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IN MASSIVE VECTOR GAUGE MODELS

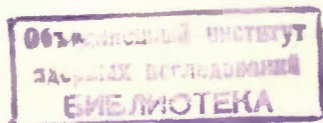
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**DEEP INELASTIC LEPTON HADRON SCATTERING  
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Глубоконеупругие лептон-адронные процессы в калибровочных моделях с массивными векторными-глюонами

Рассматривается класс моделей сильных взаимодействий, в которых дробно-заряженные цветные кварки взаимодействуют с массивными нейтральными векторными глюонами. Все векторные глюоны получают массы при помощи механизма Хиггса. Предполагается, что калибровочная эффективная константа связи  $\bar{g}$  и эффективная константа связи четверного скалярного взаимодействия  $\bar{h}$  стремятся к замкнутому циклу в области больших пространственно-подобных импульсов. Анализируется поведение моментов для структурных функций глубоконеупругих лептон-адронных процессов в области больших  $Q^2$ . Показано, что бьеркеновский скейлинг нарушается степенными по  $Q^2$  членами, умноженными на осциллирующую (по  $Q^2$ ) функцию.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Deep Inelastic Lepton Hadron Scattering in Massive Vector Gauge Models

A class of strong interaction models, in which the interactions between fractionally charged colored quarks are mediated by massive neutral vector gluons, is considered. All the vector gluons acquire masses via the usual Higgs mechanism. The effective coupling constants  $\bar{g}$  (gauge coupling) and  $\bar{h}$  (quartic-self coupling) are supposed to approach a limit cycle in the limit of large space-like momenta. The large  $Q^2$  behaviour of the moments of the deep inelastic lepton hadron structure functions is analysed using this hypothesis. It is shown that Bjorken scaling is violated by power terms of  $Q^2$  multiplied by an oscillating function of  $Q^2$ .

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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## INTRODUCTION

The discovery of asymptotic freedom in non-abelian gauge theories<sup>/1/</sup> revived the hopes to describe the physical phenomena at high energies and high momentum transfers by means of quantum field theory. If the strong interactions are described by a field theory of this kind, it is possible to calculate some dynamical quantities using perturbation theory methods suitably improved with the help of renormalization group. These ideas have been used to analyze  $e^+e^-$  annihilation into hadrons<sup>/2,3/</sup>, deep inelastic lepton hadron scattering<sup>/4,5/</sup> and also to explain the mass spectrum, decay widths, etc., of the new particles<sup>/6,7,8/</sup>. In all these papers quantum chromodynamics (QCD) (a gauge theory in which the interactions between colored quarks are mediated by massless vector colored gluons) as the best candidate for a theory of strong interaction has been used. It is conjectured that if the gauge group of QCD is an exact local symmetry, then, first, the color-triplet quarks and color-octet gluons of the theory are not existing as real particles and, second, only color singlet hadrons do exist. Alas, a proof of color and quark confinement in QCD does not exist. In particular, it has been speculated that confinement is connected with the existence of severe infrared singularities of a Yang-Mills theory in the framework of perturbation theory. However, to any finite order, it has been proved<sup>/9/</sup> that there are no singularities in experimentally acces-

sible transition probabilities. Moreover, it is by far not clear that massless gluon fields are indeed required to explain the confinement of quarks, since model calculations have shown that this confinement could also be achieved by admitting massive scalar gluons /10/.

Therefore, we think it is worthwhile to consider models in which interactions between colored quarks are mediated by massive vector gauge fields. The usual mechanism to ascribe masses to vector gluons in a gauge invariant way (only in this case the model will be renormalizable) is the Higgs mechanism /11/. This mechanism introduces new scalar fields and their quartic self-interaction. In papers /4,12/ a wide class of gauge models including scalar fields was considered. Unfortunately, the authors have not succeeded in finding any physically acceptable model which is asymptotically free (in the sense that all coupling constants involved are driven towards zero in the deep Euclidean region). This is due to the fact that the scalar fields destabilize the originally ultraviolet stable origin  $g=0$ , where  $g$  is the coupling constant of the gauge fields. In papers /13/ spontaneously broken gauge models which are not asymptotically free (the effective gauge coupling constant  $\bar{g}$  vanishes asymptotically and the effective quartic self-coupling  $\bar{h}$  tends to a finite asymptotic value) were considered. The consequences of these massive vector gauge models were analyzed. In particular, it was shown /14/ that Bjorken-scaling for the moments of the deep inelastic lepton hadron structure functions is violated by powers of logarithms (as in the case of asymptotically free gauge models).

