquad (19)$$

and further to determinant (for the Fermi-Dirac statistics) or permanent (for the Bose-Einstein statistics) of "one-particle" matrix elements of $i\hat{\bar\psi}K\hat\psi+\mathcal{N}$:*)

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*) After taking into account the connection between I and $\bar{I}$, K vanishes, and, e.g., in eq. (20.b) the factors cancel due to the relation $1+S^{(-)}IS^{(+)}\bar{I}=(1+iS^{(-)}I)(1-iS^{(-)}\bar{I})$ (in other cases analogously). The $\mathcal{N}$ gives no contribution to eqs. (22) and (23).

$$\langle 0|\hat{\psi}^{(-)}(x):e^{i\hat{\bar{\psi}}K\hat{\psi}}:\hat{\bar{\psi}}^{(+)}(y)|0\rangle = \langle 0|\hat{\psi}^{(-)}(x):i\hat{\bar{\psi}}K\hat{\psi}+\mathcal{N}:\hat{\bar{\psi}}^{(+)}(y)|0\rangle = \tag{20.a}$$

$$=\mathcal{S}^{(-)}+i\mathcal{S}^{(-)}K\mathcal{S}^{(-)}=(1-i\mathcal{S}^{(-)}\bar{I})\frac{1}{1+\mathcal{S}^{(-)}I\mathcal{S}^{(+)}\bar{I}}(1+i\mathcal{S}^{(-)}I)\mathcal{S}^{(-)}= \tag{20.b}$$

$$=\mathcal{S}^{(-)}\bar{J}\frac{1}{1+\mathcal{S}^{(-)}J\mathcal{S}^{(+)}\bar{J}}\mathcal{S}^{(-)}J\mathcal{S}^{(-)}= \tag{20.c}$$

$$=\langle 0|\hat{\psi}^{(-)}(x):\overline{G^{+}G}+\overline{G^{+}GG^{+}G}+\overline{G^{+}GG^{+}GG^{+}G}+\cdots:\hat{\bar{\psi}}^{(+)}(y)|0\rangle=\mathcal{S}^{(-)}(x-y) \tag{20.d}$$

$$\langle 0|\hat{\bar{\psi}}^{(-)}(y):e^{i\hat{\bar{\psi}}K\hat{\psi}}:\psi^{(+)}(x)|0\rangle=\langle 0|\hat{\bar{\psi}}^{(-)}(y):i\hat{\bar{\psi}}K\hat{\psi}+\mathcal{N}:\hat{\psi}^{(+)}(x)|0\rangle= \tag{21.a}$$

$$=\mathcal{S}^{(+)}-i\mathcal{S}^{(+)}K\mathcal{S}^{(+)}=\mathcal{S}^{(+)}(1-iI\mathcal{S}^{(+)})\frac{1}{1+\bar{I}\mathcal{S}^{(-)}I\mathcal{S}^{(+)}}(1+i\bar{I}\mathcal{S}^{(+)})= \tag{21.b}$$

$$=\mathcal{S}^{(+)}J\mathcal{S}^{(+)}\frac{1}{1+\bar{J}\mathcal{S}^{(-)}J\mathcal{S}^{(+)}}\bar{J}\mathcal{S}^{(+)}= \tag{21.c}$$

$$=\langle 0|\hat{\bar{\psi}}^{(-)}(y):\overline{GG^{+}}+\overline{GG^{+}GG^{+}}+\overline{GG^{+}GG^{+}GG^{+}}+\cdots:\hat{\psi}^{(+)}(x)|0\rangle=\mathcal{S}^{(+)}(x-y) \tag{21.d}$$

$$\langle 0|:e^{i\hat{\bar{\psi}}K\hat{\psi}}:\hat{\bar{\psi}}^{(+)}(y)\hat{\psi}^{(+)}(x)|0\rangle=\langle 0|:i\hat{\bar{\psi}}K\hat{\psi}+\mathcal{N}:\hat{\bar{\psi}}^{(+)}(y)\hat{\psi}^{(+)}(x)|0\rangle= \tag{22.a}$$

$$=i\mathcal{S}^{(+)}K\mathcal{S}^{(-)}=i\mathcal{S}^{(+)}I\mathcal{S}^{(-)}-(1-iI\mathcal{S}^{(+)})\bar{I}\frac{1}{1+\mathcal{S}^{(-)}I\mathcal{S}^{(+)}\bar{I}}(1+i\mathcal{S}^{(-)}I)\mathcal{S}^{(-)}= \tag{22.b}$$

$$=i\mathcal{S}^{(+)}J\mathcal{S}^{(-)}-i\mathcal{S}^{(+)}J\mathcal{S}^{(+)}\bar{J}\frac{1}{1+\mathcal{S}^{(-)}J\mathcal{S}^{(+)}\bar{J}}\mathcal{S}^{(-)}J\mathcal{S}^{(-)}= \tag{22.c}$$

$$=\langle 0|:G+\overline{GG^{+}G}+\overline{GG^{+}GG^{+}G}+\cdots:\hat{\bar{\psi}}^{(+)}(y)\hat{\psi}^{(+)}(x)|0\rangle=0 \tag{22.d}$$

$$\langle 0|\hat{\psi}^{(-)}(x)\hat{\bar{\psi}}^{(-)}(y):e^{i\hat{\bar{\psi}}K\hat{\psi}}:|0\rangle=\langle 0|\hat{\psi}^{(-)}(x)\hat{\bar{\psi}}^{(-)}(y):i\hat{\bar{\psi}}K\hat{\psi}+\mathcal{N}:|0\rangle= \tag{23.a}$$

$$=-i\mathcal{S}^{(-)}\bar{I}\mathcal{S}^{(+)}+i\mathcal{S}^{(-)}(1-i\bar{I}\mathcal{S}^{(-)})I\frac{1}{1+\mathcal{S}^{(+)}\bar{I}\mathcal{S}^{(-)}I}(1+i\mathcal{S}^{(+)}\bar{I})\mathcal{S}^{(+)}= \tag{23.b}$$

$$=-i\mathcal{S}^{(-)}\bar{J}\mathcal{S}^{(+)}+i\mathcal{S}^{(-)}\bar{J}\mathcal{S}^{(-)}J\frac{1}{1+\mathcal{S}^{(+)}\bar{J}\mathcal{S}^{(-)}J}\mathcal{S}^{(+)}\bar{J}\mathcal{S}^{(+)}= \tag{23.c}$$

$$=\langle 0|\hat{\psi}^{(-)}(x)\hat{\bar{\psi}}(y):G^{+}+\overline{G^{+}GG^{+}}+\overline{G^{+}GG^{+}GG^{+}}+\cdots:|0\rangle=0 \tag{23.d}$$

