ОБЪЕДИНЕННЫЙ
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E2 - 11179

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REMARKS CONCERNING NONRELATIVISTIC LORENTZ-DIRAC EQUATION

1978

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Submitted to ЯФ

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О нерелятивистском уравнении Лоренца-Дирака

Анализируются причины возникновения нефизического решения нерелятивистского уравнения Лоренца-Дирака. Показано, что одной из причин являются так называемые обратимые потери на излучение. Источником этих потерь является бесконечная масса точечного электрона. Качественно исследуется уравнение Лоренца-Дирака для одномерного кулоновского случая. Показано, что причиной возникновения нефизических ситуаций является не сингулярность кулоновского внешнего источника, а точечность электрона. Неточечность электрона учитывается по методу Герглотца, который в случае осцилляторного внешнего поля допускает точное решение. Учет неточечности электрона устраняет нефизические решения. Обсуждаются свойства получаемых физических решений.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

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Remarks Concerning Nonrelativistic Lorentz-Dirac Equation

Reasons for the nonphysical solutions of the nonrelativistic Lorentz-Dirac equation are examined. It is shown that these are due to the reversible radiation energy losses. The source of those losses is the infinite mass of the point electron. The Lorentz-Dirac one-dimensional equation for the Coulomb external force is analysed. It is shown the nonphysical character of the solutions obtained is due to the point electron structure, but not to the singularity of the external Coulomb force. The extension of the electron is taken into account using the well-known Herglotz method, which for the oscillator force allows the analytical solution. There are only physical damping solutions in this case. For special values of the oscillator constant the stationary radiationless motions are possible. In this case the energy becomes an approximate integral of the motion and takes the discrete values.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1978

1. We want to analyse a somewhat controversial situation with the Lorentz-Dirac nonrelativistic equation. The matter is that this equation can be solved explicitly only for quite limited cases which have physical meaning. First, it is the motion of the free charged particle. In this case the physical solution (corresponding to the motion of the charged particle with the constant velocity) is coexisting with the unphysical one (which corresponds to the motion with selfacceleration). Second, it is uniformly accelerated motion in the constant field. Third, it is the motion with increasing or decreasing (in time) amplitude in the field of the harmonic oscillator. The most interesting and controversial is the particle motion in the Coulomb external field. It was proved by C.Eliezer analytically in refs. [1,2] that for the one-dimensional motion (more exactly for the motion with the zero angular momentum) in a field of the Coulomb center the physically unreasonable situation always takes place. For example, let at the initial moment the particle be placed at distance r_0 from the attractive Coulomb center and let the initial velocity be directed towards this center. It was shown in the same refs. [1,2] that the particle initially approaches to the center at the finite distance, then stops and goes to infinity. For the Coulomb repulsion with the velocity off the center the situation is opposite: the charged particle firstly goes away from center, at finite distance stops and then falls on it. In this case there are no physically reasonable solutions.

This is in contrast to the case mentioned above, where the physical solutions coexisted side by side with the nonphysical ones.

The two-dimensional Coulomb case was also treated in references cited above. Intuitively we expect that the charged particle should rotate around the attractive Coulomb center with constantly decreasing (due to the radiation) radius. However C.Eliezer prooved the existence of the solutions, corresponding to the departure of the particle infinity for the attractive case and to the continuous falling at the center for the repulsive case. These results were criticized by G.Plass/3/ and P.Clavier/4/. At first they noted that although C.Eliezer has found solutions with mentioned above properties, he has not prooved (for the two-dimensional case) their uniqueness. P.Clavier has been able to show the existence (for $r \to 0$) of the solutions corresponding to the charged particle motion along the spiral shrinked towards the center. G.Plass calculated numerically a very small part of the particle trajectory and found that it goes inside the radiationless ellipse. From this he concluded (although with some reservations) that this solution goes asymptotically (for $t \to \infty$) to the class solutions found by P.Clavier. (At this point we note that there are no guarantee that particle being catched once inside the radiationless ellipse, does not leave it later). The results, obtained by C.Eliezer for the one-dimensional Coulomb case were rejected by P.Clavier and G.Plass on those grounds that this case has no physical meaning and that singularity $\frac{1}{r}$ is too strong for the one dimensional case. So they suggested to find the physical reasonable solutions of the Lorentz-Dirac equation on the class of the generalized functions. We believe, however, that this argumentation is not very convincing because there are physically reasonable solutions on the class of the usual functions for the radiationless motion with zero angular momentum. Finally, we mention the

recent numerical solutions of the Lorentz-Dirac equation for the Coulomb one-dimensional case /5/. These calculations support the analysis given by C.Eliezer. Also they present an argument for working with the usual functions. In fact, one may change the singular Coulomb potential $\frac{1}{r}$ by the potential without singularity: $\frac{1}{\sqrt{r^2+a^2}}$. As the numerical calculations show that above-mentioned turning point is on the finite distance from the Coulomb center then substitution $\frac{1}{r} \to \frac{1}{\sqrt{r^2+a^2}}$ does not change the results of the numerical calculation if parameter a is sufficiently small. So, the nonphysical character of the solutions is not due to the irregularity of the Coulomb potential.