In this note a possibility for effective coupling constants ( $\bar{g}$  and  $\bar{h}$ ) to approach a limit cycle in the limit of large space-like momenta is explored. A class of strong interaction models, in which interactions between fractionally charged colored quarks are mediated by massive vector colored gluons, is

considered. All the vector gluons acquire masses via the Higgs mechanism. All the scalar mesons are massive. The vector and scalar mesons are neutral with respect to ordinary flavor group. The local gauge symmetry is broken and if the strong interactions are described by models of this kind then colored hadrons, free colored gluons and quarks may exist. In the paper /17/ this possibility is discussed in detail. Our purpose is to analyze deep inelastic lepton hadron processes in the framework of this class of models. It is shown that Bjorken scaling for the moments of the structure functions of these processes is violated by power terms of  $Q^2$  multiplied by an oscillating function of  $Q^2$ .

#### QFT MODEL OF STRONG INTERACTIONS WITH MASSIVE VECTOR GLUONS

Consider a local  $SU_c(n)$  group with gauge fields  $A_\mu^a(x)$ . The Higgs scalars whose different from zero vacuum expectation values generate masses for these vector fields are chosen to transform as  $m$  complex fundamental representation of  $SU_c(n)$ . We write them as an  $n \times m$  matrix  $\phi$  transforming from the left under the local gauge group  $SU_c(n)$  and from the right under the global group  $SU'(m)$ . All the vector gluons and quarks are singlets with respect to this primed group. If  $m \geq n-1$ , this is sufficient that all the vector gluons acquire masses /18/. The colored gauge group  $SU_c(n)$  commute with the ordinary approximate  $SU(4)$  group of strong interactions. Therefore, all the vector gluons and scalar mesons are neutral with respect to this group. The quarks can be represented by an  $n \times 4$  matrix  $\psi$ . The generators of  $SU(4)$  transform the columns of this matrix, whereas the generators of  $SU_c(n)$  its rows. The Lagrangian may be written as

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu}_a + \text{Tr} \{ (\nabla_\mu \phi)^\dagger (\nabla^\mu \phi) \} + \frac{m_0^2}{2} \text{Tr} \{ \phi^\dagger \phi \} - \\
& -\frac{1}{4} h_1 \{ \text{Tr}(\phi^\dagger \phi) \}^2 - \frac{h_2}{4} \text{Tr} \{ (\phi^\dagger \phi)^2 \} + i \bar{\psi} \gamma_\mu D^\mu \psi - \\
& - \sum_{i=1}^4 m_i \bar{\psi}^{a(i)} \psi_{a(i)} ,
\end{aligned} \tag{1}$$

where

$$\begin{aligned}
F_{\mu\nu}^a &= \partial_\mu V_\nu^a - \partial_\nu V_\mu^a + g f^{abc} V_\mu^b V_\nu^c , \\
\nabla_\mu \phi &= \partial_\mu \phi - i g V_\mu^a F^a \phi , \\
D_\mu \psi &= \partial_\mu \psi - i g V_\mu^a T^a \psi .
\end{aligned} \tag{2}$$

The  $f^{abc}$  are the  $SU_c(n)$  group structure constants and  $F^a$  and  $T^a$  are the representation matrices corresponding to the scalar and fermion multiplets, respectively. After spontaneous symmetry breaking, the Lagrangian in eq. (1) will be invariant under a global  $SU(m)$ , a subgroup of  $SU(n) \times SU'(m)$ . With the exception of the massless Goldstone bosons (which with a suitable choice of the gauge, can be excluded from the spectrum of physical states) all the Higgs scalars are massive. All the mass parameters of the Lagrangian (1) are supposed to be small (less than 1-2 GeV). This assumption does not contradict the fact that the masses of free quarks and gluons can be found to be very large as it was shown in paper <sup>/17/</sup>. In what follows  $SU_c(3)$  group is chosen as a color group, i.e., it is supposed that usual hadrons are bound states of four color-triplet

quarks and interactions between them are mediated by massive vector colored gluons and flavor-singlet Higgs scalars. The results, however, are not very sensitive to this special choice of the color gauge group.