Hence

$$\langle j_1\cdots j_k\bar{j}_1\cdots\bar{j}_\ell|\mathcal{S}^{+}\mathcal{S}|i_1\cdots i_m\bar{i}_1\cdots\bar{i}_n\rangle=$$

$$=\delta_{km}\delta_{\ell n}\int d^3x_{j_1}\cdots d^3x_{j_k}d^3x_{\bar{i}_1}\cdots d^3x_{\bar{i}_n}d^3x_{i_1}\cdots d^3x_{i_m}d^3x_{\bar{j}_1}\cdots d^3x_{\bar{j}_\ell}\bar{u}_{j_1}\gamma_4\cdots\bar{u}_{j_k}\gamma_4$$

$$\bar{u}_{\bar{i}_1}\gamma_4\cdots\bar{u}_{\bar{i}_n}\gamma_4 \begin{vmatrix} \mathcal{S}^{(-)}_{j_1 i_1}\cdots\mathcal{S}^{(-)}_{j_1 i_m} & 0\;\cdots\;0 \\ \hdashline \mathcal{S}^{(-)}_{j_k i_1}\cdots\mathcal{S}^{(-)}_{j_k i_m} & 0\;\cdots\;0 \\ 0\;\cdots\;0 & \mathcal{S}^{(+)}_{\bar{i}_1\bar{j}_1}\cdots\mathcal{S}^{(+)}_{\bar{i}_1\bar{j}_l} \\ \hdashline 0\;\cdots\;0 & \mathcal{S}^{(+)}_{\bar{i}_n\bar{j}_1}\cdots\mathcal{S}^{(+)}_{\bar{i}_n\bar{j}_l} \end{vmatrix}_{\mp} \gamma_4 u_{i_1}\cdots\gamma_4 u_{i_m}\gamma_4 u_{\bar{j}_1}\cdots\gamma_4 u_{\bar{j}_l} \qquad (24)$$

what means the S-matrix unitarity (here notations are the same as in eqs. (I.15), (I.21) and (I.22)). Any non-diagonal in the particle (antiparticle) number matrix element of $\mathcal{S}^+\mathcal{S}$ vanishes due to the factors (22) or (23) .

Because the amplitudes, and hence, probabilities are expressed in terms of G and G^+, but not of C and C^+ (or I and $\bar{I}$), eqs. (20)-(23) are represented in these terms, too. To this end the quantities $\mathfrak{J}$ and $\bar{\mathfrak{J}}$ were introduced like I and $\bar{I}$ ($C=i\hat{\bar{\varphi}}I\hat{\varphi}$, $C^+=-i\hat{\bar{\varphi}}\bar{I}\hat{\varphi}$):

$$G=C+\mathcal{N}=i\hat{\bar{\varphi}}\,\mathfrak{J}\hat{\varphi}\,,\quad G^+=C^++\mathcal{N}=-i\,\hat{\bar{\varphi}}\,\bar{\mathfrak{J}}\hat{\varphi}\,. \qquad (25)$$

The $\mathfrak{J}$ and $\bar{\mathfrak{J}}$ have properties

$$\begin{aligned} i\mathcal{S}^{(-)}\mathfrak{J}\mathcal{S}^{(-)}&=(1+i\mathcal{S}^{(-)}I)\mathcal{S}^{(-)}\,, & -i\mathcal{S}^{(+)}\mathfrak{J}\mathcal{S}^{(+)}&=\mathcal{S}^{(+)}(1-iI\mathcal{S}^{(+)}) \\ \mathcal{S}^{(-)}\mathfrak{J}\mathcal{S}^{(+)}&=\mathcal{S}^{(-)}I\mathcal{S}^{(+)}\,, & \mathcal{S}^{(+)}\mathfrak{J}\mathcal{S}^{(-)}&=\mathcal{S}^{(+)}I\mathcal{S}^{(-)} \\ -i\mathcal{S}^{(-)}\bar{\mathfrak{J}}\mathcal{S}^{(-)}&=(1-i\mathcal{S}^{(-)}\bar{I})\mathcal{S}^{(-)}\,, & i\mathcal{S}^{(+)}\bar{\mathfrak{J}}\mathcal{S}^{(+)}&=(1+i\mathcal{S}^{(+)}\bar{I})\mathcal{S}^{(+)} \\ \mathcal{S}^{(-)}\bar{\mathfrak{J}}\mathcal{S}^{(+)}&=\mathcal{S}^{(-)}\bar{I}\mathcal{S}^{(+)}\,, & \mathcal{S}^{(+)}\bar{\mathfrak{J}}\mathcal{S}^{(-)}&=\mathcal{S}^{(+)}\bar{I}\mathcal{S}^{(-)} \end{aligned} \qquad (26)$$

The results of transition to $\mathfrak{J}$ and $\bar{\mathfrak{J}}$ are given by eqs. (20.c) (21.c) , (22.c) and (23.c) . Hence expressions in terms of G and G^+ are clear. Another way of this transformation is the use of the $\hat{\bar{\varphi}}K\hat{\varphi}$ in terms of C and C^+ (see eq. (1) , i.e.,$i\hat{\bar{\varphi}}K\hat{\varphi}=$ $=C^++C+\overline{C^+C}+\overline{C\,C^+}+\cdots$) and the fact that all C and C^+, not being extreme, can be directly substituted by G and $G^{+*)}$ ($\mathcal{N}$ gives no contribution, since these C and C^+ create and annihilate pairs, respectively). The transformation of the extreme C and C^+ into G and G^+ is illustrated in Appendix .