2. Here we give an analysis of the Lorentz-Dirac equation for the Coulomb case.

$$m_0 \frac{d^2 x_i}{dt^2} - \frac{\gamma x_i}{r^3} = \frac{2}{3} \frac{e^2}{c^3} \frac{d^3 x_i}{dt^3}. \tag{2.1}$$

Multiplying (2.1) by $\frac{dx_i}{dt}$, summing and transforming the right-hand side of (2.1) one has

$$\frac{d}{dt}\left[\frac{1}{2} m_0 \left(\frac{dx_i}{dt}\right)^2 + \frac{\gamma}{r} - \frac{2}{3} \frac{e^2}{c^3} \frac{d^2 x_i}{dt^2} \cdot \frac{dx_i}{dt}\right] = -\frac{2}{3} \frac{e^2}{c^3} \left(\frac{d^2 x_i}{dt^2}\right)^2 < 0. \tag{2.2}$$

The right-hand side of (22) corresponds to the so-called irreversible radiation losses, whereas the last term at the left-hand side corresponds to the reversible losses/6/. The origin of these terms is due to the radial dependence of the energy flux produced by the accelerated charge/7,8/. In fact, the energy flux contains three parts. First there are terms which depend only upon the charge velocity. They are exactly the same as for the uniformly moving charge. It is known that such charge carries the electromagnetic field with itself. The disturbance of the electromagnetic field caused by the charge motion is called sometimes as electromagnetic "furrow". Second, there are terms which contain both the velocity and acceleration of charge. These are needed to rebuild the electromagnetic furrow from one velocity to another. Both of these terms decreasing sufficiently rapidly do not contribute to the energy flux at infinity. So, the energy flux produced by these term is contained in a finite region of space, surrounding the moving charge. At last, the energy flux contains the terms which depend only upon the charge acceleration. Being integrated over the sphere of the infinite radius they do not disappear. So, these terms are responsible for the energy flux to infinity and they give rise to the origin of the right-hand side of eq. (2.2).

Forget for the moment about the terms containing the acceleration in eq. (2.2). Then we have for one-dimensional Coulomb repulsive case:

$$\frac{d}{dt}\left[\frac{1}{2}m_0\left(\frac{dr}{dt}\right)^2 + \frac{\gamma}{r}\right] = 0. \qquad (2.3)$$

If at the initial time the electron has velocity directed towards the Coulomb center, then the motion takes place in the following way. At first, the distance of the electron from the Coulomb center is decreasing. This results in increasing the potential energy $\frac{\gamma}{r}$. But the expression in square brackets has the constant value, so the kinetic energy is decreasing and the electron is slowing down. At some moment it stops, and then goes to infinity with the velocity growing up to the value $\sqrt{\frac{2E}{m_0}}$. Now include the right-hand side of the (2.2):

$$\frac{d}{dt}\left[\frac{1}{2}m_0\left(\frac{dr}{dt}\right)^2 + \frac{\gamma}{r}\right] = -\frac{2}{3}\frac{e^2}{c^3}w^2 < 0. \qquad (2.4)$$

Initial conditions being the same as in the previous case, the motion has also same qualitative features. Due to the negativity of the right-hand side of eq. (2.4) the electron stops at an earlier time and at a greater distance from the Coulomb center than in the previous case. There are no anomalies in this case and it is possible to show their absence also for the attractive case. So, nonphysical behaviour of the solutions is only due to the term corresponding to the reversible energy losses. Or, in other words: rebuilding of the electromagnetic furrow requires too much energy and this exceeds the energy losses due to the radiation. There is the obvious reservoir of energy: the infinite electromagnetic mass of the point charge.