## DEEP INELASTIC LEPTON HADRON SCATTERING IN MASSIVE VECTOR GAUGE MODELS

The behaviour of the cross sections of the deep inelastic lepton hadron processes is governed by the behaviour of the structure functions  $F_k(x, Q^2)$  in the region:  $Q^2 = -q^2 \rightarrow \infty$ ,  $\nu \rightarrow \infty$  and  $x = \frac{Q^2}{2M\nu}$  fixed ( $k = 1, 2$  for electron hadron and  $k = 1, 2, 3$  for neutrino hadron processes). These functions are defined by the matrix element of the commutator of two electromagnetic or weak hadron currents. Using the Wilson operator product expansion at light-like distances <sup>/19/</sup>, sum rules for the structure functions can be obtained <sup>/10/</sup> in the limit of large  $Q^2$  ( $Q^2 \gg M^2$ , where  $M$  is the hadron mass). In the framework of our models these sum rules take the following form in the case of deep inelastic electron hadron scattering

$$\begin{aligned}
M_k^{(n)}(Q^2) &\equiv \int_0^1 dx x^{n-2} F_k(x, Q^2) = \\
&= \langle e^2 \rangle \sum_i c_i^{(n)}(\mu^2) E_{i(k)}^{(n)} \left( \frac{Q^2}{\mu^2}; g_\mu^2, h_\mu \right) + \\
&+ \sum_a (e_a^2 - \langle e^2 \rangle) c_a^{(n)}(\mu^2) E_{NS(k)}^{(n)} \left( \frac{Q^2}{\mu^2}; g_\mu^2, h_\mu \right) + O\left(\frac{M^2}{Q^2}\right), \\
n = 2, 4, \dots, \quad k = 1, 2, \quad i = V, \psi, S.
\end{aligned} \tag{3}$$

In eq.(3)  $F_1(x, Q^2) \equiv 2MxW_1(x, Q^2)$ ,  $F_2(x, Q^2) \equiv vW_2(x, Q^2)$ , where  $W_1$  and  $W_2$  are the usual structure functions, and  $e_a$  are the quark charges. The constants  $c_{i(a)}^{(n)}(\mu^2)$  and the functions  $E_{i(NS)k}^{(n)}$  are connected with a strong interaction dynamics at large and small distances, respectively. The quantity  $1/\mu$  with dimension of length can be considered as a boundary between small and large distances. It is reasonable to choose  $\mu > M_{\text{hadr}}$ , but the particular choice of parameter  $\mu$  within this region is arbitrary. The unknown constants  $C_{i(a)}^{(n)}$  are determined by the hadronic matrix elements of the operators  $O_{a_1 \dots a_n}^{i(a)}$

$$\frac{1}{2} \sum_{s=\pm 1} \langle p, s | O_{a_1 \dots a_n}^{i(a)}(0) | p, s \rangle = C_{i(a)}^{(n)}(\mu^2) P_{a_1} \dots P_{a_n} + (4)$$

+ (terms containing  $\delta_{a_i a_j}$ ).

Note that only the term proportional to  $P_{a_1} \dots P_{a_n}$  in eq. (4) contributes to the leading terms in the moments of structure functions (eq.(3)). For the different hadrons the constants  $c_a^{(n)}(\mu^2)$  are different. For our model the dominant gauge invariant operators of twist two appearing in the operator product expansion of two electromagnetic currents near the light cone will be:

$$O_{a_1 \dots a_n}^V = \frac{i^n}{2} S \{ \text{Tr } F_{aa_1} \nabla_{a_2} \dots \nabla_{a_{n-1}} F_{a_n}^a \} - \text{trace terms}, \quad (5a)$$

$$O_{a_1 \dots a_n}^\psi = \frac{i^{n-1}}{2} \sum_{a=1}^4 S \{ \bar{\psi}_a \gamma_{a_1} D_{a_2} \dots D_{a_n} \psi_a \} - \text{trace terms}, \quad (5b)$$