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*) This can be done everywhere in the closed loops, considered in Sec. I.3.

3. DISTRIBUTION GENERATED BY EXTERNAL FIELDS

Let us decompose $:e^{C^+}::e^{C}:$ into N-products once again, directly in terms of G and G^+

$$:e^{C^+}::e^{C}:=\sum_{p,q=0}^{\infty}\frac{1}{p!}\frac{1}{q!}\sum_{\substack{f_1\cdots f_p\\ \bar{f}_1\cdots\bar{f}_q}}:e^{G^+-\mathcal{N}}:|f_1\cdots f_p\bar{f}_1\cdots\bar{f}_q\rangle\langle f_1\cdots f_p\bar{f}_1\cdots\bar{f}_q|:e^{G-\mathcal{N}}:= \tag{27.a}$$

$$=\sum_{p,q=0}^{\infty}\frac{1}{p!}\frac{1}{q!}\sum_{\substack{f_1\cdots f_p\\ \bar{f}_1\cdots\bar{f}_q}}[\ldots\{[:e^{G^+}:,a^+_{f_1}]a^+_{f_2}\}\cdots a^+_{\bar{f}_q}]_{\pm}|0\rangle\langle 0|[a_{\bar{f}_q}\cdots\{a_{f_2}[a_{f_1},:e^{G}:]\}\ldots]_{\pm}= \tag{27.b}$$

$$=\sum_{p,q=0}^{\infty}\sum_{\substack{p\ \mathrm{pair.}\ \mathcal{S}^{(-)}\\ q\ \mathrm{pair.}\ \mathcal{S}^{(+)}}}\overbracket{:e^{G^+}:|0\rangle\langle 0|:e^{G}:}^{\cdots}=\sum_{p,q=0}^{\infty}\sum_{\substack{p\ \mathrm{pair.}\ \mathcal{S}^{(-)}\\ q\ \mathrm{pair.}\ \mathcal{S}^{(+)}}}\overbracket{:e^{G^+}::e^{-\mathcal{N}}::e^{G}:}^{\cdots}= \tag{27.c}$$

$$=\sum_{p,q=0}^{\infty}\sum:\left(\overbracket{e^{G^+}e^{G}}^{\cdots}\right)_{\star}e^{-\mathcal{N}}: \quad . \tag{27.d}$$

In eqs. (27.c) and (27.d) the sum over $f_1\cdots f_p\bar{f}_1\cdots\bar{f}_q$ is represented by a sum of all possible terms with p pairings $\mathcal{S}^{(-)}$ and q pairings $\mathcal{S}^{(+)}$ simultaneously. Enumeration of the number of pairings of a given type for G and G^+ is the same as for C and C^+, i.e., is given by the coefficient (I.B.8) of Appendix I.B. The $\star$ in (27.d) means that only creation operators have survived as the free ends of G^+, and only annihilation ones have survived as the free ends of G (due to contact with the vacuum)*). Expression (27.c) is N-ordered, and therefore is equivalent to (1) or (16), the necessary factor $e^{-\mathcal{N}}$ being arised.

Note that in operator terms eqs. (20.d), (21.d), (22.d) and (23.d) are represented as follows

$$(\overbracket{G^+G}+\overbracket{G^+G}\overbracket{G^+G}+\cdots)_{\star}=\mathcal{N}_p=\overbracket{\mathcal{N}\mathcal{N}} \tag{28}$$

$$(\overbracket{GG^+}+\overbracket{GG^+}\overbracket{GG^+}+\cdots)_{\star}=\mathcal{N}_a=\overbracket{\mathcal{N}\mathcal{N}} \tag{29}$$

$$(G+\overbracket{GG^+}G+\cdots)_{\star}=0 \tag{30}$$

$$(G^++\overbracket{G^+G}G^++\cdots)_{\star}=0\ , \tag{31}$$

where $\mathcal{N}_p$ and $\mathcal{N}_a$ are particle and antiparticle number operators, respectively ($:\mathcal{N}:=\mathcal{N}_p+\mathcal{N}_a$). Note that eqs. (28) and

---

*) This can be also represented by pairings of the extreme G and G^+ with $\mathcal{N}$ (cf. eqs. (A.2) of Appendix).

(29) give the decomposition of the particle number operator in terms of G and G^+.

A diagrammatical representation of eqs. (20)-(23) (or (28)-(31)) in terms of C, C^+ and G, G^+ is given in Figs. 1 and 2.

Consider diagonal terms of eq. (27.d) :

one-particle $$e^D:\left(\overline{G^+G}+\overline{G^+G}\,\overline{G\,G^+}\,\overline{G^+G}+\cdots\right)_\star e^{-\mathcal{N}}: = e^D:\mathcal{N}_p e^{-\mathcal{N}}: \tag{32}$$

two-particle

$$\frac{c}{(2!)^2}:\overline{G^+\overline{G^+:|0\rangle\langle 0|:G}G}:+\frac{1}{(3!)^2}\left(c':\overline{G^+\overline{G^+\overline{G^+:|0\rangle\langle 0|:G}G}G}:+c'':\overline{G^+\overline{G^+\overline{G^+:|0\rangle\langle 0|:G}G}G}:\right)+\cdots \tag{33.a}$$

$$=e^D\frac{1}{2!}:\left(\overline{G^+G}+\overline{G^+G}\,\overline{G\,G^+}\,\overline{G^+G}+\cdots\right)^2_\star e^{-\mathcal{N}}: = e^D\frac{1}{2!}:\mathcal{N}_p^2 e^{-\mathcal{N}}: \tag{33.b}$$

