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WEAK RADIATIVE DECAYS OF K_L-MESONS IN CHIRAL THEORY

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WEAK RADIATIVE DECAYS OF K_L-MESONS IN CHIRAL THEORY

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Слабые радиационные распады K_L мезонов в киральной теории

С помощью полюсных диаграмм с виртуальными π°- и η- мезонами радиационные распады K_L- мезонов выражаются через аналогичные распады η- мезонов, вычисленные в однопетлевом приближении киральной теории поля. Результаты согласуются с экспериментальными данными.

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Weak Radiative Decays of K_L-Mesons in Chiral Theory

Radiative decays of K_L -mesons are expressed in terms of analogous decays of η -mesons calculated in the one-loop approximation of chiral field theory. These decays are connected through the pole diagrams with virtual η and π° mesons. The results are consistent with experiment.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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In paper[1] the suppression of the probability of decay $\eta \to \pi^+ \pi^- \gamma$ by one order of magnitude as compared to that of $\eta \to \gamma\gamma$ is explained within the one-loop approximation of chiral quantum field theory. In this paper a similar consideration is applied to decays $K_L \to \pi^+\pi^-\gamma$ and $K_L \to \gamma\gamma$.

An attempt of the theoretical interpretation of decays $K_L \to \pi^+ \pi^- \gamma$ and $K_L \to \gamma\gamma$ has been undertaken in recent paper [2]. The author used the Weinberg-Salam model and derived some relations for probabilities of these decays. One of the results [2] says that for these decays of much importance are contributions of large distances where processes proceed through certain hadron states. The Weinberg-Salam model fails to explain such processes while the chiral theory is much more appropriate for this purpose [3].

In this paper we show that the chiral theory allows one to calculate the absolute values of widths of decays $K_L \to \pi^+\pi^-\gamma$ and $K_L \to \gamma\gamma$ by relating them to analogous decays of η mesons calculated in ref. [1].

The probabilities of K_L decays are expressed in terms of probabilities of similar decays of η mesons if one uses the pole diagrams of virtual transitions of K_L into η and π°-mesons (see ref. /4/ and Figs.1,2). A special note should be made here that the formula obtained in /1/ for the probability of decay $\eta \to \pi^+\pi^-\gamma$ is very sensitive to a negligible change in the mass of the decaying particle. For instance, $\Gamma_{(\eta\to\pi^+\pi^-\gamma)}$ decreases by a factor of three if the

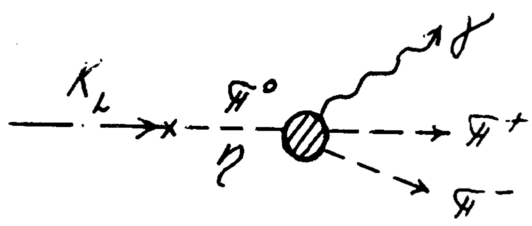

Fig. 1

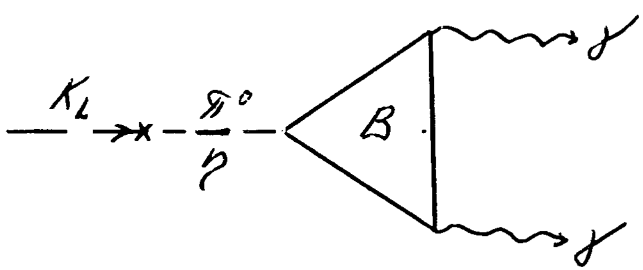

Fig. 2

mass of η-meson is changed by the mass close to that of K_L-meson.

To calculate the widths of decay $K_L \to \pi^+\pi^-\gamma$, it is necessary to consider the following Lagrangians. The Lagrangian of transitions of K_L-meson into π°, η_8, η_0 and η_c mesons (see /4/)

$$\mathfrak{L}^{(KL)} = a : K_L(\pi^\circ + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 - \sqrt{2}\eta_c): \qquad (1)$$

From our calculations it follows that the consideration of the pole diagrams with π° and η_8 mesons gives the satisfactory results for the probability of decays $K_L \to \gamma\gamma$ and $K_L \to \pi^+\pi^-\gamma$. The consideration of the pole diagrams with singlet and charmed mesons is less essential. We shall discuss these contributions at the end of this article.

The constant a can be evaluated either on the basis of chiral Lagragian satisfying the rule $|\Delta T|=1/2$ (see /5/), or from current algebra by using the relation between the amplitudes of transition $K_L \to \pi^\circ$ and $K_S \to \pi^\circ\pi^\circ$ /4,6/. This gives the value *

$$a \approx 4.5\times 10^{-2}\ (\mathrm{MeV})^2 . \qquad (2)$$

* The close value for a follows from consideration of an interaction of the type "current-current" with the universal weak coupling constant /7/

$$\mathfrak{L}_W = \sqrt{2}\, G F_\pi^2 : \partial_\mu K_L [\partial_\mu \pi^\circ + \frac{1}{\sqrt{3}} \partial_\mu \eta] :$$