$$O_{a_1 \dots a_n}^S = \frac{i}{2} \sum_{i=1}^m S \{ \phi_i^+ \nabla_{a_1} \dots \nabla_{a_n} \phi_i \} - \text{trace terms}, \quad (5c)$$

$$O_{a_1 \dots a_n}^a = \frac{i^{n-1}}{2} S \{ \bar{\psi}^a \gamma_{a_1} D_{a_2} \dots D_{a_n} \psi^a \} - \text{trace terms}, \quad (5d)$$

where  $\nabla_a$  and  $D_a$  are covariant derivatives,  $S$  denotes the symmetrization over  $a_1 \dots a_n$  and  $a$  denotes quark flavor.

The coefficients  $c_{i(a)}^{(n)}(\mu^2)$  can be expressed in terms of parton distributions as follows:

$$\begin{aligned} c_i^{(n)}(\mu^2) &= \int_0^1 dx x^{n-1} f_i(x, \mu^2), \quad i = V, S, \\ c_a^{(n)}(\mu^2) &= \int_0^1 dx x^{n-1} \{ f_a(x, \mu^2) + f_a^-(x, \mu^2) \}, \quad (6) \\ c_\psi^{(n)}(\mu^2) &= \sum_a c_a^{(n)}(\mu^2), \end{aligned}$$

where  $f_V, f_S, f_a, f_a^-$  are the gluon, scalar, quark and antiquark distributions in hadrons at momentum transfer  $Q^2 = \mu^2$ , respectively.

If  $Q^2 \geq \mu^2 \gg m^2$  ( $m$  denotes all the mass parameters of the Lagrangian (1)), the coefficient functions  $E_{i(NS)k}^{(n)}(Q^2/\mu^2; g_\mu^2, h_\mu)$  satisfy the following renormalization group equations:

$$\left[ \left( \frac{\partial}{\partial t} - \beta_g(g_\mu^2, h_\mu) \frac{\partial}{\partial g_\mu^2} - \beta_h(g_\mu^2, h_\mu) \frac{\partial}{\partial h_\mu} \right) \delta_{ij} + \right. \quad (7a)$$

$$\left. + \gamma_{ji}^{(n)}(g_\mu^2, h_\mu) \right] E_{j(k)}^{(n)}(Q^2/\mu^2; g_\mu^2, h_\mu) = 0,$$

$$\left[ \frac{\partial}{\partial t} - \beta_g(g_\mu^2, h_\mu) \frac{\partial}{\partial g_\mu^2} - \beta_h(g_\mu^2, h_\mu) \frac{\partial}{\partial h_\mu} + \right. \quad (7b)$$

$$\left. + \gamma_{NS}^{(n)}(g_\mu^2, h_\mu) \right] E_{NS(k)}^{(n)}(Q^2/\mu^2; g_\mu^2, h_\mu) = 0,$$

$$\text{where } t = \frac{1}{2} \ln \frac{Q^2}{\mu^2}.$$

Here  $\gamma^{(n)}$  is the matrix of anomalous dimensions of the flavor-singlet operators  $0_{a_1 \dots a_n}^{V, \psi, S}$  defined by eqs. (5a-5c) and  $\gamma_{NS}^{(n)}$  is the anomalous dimension of the flavor nonsinglet operators  $0_{a_1 \dots a_n}^a$  defined by eq. (5d). Note that for simplicity  $h_1 = h_2 = h$  (in the Lagrangian (1)) is supposed.

The solutions of eq. (7a, 7b) are

$$E_{i(k)}^{(n)} \left( \frac{Q^2}{\mu^2}; g_\mu^2, h_\mu \right) = \{ T \exp - \int_0^t \gamma^{(n)}(\bar{g}^2, \bar{h}) dx \}_{ji} E_{j(k)}^{(n)}(1, \bar{g}^2, \bar{h}), \quad (8a)$$

$$E_{NS(k)}^{(n)} \left( \frac{Q^2}{\mu^2}; g_\mu^2, h_\mu \right) = \{ \exp - \int_0^t \gamma_{NS}^{(n)}(\bar{g}^2, \bar{h}) dx \} E_{NS(k)}^{(n)}(1, \bar{g}^2, \bar{h}), \quad (8b)$$