n-particle $$e^D\frac{1}{n!}:\left(\overline{G^+G}+\overline{G^+G}\,\overline{G\,G^+}\,\overline{G^+G}+\cdots\right)^n_\star e^{-\mathcal{N}}: = e^D\frac{1}{n!}:\mathcal{N}_p^n e^{-\mathcal{N}}: \tag{34}$$

n-antiparticle $$e^D\frac{1}{n!}:\left(\overline{G\,G^+}+\overline{G\,G^+}\,\overline{G^+G}\,\overline{G\,G^+}+\cdots\right)^n_\star e^{-\mathcal{N}}: = e^D\frac{1}{n!}:\mathcal{N}_a^n e^{-\mathcal{N}}: \tag{35}$$

particle-antiparticle $$:G^+:|0\rangle\langle 0|:G:+\frac{1}{(2!)^2}\Big(c:\overline{G^+\overline{G^+:|0\rangle\langle 0|:G}G}:+ \tag{36.a}$$

$$+c':\overline{G^+\overline{G^+:|0\rangle\langle 0|:}G}G:+c'':G^+\overline{G^+:|0\rangle\langle 0|:G}G:+c''':G^+\overline{G^+:|0\rangle\langle 0|:G}G:\Big)+\cdots=$$

$$=e^D:\Big(\left(G^++\overline{G^+G}\,G^++\cdots\right)\left(G+G\,\overline{G^+G}+\cdots\right)+\left(\overline{G^+G}+\overline{G^+G}\,\overline{G\,G^+}\,\overline{G^+G}+\cdots\right)\cdot$$

$$\cdot\left(\overline{G\,G^+}+\overline{G\,G^+}\,\overline{G^+G}\,\overline{G\,G^+}+\cdots\right)\Big)_\star e^{-\mathcal{N}}:= e^D:\mathcal{N}_p\mathcal{N}_a e^{-\mathcal{N}}: \tag{36.b}$$

m-particle and n-antiparticle

$$\frac{e^D}{(m!n!)^2}:\begin{array}{r} m\left\{\begin{array}{c} \\ \\ \\ \end{array}\right. \\ n\left\{\begin{array}{c} \\ \\ \\ \end{array}\right. \end{array}\left|\begin{array}{cc} \overbrace{(\overline{G^+G}+\ldots)\cdots(\overline{G^+G}+\cdots)}^{m} & \overbrace{(G^++\ldots)\cdots(G^++\ldots)}^{n} \\ \cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots \\ (\overline{G^+G}+\cdots)\cdots(\overline{G^+G}+\cdots) & (G^++\ldots)\cdots(G^++\cdots) \\ (G+\cdots)\cdots(G+\cdots) & (\overline{G\,G^+}+\ldots)\cdots(\overline{G\,G^+}+\cdots) \\ \cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots \\ (G+\cdots)\cdots(G+\cdots) & (\overline{G\,G^+}+\ldots)\cdots(\overline{G\,G^+}+\cdots) \end{array}\right|_{+\star} e^{-\mathcal{N}}:$$

$$=e^D\sum_{k=0}^{\min(m,n)}:\frac{(\overline{G^+G}+\cdots)^{m-k}_\star(\overline{G\,G^+}+\cdots)^{n-k}_\star(G+\cdots)^k_\star(G^++\cdots)^k_\star}{(m-k)!(n-k)!(k!)^2}:=e^D\frac{1}{m!n!}:\mathcal{N}_p^m\mathcal{N}_a^n e^{-\mathcal{N}}: \qquad \begin{array}{r}(37.a)\\(37.b)\end{array}$$

Eq. (33.a) gives as an example several terms as they follow from decomposition (27.c). The numbers $c, c', c'', \ldots$ of identical pairings can be calculated either directly or using general formula (I.B.8) for the coefficient c ($c=\frac{2!2!}{2!}=2$, $c'=3!3!$, $c''=\frac{3!3!}{2}$, ...)*). Another example is expression (36.a), where the same ways give $c = c' = c'' = c''' = 4$.

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*) Of course, they can be obtained from the decomposition (27.a), too. For illustration see Appendix.