The Lagrangians of strong meson-baryon interactions

$$\mathcal{L}_1 = 2g[\alpha d_{ijk} - i(1-\alpha) f_{ijk}]\bar{B}_i \gamma_5 B_j \Phi_k , \tag{3}$$

$$\mathcal{L}_2 = \frac{i}{2F_\pi^2}\bar{B}_i \gamma_\mu B_\ell \Phi_j \partial^\mu \Phi_k \{(g_A^2-1) f_{i\ell m} f_{kjm} + g_A^2 [\frac{2}{3}\alpha^2(\delta_{ij}\delta_{k\ell} - \delta_{ik}\delta_{j\ell}) + 2\alpha(\alpha-1) f_{kjm}(f_{i\ell m} - i d_{i\ell m})]\}, \tag{4}$$

were discussed in detail and applied to the description of decay $\eta \to \pi^+\pi^-\gamma$ in ref. /1/. In these formulae, B_i and Φ_i are the fields of the baryon and meson octets; g is the strong interaction constant, $g^2/4\pi \approx 14{,}7$; F_π is the constant of π meson decay $F_\pi = 92$ MeV; $g_A = 1.25$ is the renormalization constant of axial current; $\alpha \approx 2/3$ is the parameter of mixing of f and d coupling in SU(3) theory.

The electromagnetic interaction Lagrangian is

$$\mathcal{L}^{(A)} = -e:A_\mu[\bar{p}\gamma^\mu p - \bar{\Xi}^- \gamma^\mu \Xi^- + \bar{\Sigma}^+\gamma^\mu\Sigma^+ - \bar{\Sigma}^-\gamma^\mu\Sigma^-]: \tag{5}$$

By using the Lagrangians (4),(5),(3) in ref. /1/ the width of decay $\eta \to \pi^+\pi^-\gamma$ in the one-loop approximation was found in the form

$$\Gamma_{(\eta\to\pi^+\pi^-\gamma)} = \bar{\alpha}\left(\frac{g_A c_\eta}{3}\right)^2\left(\frac{m_\eta}{\sqrt{2}\,4\pi F_\pi}\right)^6\left[1-\left(\frac{2m_\pi}{m_\eta}\right)^2\right]^5 m_\eta \mathcal{J}_\eta \approx 47 \text{ eV}. \tag{6}$$

Here $\bar{\alpha} = 1/137$ is the electromagnetic coupling constant ($\bar{\alpha} = e^2/4\pi$),

$$C_\eta = 6\alpha - g_A^2[1+(2\alpha-1)^3] = 2.4 \tag{7}$$

is the SU(3) factor arising from the consideration of contributions from all one-loop baryon diagrams (see Fig.3), $\mathcal{J}_\eta$ is the phase integral

$$\mathcal{J}_\eta = \int_0^1 dx\, x^3(1-x)^{3/2}\left\{\left[1-\left(\frac{2m_\pi}{m_\eta}\right)^2\right]^{-1} - x\right\}^{-1/2} \approx 0.034. \tag{8}$$

The probability of decay $K_L \to \pi^+\pi^-\gamma$ is defined by two diagrams of Fig.1. The contributions of these

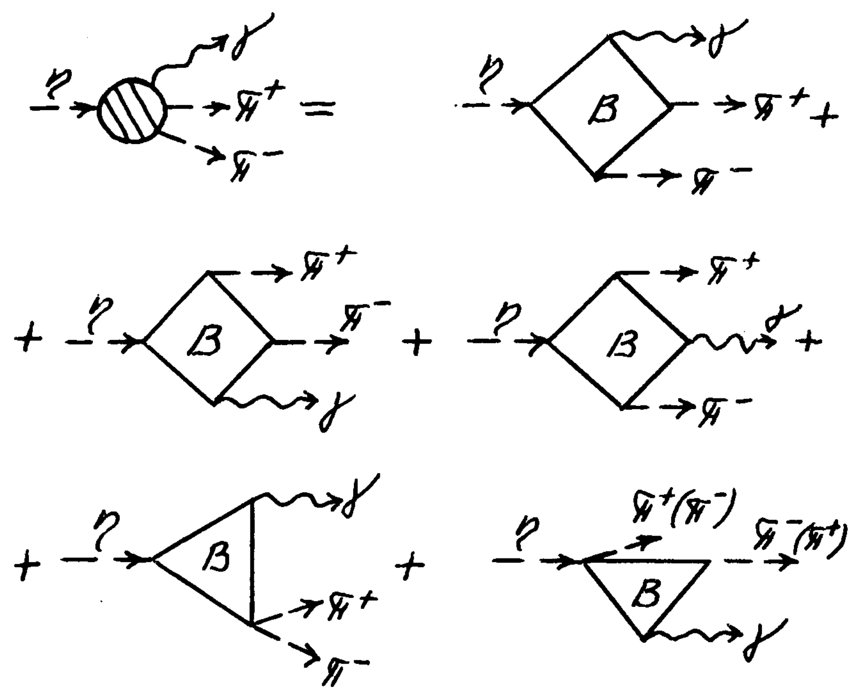

Fig. 3.