where T implies that the exponential is to be t-ordered. The large  $Q^2$  behaviour of  $E_{i(k)}^{(n)}$ ,  $E_{NS(k)}^{(n)}$  is governed by the large  $Q^2$  behaviour of effective coupling constants  $\bar{g}$  and  $\bar{h}$  which satisfy the following system of renormalization group equations

$$\frac{d\bar{g}^2(t, g_\mu^2, h_\mu)}{dt} = \beta_g(\bar{g}^2, \bar{h}), \quad \frac{d\bar{h}(t, g_\mu^2, h_\mu)}{dt} = \beta_h(\bar{g}^2, \bar{h}) \quad (9)$$

and

$$\bar{g}^2(0, g_\mu^2, h_\mu) = g_\mu^2, \quad \bar{h}(0, g_\mu^2, h_\mu) = h_\mu. \quad (10)$$

In the deep Euclidean region ( $Q^2 \geq \mu^2 \gg m^2$ ) the effective coupling constants are assumed to approach a limit cycle

$$\begin{aligned} \bar{g}(t, g_\mu, h_\mu) &\rightarrow g_0 \cos \omega t, \\ \bar{h}(t, g_\mu, h_\mu) &\rightarrow h_0 \sin \omega t, \end{aligned} \quad (11)$$

where  $g_0$ ,  $h_0$  and  $\omega$  are unknown parameters. A period of this limit cycle is given by  $\tau = 2\pi/\omega$ . We suppose also that

$$h_0 \sim g_0^2 \ll 1. \quad (12)$$

Then in our model the anomalous dimension matrix  $\gamma^{(n)}(g^2, \bar{h})$  for large  $Q^2$  takes the form

$$\gamma^{(n)} = \tilde{\gamma}^{(n)} \frac{g_0^2}{4\pi^2} \cos^2 \omega t + O(g_0^4) = \begin{pmatrix} n \gamma_{VV}^V & n \gamma_{\psi\psi}^V & n \gamma_{SS}^V \\ n \gamma_{VV}^\psi & n \gamma_{\psi\psi}^\psi & 0 \\ n \gamma_{VV}^S & 0 & n \gamma_{SS}^S \end{pmatrix} \frac{g_0^2}{4\pi^2} \cos^2 \omega t + O(g_0^4) \quad (13)$$

with the matrix elements <sup>/4.5.21/</sup>

$$n_{\gamma_{VV}}^V = \frac{1}{2} \left\{ \frac{11}{3} - \frac{12}{n(n-1)} - \frac{12}{(n+1)(n+2)} + 12 \sum_{j=2}^n \frac{1}{j} + \frac{m}{6} \right\},$$

$$n_{\gamma_{\psi\psi}}^V = \frac{2}{3} \frac{n^2 + n + 2}{n(n^2 - 1)},$$

$$n_{\gamma_{SS}}^V = \frac{4}{3} \frac{1}{n(n-1)},$$

$$n_{\gamma_{VV}}^{\psi} = 8 \frac{n^2 + n + 2}{n(n+1)(n+2)},$$

$$n_{\gamma_{\psi\psi}}^{\psi} = \gamma_{NS}^{(n)} = \frac{2}{3} \left[ 1 - \frac{2}{n(n+1)} + 4 \sum_{j=2}^n \frac{1}{j} \right],$$

$$n_{\gamma_{VV}}^S = 2 \frac{m}{(n+1)(n+2)},$$

$$n_{\gamma_{SS}}^S = \frac{8}{3} \sum_{j=2}^n \frac{1}{j}.$$

The perturbation expansion of  $\gamma^{(n)}(\bar{g}^2, \bar{h})$  (eq.(13)) has been received using the following condition:

$$\frac{n_{\gamma_{SS}}^S}{n_{\gamma_{SS}}^S} \operatorname{tg} \frac{\omega L}{2} \leq 1, \quad L = \ln \frac{Q^2}{\mu^2}. \quad (14)$$

Here  $n_{\gamma_{SS}}^S$  is the coefficient in front of the term  $\left(\frac{h_0}{16\pi^2}\right)^2 \sin^2 \frac{\omega L}{2}$  in the expansion of  $\gamma$  matrix and

$$n_{\gamma_{SS}}^S = \frac{1}{16} \left(1 - \frac{6}{n(n+1)}\right) (13m+17). \quad (15a)$$

In eqs. (13,15a)  $m$  is the number of the Higgs multiplets.