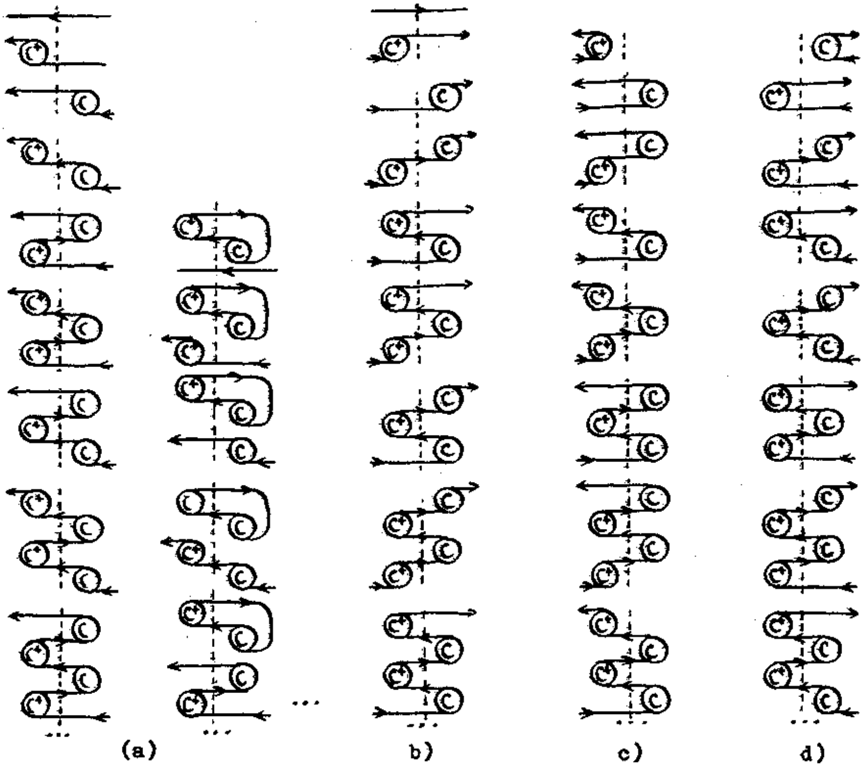
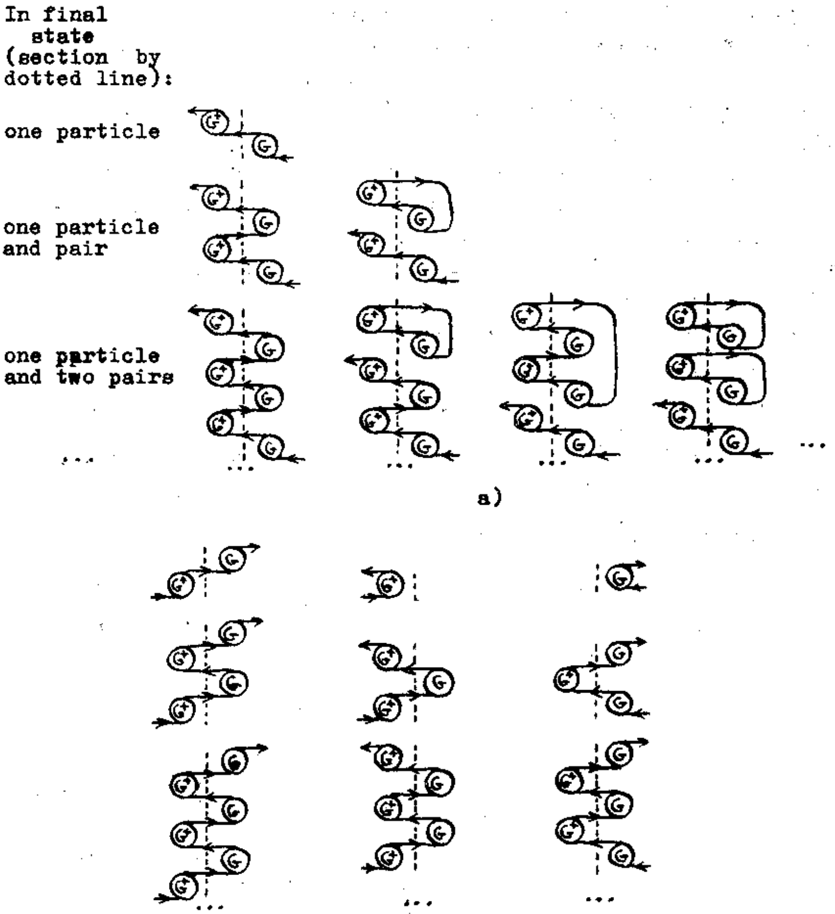


Fig. 1. A diagrammatical representation of the S-matrix unitarity condition in terms of C and C^+. a) One-particle sector of $:e^{C^+}::e^{C}:$. Sum of diagrams of the first column corresponds to eq. (20.b) , only the first diagram (—←—) giving a contribution. Sums of diagrams b), c) and d) correspond to eqs. (21.b), (22.b) and (23.b), respectively. Being accompanied by closed loops (as in the case a)), b), c) and d) represent antiparticle, vacuum-pair, pair-vacuum sectors of $:e^{C^+}::e^{C}:$. In all the cases diagrams are in fact the same, except for directions of ends and presence of additional diagrams —←— and —→— in a) and b), which only give non-zero contribution.



a)

b) c) d)

Fig. 2. A diagrammatical representation of the S-matrix unitarity condition in terms of G and G^+. a) One-particle sector of $:e^{C^+}::e^{C}:$. Sum of the diagrams of the first column corresponds to eq. (20.d). Sums of diagrams b), c) and d) correspond to eqs. (21.d), (22.d) and (23.d) , respectively. Being accompanied by closed loops (as in the case a)), b) c) and d) represent antiparticle vacuum-pair, pair-vacuum sectors of $:e^{C^+}::e^{C}:$.

Equation (37) is written, using permanent which in non-operator terms leads either to the determinant or to the permanent according to statistics and to eq. (24).

Non-diagonal matrix elements of (27) vanish due to expressions (30) or (31), entering as common factors (however, prior to the summation over final states $\langle j|\hat{S}^+|f\rangle\langle f|\hat{S}|i\rangle \neq 0$ even if i, j and f differ from each other in any number of pairs; if there is such a distinction of i and j, then after summation each additional pair leads to one of two mentioned common factors, and the sum vanishes).

Hence, there again follows the unitarity of S-matrix: eq. (27.d) reduces to

$$e^D \sum_{m,n=0}^{\infty} \frac{1}{m!n!} :\mathcal{N}_p^m \mathcal{N}_a^n e^{-\mathcal{N}}: = e^D : e^{\mathcal{N}} e^{-\mathcal{N}} : = e^D. \qquad (38)$$

Expression (37.a) is the relevant diagonal in particle and (independently) antiparticle numbers part of the operator

$$\frac{1}{(m+n)!} :\left(i\hat{\bar{\psi}} K \hat{\psi} + \mathcal{N}\right)^{m+n} e^{-\mathcal{N}}: =$$

$$= \frac{1}{(m+n)!} :\Big((G+\cdots)+(G^++\cdots)+(\overbracket{G^+G}+\cdots)+(\overbracket{GG^+}+\cdots)\Big)_{\star}^{m+n} e^{-\mathcal{N}}: \qquad (39)$$

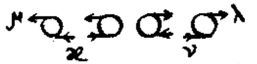
which in turn is the (m+n)-th term of the expansion of

$$:e^{C^+}::e^{C}: = e^{D} :e^{i\hat{\bar{\psi}} K \hat{\psi}}: = e^D : \exp\left(i\hat{\bar{\psi}} K\hat{\psi} + \mathcal{N}\right) \cdot e^{-\mathcal{N}} : =$$