diagrams to the decay width can be easily calculated by using the Lagrangian (1),(3)-(5) and the width of decay $\eta\to\pi^+\pi^-\gamma$ (6). The result is

$$\Gamma_{(K_L\to\pi^+\pi^-\gamma)} = \frac{a^2}{3m^4_{K_L}}\left[\left(\frac{m^2_\eta}{m^2_{K_L}}-1\right)^{-1} - 3\left(1-\frac{m^2_\pi}{m^2_{K_L}}\right)^{-1}\right]^2 \Gamma^{(m_\eta\to m_{K_L})}_{(\eta\to\pi^+\pi^-\gamma)} . \qquad (9)$$

$\Gamma^{(m_\eta\to m_{K_L})}_{(\eta\to\pi^+\pi^-\gamma)}$ is the width of the decay of η meson into $\pi^+\pi^-\gamma$ (formula (6)) but with the mass of the decaying particle equal to m_{K_L} instead of m_η. It turns out that

$$\Gamma^{(m_\eta\to m_{K_L})}_{(\eta\to\pi^+\pi^-\gamma)} \approx 16\ \mathrm{eV}, \qquad (10)$$

i.e. it decreases by a factor of three as compared to (6). And finally, we get the following estimate of the probability of decay $K_L\to\pi^+\pi^-\gamma$

$$W_{(K_L\to\pi^+\pi^-\gamma)} = 0.6\times 10^3\,\mathrm{sec}^{-1}$$

while the experimental data /8/ are

$$W^{(exp)}_{(K_L\to\pi^+\pi^-\gamma)} = 1.2\times 10^3\,\mathrm{sec}^{-1} .$$

Since the calculation of amplitude in chiral theory is accurate within 20-30%, the agreement with experiment can be considered to be satisfactory.

Now we briefly discuss what gives the consideration of pole diagrams with η_0 and η_c mesons. The consideration of $\eta_0(\eta')$ meson results in the appearance of a new term

$$\Delta = \sqrt{6}\,\frac{C_{\eta'}}{C_\eta}\,\frac{g_{\eta' pp}}{g_{\pi^\circ pp}}\left(\frac{m^2_{\eta'}}{m^2_{K_L}}-1\right)^{-1} \qquad (11)$$

in the brackets in formula (9). Here $C_{\eta'}\approx 4.8$ is the SU(3) factor for η' meson, and $g_{\eta' pp}$ is the coupling constant of η'-meson-proton strong interaction. The decay $\eta'\to\gamma\gamma$ gives the estimate $g_{\eta' pp}\approx 0.5\, g_{\pi^\circ pp}$. Thus, the term (11) equals $\Delta\sim 0.87$.

After the addition of this term the value of the probability of the $K_L\to\pi^+\pi^-\gamma$ decay increases and better fits the experimental value, $\overline{W}_{(K_L\to\pi^+\pi^-\gamma)}\approx 1.3\ \mathrm{sec}^{-1}$. However as far as our estimates are very rough we notice only that the contribution of η'-meson is less than the corresponding contributions of η and π° mesons, and the addition of it can improve, in certain extent, the previous result. The contribution of the heavy η^c-meson is not essential that could be seen from eq. (9).

We conclude this note with the one-loop approximation formula obtained for $K_L\to\gamma\gamma$ (see also ref. /4/ and Fig. 2)

$$\Gamma_{(K_L\to\gamma\gamma)} = \frac{a^2}{3m^4_{K_L}}\left[\left(\frac{m^2_\eta}{m^2_{K_L}}-1\right)^{-1} - 3\left(1-\frac{m^2_\pi}{m^2_{K_L}}\right)^{-1}\right]^2\left(\frac{m_{K_L}}{m_\eta}\right)^3\Gamma_{(\eta\to\gamma\gamma)},$$

where

$$\Gamma_{(\eta\to\gamma\gamma)} = \frac{m_\eta}{3\pi}\left(\frac{\bar{a}\, g_A\, a\, m_\eta}{4\pi F_\pi}\right)^2$$

is the width of decay $\eta \to \gamma\gamma$ found in the one-loop approximation. Then the result for the probability of decay $K_L \to \gamma\gamma$ is

$$W_{(K_L \to \gamma\gamma)} = 11 \times 10^3 \ \mathrm{sec}^{-1} ,$$

while experiment gives /8/

$$W^{(exp)}_{(K_L \to \gamma\gamma)} = 9.5 \times 10^3 \ \mathrm{sec}^{-1} .$$

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REFERENCES

1. Volkov M.K. Yadern. Fiz., 1978, 27, p.758.
2. Malakian Yy.P., Yadern Fiz., 1977, 26, p.634.
3. Volkov M.K., Pervushin V.N. UFN, 1976, v.120, p.363.
4. Gaillard M.K., Lee B.W. Phys.Rev., 1974, D10, p.897.
5. Volkov M.K., Pervushin V.N. Phys.Lett., 1974, 51B, p.499.
6. Adler S.L., Dashen R.F. Current Algebra, Benjamin, New York 1968.
7. Kalinovski Yu.L., Pervushin V.N. JINR, P2-10175, Dubna, 1976.
8. Rev. of Mod.Phys., 48, No.2, part II (1976).

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