The solution of the system of equations (8a) in the limit of large  $Q^2$ , taking into account the explicit form (13) of  $\gamma^{(n)}$  is

$$E_{i(k)}^{(n)} \left( \frac{Q^2}{\mu^2}; g_{\mu}^2, h_{\mu} \right) = \sum_{\ell=1}^3 \exp \left\{ - \frac{\lambda_{\ell}^{(n)}}{4\omega} \frac{g_0^2}{4\pi^2} (\omega L + \sin \omega L) \right\} \times \\ \times \{ P_{\ell}^{(n)} (I + 0(g_0^4 L, \frac{g_0^4}{\omega}) \}_{i\psi} (1+0(g_0^2)) \\ (i = V, \psi, S; \quad k=1,2), \quad (15)$$

Where  $\lambda_{\ell}^{(n)}$  are eigenvalues of the matrix  $\tilde{\gamma}^{(n)T}$  ( $\tilde{\gamma}^T$  is a transposed  $\tilde{\gamma}$ )

$$\tilde{\gamma}^{(n)T} = \sum_{\ell=1}^3 \lambda_{\ell} P_{\ell}^{(n)} \quad (16)$$

and  $P_{\ell}^{(n)}$  are projection matrices

$$P_k^{(n)} P_{\ell}^{(n)} = \delta_{k\ell} P_{\ell}^{(n)}, \quad \sum_{\ell} P_{\ell} = I. \quad (17)$$



To obtain eq.(15) we have used also the perturbation theory expansions of the functions  $E_{i(k)}^{(n)}(1, \bar{g}^2, \bar{h})$

$$\begin{aligned} E_{V(k)}^{(n)}(1, \bar{g}^2, \bar{h}) &= 0(\bar{g}^{-2}), \\ E_{\psi(k)}^{(n)}(1, \bar{g}^2, \bar{h}) &= 1 + 0(\bar{g}^{-2}), \\ E_{S(k)}^{(n)}(1, \bar{g}^2, \bar{h}) &= 0(\bar{g}^4). \end{aligned} \quad (18)$$

Note that eq. (15) is true only for the  $Q^2 (L = \ln \frac{Q^2}{\mu^2})$  which are limited by the conditions

$$\begin{aligned} tg^2 \frac{\omega L}{2} < \frac{n \gamma_{SS}}{n \gamma'_{SS}}, \\ \frac{g_0^2}{4\pi^2} L < 1, \end{aligned} \quad (19)$$

and for  $\omega \geq g_0^2 / 4\pi^2$ .

For the non-singlet functions  $E_{NS(k)}^{(n)}(\frac{Q^2}{\mu^2}; g^2, h)$  which satisfy eq. (8b) we have

$$\begin{aligned} E_{NS(k)}^{(n)}(\frac{Q^2}{\mu^2}, g^2, h) &= \exp \left\{ -\frac{\gamma_{NS}^{(n)}}{4\omega} \frac{g_0^2}{4\pi^2} (\omega L + \sin \omega L) \right\} \times \\ &\times (1 + 0(g_0^4 L, \frac{g_0^4}{\omega})) (1 + 0(g^2)). \end{aligned} \quad (20)$$

So, in the region:  $Q^2 \geq \mu^2 \gg m^2$ ,  $Q^2 \gg M^2$  and limited by the conditions (19), we obtain the following results for the singlet  $M_{S(k)}^{(n)}$  and non-singlet  $M_{NS(k)}^{(n)}$  pieces of the moments of the structure functions:

$$M_{S(k)}^{(n)}(Q^2) = \langle e^2 \rangle \sum_{\ell} \sum_i \exp \left\{ -\frac{\lambda^{(n)}}{4\omega} \frac{g_0^2}{4\pi^2} (\omega L + \sin \omega L) \right\} \times$$

$$+ \sin \omega L \} c_i^{(n)}(\mu^2) (P_{\ell}^{(n)})_{i\psi} \times \left\{ 1 + 0(g_0^4 L, \frac{g_0^4}{\omega}) \right\} + 0(\frac{M^2}{Q^2}), \quad (21a)$$