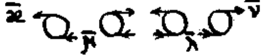
$$= e^D : \exp\Big((G+\cdots)+(G^++\cdots)+(\overbracket{G^+G}+\cdots)+(\overbracket{GG^+}+\cdots)\Big)_{\star} e^{-\mathcal{N}}: \qquad (40)$$

Let us construct a generating function for probabilities of transitions between states with given numbers of particles and antiparticles. To this end we make the substitutions

$$G = G_1+G_2+G_3+G_4 \rightarrow \tilde{G} = \varkappa\mu G_1 + \mu\nu G_2 + \varkappa\lambda G_3 + \lambda\nu G_4 \qquad (41)$$



$$G^+ = G_1^+ + G_2^+ + G_3^+ + G_4^+ \rightarrow \tilde{G}^+ = \bar{\varkappa}\bar{\mu} G_1^+ + \bar{\mu}\bar{\nu} G_2^+ + \bar{\varkappa}\bar{\lambda} G_3^+ + \bar{\lambda}\bar{\nu} G_4^+ \qquad (42)$$



in eq. (40) (and, therefore, in eq. (37.a)), G_1, G_2, G_3, and G_4 being four terms of eq. (I.7.b), and $\varkappa, \lambda, \mu, \nu$ being complex c-number parameters. Then the operators $(\overbracket{G^+G}+\cdots)_{\star}$, $(\overbracket{GG^+}+\cdots)_{\star}$, $(G+\cdots)_{\star}$, $(G^++\cdots)_{\star}$ and (40)*) transform into:

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*) Multiplied by $e^{B^*+B} = e^{-D}$.

$$\Big(\overbracket{G^+G}+\overbracket{G^+G}\overbracket{G^+G}+\cdots\Big)^{\sim}_{\bigstar}=|\varkappa|^2|\mu|^2\Big(\overbracket{G_1^+G_1}+|\mu|^2|\nu|^2\overbracket{G_1^+G_2}\overbracket{G_2^+G_1}+\cdots\Big) \quad (43)$$

$$\Big(\overbracket{GG^+}+\overbracket{GG^+}\overbracket{GG^+}+\cdots\Big)^{\sim}_{\bigstar}=|\lambda|^2|\nu|^2\Big(\overbracket{G_4G_4^+}+|\mu|^2|\nu|^2\overbracket{G_4G_2^+}\overbracket{G_2G_4^+}+\cdots\Big) \quad (44)$$

$$\Big(G+\overbracket{GG^+}G+\cdots\Big)^{\sim}_{\bigstar}=\varkappa\lambda\Big(G_3+|\mu|^2|\nu|^2\overbracket{G_4G_2^+}G_1+|\mu|^4|\nu|^4\overbracket{G_4G_2^+}\overbracket{G_2G_2^+}G_1+\cdots\Big) \quad (45)$$

$$\Big(G^++\overbracket{G^+G}G^++\cdots\Big)^{\sim}_{\bigstar}=\bar{\varkappa}\bar{\lambda}\Big(G_3^++|\mu|^2|\nu|^2\overbracket{G_1^+G_2}G_4^++|\mu|^4|\nu|^4\overbracket{G_1^+G_2}\overbracket{G_2^+G_2}G_4^++\cdots\Big) \quad (46)$$

$$\gamma(\varkappa,\lambda,\mu,\nu)=e^{B^*+B}\,e^{\widetilde{D}}:\exp\Big((G+\cdots)+(G^++\cdots)+(\overbracket{G^+G}+\cdots)+(\overbracket{GG^+}+\cdots)\Big)^{\sim}_{\bigstar}e^{-N}: \quad (47)$$

respectively. Equation (47) is a generating function of interest, and coefficients of its power series expansion in $\varkappa, \lambda, \ldots$ give the probabilities of transitions between states with given numbers of particles and antiparticles. Thus, the power of $\varkappa$ (λ) equals the number of particles (antiparticles) in the "right" initial state, and the power of $\bar{\varkappa}$ ($\bar{\lambda}$) equals the number of particles (antiparticles) in the "left" initial state. The parameters $\mu, \bar{\mu}$ ν and $\bar{\nu}$ enter only in combinations $|\mu|^2$ and $|\nu|^2$. The power of $|\mu|^2$ ($|\nu|^2$) is equal to the number of particles (antiparticles) in the final state. Diagonal in number of particles and (independently) antiparticles terms of eq. (47) contain $\varkappa$, $\bar{\varkappa}$, λ and $\bar{\lambda}$ only in combinations $|\varkappa|^2$ and $|\lambda|^2$, powers of which are equal to numbers of initial particles and antiparticles. Thus, eq. (37.a) contains $|\varkappa|^{2m}|\lambda|^{2n}$. The power series expansion of eq. (37.a) in $|\mu|^2$ and $|\nu|^2$ (eqs. (43)-(46) are implied to be substituted into eq. (37.a)) gives*) transition probabilities (distribution over numbers of final particles) of interest.**) One can obtain probabilities for finding final particles with given quantum numbers, decomposing each pairing function $S^{(-)}$ and $S^{(+)}$ over suitable complete sets of one-particle states and taking into account statistics.

We have considered the product S^+S. The product SS^+ may be treated analogously. Only G and G^+ interchange their places in eqs. (27)-(47).

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*) Up to the factor $e^{B^*+B}=e^{-D}$.

**) We have encountered with such a series (in $\alpha=|\mu|^2|\nu|^2$) in Sec. I.3.

4. CONNECTION BETWEEN SPIN AND STATISTICS

Many people (Pauli, Feynman, Schwinger et al./1,2,5,26-28/) investigated the problem of connection between spin and statistics. Several arguments were given by Feynman who stressed an intimate connection between the relativistic Dirac equation and Pauli principle/2/. According to Feynman one may put the question as follows. Let us accept the Dirac equation and consider for many-particle wave functions the possibilities:

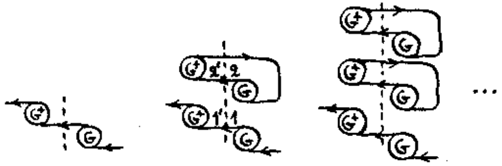
a) simple (non-symmetrized) products of one-particle wave functions,

b) symmetrized products (Bose-Einstein statistics),

c) antisymmetrized products (Fermi-Dirac statistics).

Why is the possibility c) preferable?

Let us take an initial state with one electron. In final states one can observe either electron, or electron + pair, or electron + 2 pairs, etc. When summing probabilities over all final states, in the case a) only graphs of Fig. 3 are possible.



a) b) c)

Fig. 3

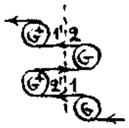


Fig. 4

Neither the "snaky" graphs, nor more complicated closed loops (including "snakes") are possible.

Symmetrization or antisymmetrization lead to infinitely many additional "snaky" graphs (see Fig. 2, the first column) and to complicated closed loops. For example, the graph b) of Fig. 3 is now accompanied by the "snaky" graph, which is drawn in Fig. 4. We have shown above that the "snaky" graphs are summed in such a manner, that

$$\langle 1|:\overbrace{G^+G}+\overbrace{G^+\overbrace{G\,G^+}G}+\cdots:|1\rangle = 1 \qquad (\langle 1|1\rangle = 1) \qquad (48)$$

This guarantees that the sum of transition probabilities for the electron into all possible final states is equal to unity[*].

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[*] If an initial electron is characterized by a normalized function $u(x)$ ($\int d^3x\,\bar{u}(x)\gamma_4 u(x) = 1$), and, therefore by the state vector $\int d^3y\,\hat{\bar{\psi}}^{(+)}(y)\gamma_4 u(y)|0\rangle$, then from eq. (20.d) we get

$$\langle 1|:\overbrace{G^+G}+\overbrace{G^+\overbrace{G\,G^+}G}+\cdots:|1\rangle=\int d^3x\,d^3y\,\bar{u}(x)\gamma_4 S^{(-)}(x-y)\gamma_4 u(y)=1.$$

However, all the terms of eq. (48), but the first one, are neither absolute, nor relative probabilities.

If simple products are assumed, the "snaky" graphs cannot arise, and the matrix element $\langle 0|\hat{\psi}^{(-)}(x):\overline{G^{+}G}:\hat{\bar{\psi}}^{(+)}(y)|0\rangle$ should be a constant multiple of $S^{(-)}(x-y)$ separately. However, it is clear from (20.d) that*)

$$\langle 0|\hat{\psi}^{(-)}(x):\overline{G^{+}G}:\hat{\bar{\psi}}^{(+)}(y)|0\rangle \neq S^{(-)}(x-y) \, . \tag{49}$$

In fact

$$\langle 0|\hat{\psi}^{(-)}(x):\overline{G^{+}G}:\hat{\bar{\psi}}^{(+)}(y)|0\rangle = (1-iS^{(-)}\bar{I})(1+iS^{(-)}I)S^{(-)} = (1+S^{(-)}\bar{I}S^{(+)}I)S^{(-)} \tag{50}$$

where the latter expression is obtained using the Schwinger relation (17). If we subtract the superfluous term from both sides we obtain

$$\langle 0|\hat{\psi}^{(-)}(x):(i\hat{\bar{\psi}}K'\hat{\psi}+\mathcal{N}):\hat{\bar{\psi}}^{(+)}(y)|0\rangle = S^{(-)}(x-y) \, , \tag{51}$$

where

$$K' = I-\bar{I}-i\bar{I}(S^{(-)}-S^{(+)})I \qquad (=0) \, . \tag{52}$$

However the term $\bar{I}S^{(+)}I$ has no reasonable meaning since it describes a creation of something with negative frequencies. Using the Schwinger relation (17), we can represent it in the form

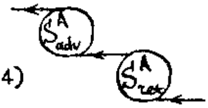
$$\bar{I}S^{(+)}I = (1-i\bar{I}S^{(-)})IS^{(+)}\frac{1}{1+\bar{I}S^{(-)}IS^{(+)}}\bar{I}(1+iS^{(-)}I) \tag{53}$$

where the above difficulty is absent (only positive frequencies are created in each term of this infinite series) at the expense of the infinite sum of the "snaky" graphs, which are interpretable only in terms of antisymmetrized or symmetrized wave functions. With eq. (53) the quantity (52) takes the form (13.e).

We refer to Feynman arguments /1,2/ to distinguish between Fermi-Dirac and Bose-Einstein statistics.

Let us discuss another possibility. One may try to interpret, as the unitarity condition, the relation

$$\int d^3y\, S^{A}_{adv}(x,y)\gamma_4 S^{A}_{ret}(y,z) = \gamma_4\delta(\vec{x}-\vec{z}) \tag{54}$$



for the retarded and advanced Green functions S^{A}_{ret} and S^{A}_{adv} of Dirac equation. However the final state

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*) And $\langle 1|:\overline{G^{+}G}:|1\rangle > 1(<1)$ for F.-D. (B.-E.) statistics /2/.

$$\Psi(x)=i\int_{x_0'=t'} d^3x'\, S^{A}_{ret}(x,x')\gamma_4\Psi(x') \tag{55}$$

together with S^A_{ret} contains both positive and negative frequencies, even in the case, when the initial state $\psi\ (x')$ is the positive-frequency one (the Klein paradox).