$$\begin{aligned} M_{NS(k)}^{(n)}(Q^2) &= \sum_a (e_a^2 - \langle e^2 \rangle) c_a^{(n)}(\mu^2) \exp \left\{ -\frac{\gamma_{NS}^{(n)}}{4\omega} \frac{g_0^2}{4\pi^2} (\omega L + \sin \omega L) \right\} \times \\ &\times \left\{ 1 + 0(g_0^4 L, \frac{g_0^4}{\omega}) \right\} + 0(\frac{M^2}{Q^2}). \end{aligned} \quad (21b)$$

Therefore, in the models of consideration the Bjorken scaling is violated by power terms of  $Q^2$  multiplied by an oscillating function of  $Q^2$ . Note that the same results hold for neutrino hadron processes.

The momentum transfers  $Q^2$  for which expressions (21) for the moments hold, are limited by conditions (19). For instance, for  $m=2$ ,  $Q_{\max}^2 \sim 3 \times 10^3 \mu^2 (\text{GeV})^2$  when  $g_0^2 / 4\pi = 0.2$  and  $\omega = 0.2$ , while  $Q_{\max}^2 \sim 1.5 \times 10^2 \mu^2 (\text{GeV})^2$  when  $g_0^2 / 4\pi = 0.2$  and  $\omega = 0.3$ , where  $Q_{\max}^2$  are maximal values for which expressions (21) are still true. To consider the region of larger  $Q^2$  we should take into account the higher perturbation theory corrections to the expressions for  $\gamma^{(n)}$  and  $E_{i(k)}^{(n)}(1, \bar{g}^2, h)$  (eqs. (13) and (18) respectively). The deep inelastic lepton hadron scattering data available at present time are not able to distinguish between the logarithmic and power in  $Q^2$  violation of Bjorken scaling. And it is even more difficult to observe the oscillating factor, predicted by our model. The following ratio:

$$R^{(n)}(Q^2) = \frac{\int_0^1 dx x^{n-2} \{ F_1(x, Q^2) - F_2(x, Q^2) \}}{\int_0^1 dx x^{n-2} F_2(x, Q^2)} = h^{(n)} \frac{g^2}{g^2} (1 + 0(\bar{g}^2)) \quad (22)$$

( $h^{(n)}$  are some known constants) is crucial for distinguishing different models. In the fixed point models

$$R_1^{(n)} = h^{(n)} g_0^2 / 4\pi^2, \quad (23)$$

while in our model

$$R_2^{(n)} = h^{(n)} g_0^2 / 4\pi^2 \cos^2 \frac{\omega L}{2}. \quad (24)$$

The deviation

$$\Delta R = \frac{R_1^{(n)} - R_2^{(n)}}{R_1^{(n)}}$$

at  $Q^2 = 200 \text{ GeV}^2$  is of 10% for  $\omega = 0.2$  and of 19% for  $\omega = 0.3$  ( $g_0^2/4\pi = 0.2$  and  $\mu^2 = 10 \text{ GeV}^2$ ). This region for  $Q^2$  will be reached in the future deep inelastic  $\mu - N$  scattering experiments<sup>/22/</sup>. The oscillations, however, could be observed at very large  $Q^2$  only, which should not be available at this experiment.

#### SUMMARY

A class of strong interaction models, in which the interactions between fractionally charged colored quarks are mediated by massive vector gluons, is considered. In the latter all the vector gluons acquire masses via the Higgs mechanism, and all the scalar mesons are massive. The effective coupling constants  $\bar{g}$  and  $\bar{h}$  are assumed to approach a limit cycle in the deep Euclidean region. Deep inelastic lepton hadron scattering is analyzed using this hypothesis. It is shown that Bjorken scaling for the moments of the structure functions is violated by power terms of  $Q^2$  multiplied by an oscillating function of  $Q^2$  (eqs.(21). The  $Q^2$ , for which expressions (21) for the moments hold, are limited by conditions (19). The present deep inelastic lepton hadron scattering data are unable, however, to distinguish between our and other (asymptotically free and fixed

point) models. We hope that the future deep inelastic  $\mu - N$  scattering experiments should apply to the resolution of this problem.

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