One remark concerning the S-matrix. We may try to take $:e^{C}:$ as S-matrix. It describes only observable processes. However, $:e^{C^+}::e^{C}:=e^{D}$, but not the unity, as one expects for sums of probabilities. We restore normalization if turn to $S=e^{B}:e^{C}:$, and this additional factor means that each observable process is accompanied by unobservable vacuum loops, and as a consequence of the unobservability we must use superposition of infinitely many such amplitudes.[1]

APPENDIX

Let us demonstrate how the extreme C and C^+ may be reduced to G and G^+ (see p. 10). Using the relation

$$:\mathcal{N}e^{-\mathcal{N}}:\ =\overline{\mathcal{N}:e^{-\mathcal{N}}:\mathcal{N}}+\overline{\mathcal{N}:e^{-\mathcal{N}}:\mathcal{N}} \tag{A.1}$$

we obtain

$$:(i\hat{\bar{\psi}}K\hat{\psi}+\mathcal{N})e^{-\mathcal{N}}:\ =i\hat{\bar{\psi}}^{(+)}K\hat{\psi}^{(+)}|0\rangle\langle0|+i\hat{\bar{\psi}}^{(+)}|0\rangle K\langle0|\hat{\psi}^{(-)}-$$

$$-i\hat{\psi}^{(+)}|0\rangle K^{T}\langle0|\hat{\bar{\psi}}^{(-)}+|0\rangle\langle0|i\hat{\bar{\psi}}^{(-)}K\hat{\psi}^{(-)}+\overline{\mathcal{N}|0\rangle\langle0|\mathcal{N}}+\overline{\mathcal{N}|0\rangle\langle0|\mathcal{N}},$$

$$i\hat{\bar{\psi}}^{(+)}|0\rangle K\langle0|\hat{\psi}^{(-)}+\overline{\mathcal{N}|0\rangle\langle0|\mathcal{N}}=$$

$$=\overline{C^{+}|0\rangle\langle0|\mathcal{N}}+\overline{\mathcal{N}|0\rangle\langle0|C}+\overline{C^{+}|0\rangle\langle0|C}+\overline{\mathcal{N}|0\rangle\langle0|\mathcal{N}}+$$

$$+\overline{\mathcal{N}|0\rangle C}\,\overline{C^{+}\langle0|\mathcal{N}}+\overline{C^{+}|0\rangle C}\,\overline{C^{+}\langle0|\mathcal{N}}+\overline{\mathcal{N}|0\rangle C}\,\overline{C^{+}\langle0|C}+\overline{C^{+}|0\rangle C}\,\overline{C^{+}\langle0|C}+\ldots=$$

$$=\overline{G^{+}|0\rangle\langle0|G}+\overline{G^{+}|0\rangle G}\,\overline{G^{+}\langle0|G}+\cdots$$

$$i\hat{\bar{\psi}}^{(+)}K\hat{\psi}^{(+)}|0\rangle\langle0|=\ :C^{+}+C+\overline{C^{+}C}+\overline{C\,C^{+}}+\cdots:|0\rangle\langle0|=$$

$$=:C^{+}:|0\rangle\langle0|+\overline{:\mathcal{N}\,\overline{\mathcal{N}:|0\rangle\langle0|}\,C}+\overline{:\mathcal{N}\,\overline{C^{+}:|0\rangle\langle0|C}}+\overline{:\mathcal{N}\,\overline{C^{+}:|0\rangle\langle0|C}}+$$

$$+\overline{:C^{+}\,\overline{C^{+}:|0\rangle\langle0|\,C}}+\ldots=\ :G^{+}:|0\rangle\langle0|+\overline{:G^{+}\,\overline{G^{+}:|0\rangle\langle0|G}}+\cdots \tag{A.2}$$

and so on.

Now illustrate transformation of sums over final states into pairings

$$\int\limits_{f_1 f_2}\frac{1}{2!}:G^+G^+e^{-\mathcal{N}}:\frac{1}{2!}|e^-_{f_1}e^-_{f_2}\rangle\langle e^-_{f_1}e^-_{f_2}|\frac{1}{2!}:GG\,e^{-\mathcal{N}}:=$$

$$=\left(\frac{1}{2!}2:\overline{G^+\overline{G^+:}}\right)\frac{1}{2!}:e^{-\mathcal{N}}:\left(\frac{1}{2!}2:\overline{\overline{G}\;G}:\right)=\left(\frac{1}{2!}\right)^3 4\;:\overline{G^+\overline{G^+::e^{-\mathcal{N}}::G}\,G}:$$

$$\int\limits_{f_1 f_2}\frac{1}{2!}:G^+G^+e^{-\mathcal{N}}:\frac{1}{2!}|e^+_{f_1}e^+_{f_2}\rangle\langle e^+_{f_1}e^+_{f_2}|\frac{1}{2!}:GG\,e^{-\mathcal{N}}:=$$

$$=\left(\frac{1}{2!}2:\overline{G^+\overline{G^+:}}\;\right)\frac{1}{2!}:e^{-\mathcal{N}}:\left(\frac{1}{2!}2:\overline{\overline{G}\,G}:\right)=\left(\frac{1}{2!}\right)^3 4:\overline{G^+\overline{G^+::e^{-\mathcal{N}}::G}G}:$$

$$\int\limits_{f_1 f_2}\frac{1}{2!}:G^+G^+e^{-\mathcal{N}}:|e^-_{f_1}e^+_{f_2}\rangle\langle e^-_{f_1}e^+_{f_2}|\frac{1}{2!}:GG\,e^{-\mathcal{N}}:=$$

$$=\frac{1}{2!}2:\overline{G^+\overline{G^+}}+G^+\overline{\overline{G^+}}::e^{-\mathcal{N}}:\frac{1}{2!}2:\overline{\overline{G}\,G}+\overline{\overline{G}}G:=\left(\frac{1}{2!}\right)^2 4\Big(:\overline{G^+\overline{G^+::e^{-\mathcal{N}}:G}\,G}:+$$

$$+:\overline{G^+\,\overline{G^+::e^{-\mathcal{N}}::G}}G:+:G^+\overline{\overline{G^+::e^{-\mathcal{N}}::G}G}:+:G^+\overline{\overline{G^+::e^{-\mathcal{N}}::G}}G:\Big)$$

(A.3)

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