3. Here we consider the exactly soluble motion of the charged particle in the field of one and two-dimensional oscillator. The same physically unreasonable situations are present in this case but there is possibility for the analytic analysis of the solutions obtained. Nonrelativistic Lorentz-Dirac equation has the following form in this case:

$$m_0\frac{d^2x}{dt^2} - \frac{2}{3}\frac{e^2}{c^3}\frac{d^3x}{dt^3} + kx = 0. \qquad (3.1)$$

It is convenient to measure distance in units of the classical electron radius $r_0 = \frac{e^2}{m_0c^2}$, time in units of time needed for the light to cover the distance equal to electron radius $r_0 = \frac{e^2}{m_0c^3}$, velocity in units of the light velocity c, etc. Then we have

$$\frac{d^2x}{dt^2} - \frac{2}{3}\frac{d^3x}{dt^3} + \gamma x = 0.$$

Where γ is dimensionless constant equal to the ratio of the oscillator constant to that of radiation reaction:

$$\gamma = k \cdot \left(\frac{e^2}{m_o c^3}\right)^2 .$$

The solution of eq. (3.1) is a combination of three exponents:

$$x(t) = A_1 \cdot \exp(\alpha_1 t) + A_2 \cdot \exp(-\alpha_2 t) \cdot \cos \omega t + A_3 \cdot \exp(-\alpha_3 t) \cdot \sin \omega t, \tag{3.2}$$

where

$$\alpha_1 = \beta_1 + \beta_2, \quad \alpha_2 = \frac{\beta_1 + \beta_2 - 1}{2}, \quad \omega = \frac{\sqrt{3}}{2}(\beta_1 - \beta_2),$$

$$\beta_{1,2} = \left[\frac{\gamma}{2} + \frac{1}{8} \pm \sqrt{\frac{\gamma}{2}\left(\frac{\gamma}{2} + \frac{1}{4}\right)}\right]^{1/3} \qquad (\alpha_1, \alpha_2 > 0).$$

The coefficients A_1, A_2, A_3 are determined by the initial coordinates x_o, v_o, w_o

$$A_1 = \frac{w_o + x_o(\alpha_2^2 + \omega^2) + 2\alpha_2 v_o}{(\alpha_1 + \alpha_2) + \omega^2}, \quad A_2 = x_o - A_1,$$

$$A_3 = \frac{v_o + \alpha_2 x_o - A_1(\alpha_1 + \alpha_2)}{\omega} .$$

It is clear that in general case the charge particle moves away from the center of force (located at $x = 0$). Now consider how the motion depends upon the relation between A_1, A_2, A_3. For A_1 sufficiently large the charged particle gives away from the origin to infinity at once. For smaller values of A_1 the interference between increasing exponents takes place: particle initially approaches the origin, stops and then goes awy to infinity (fig. 1). For still smaller values of A_1 the particle oscillates with decreasing in time amplitude, then the growing exponent begins to play, after that particle goes to

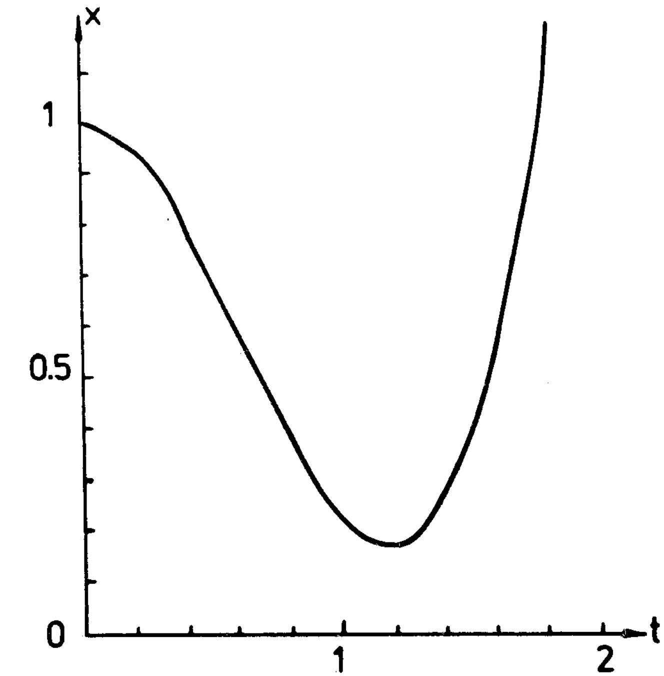

Fig. 1. One-dimensional motion of the charged radiating particle which is attracted to the origin with the force proportional to the first degree of the distance $(F = -kx)$. In this figure and in the following ones the distance is measured in units of the classical electron radius $r_o = \frac{e^2}{m_o c^2}$; time, in units $\frac{e^2}{m_o c^3}$, etc. The particle approaches the force center, then stops and goes to infinity.

infinity (fig. 2). At last for $A_1 = 0$ the particle oscillates around the origin with a decreasing in time amplitude. The period of the oscillation is determined by the constant γ. So, for one-dimensional case the situation in the oscillator and Coulomb cases is very similar.

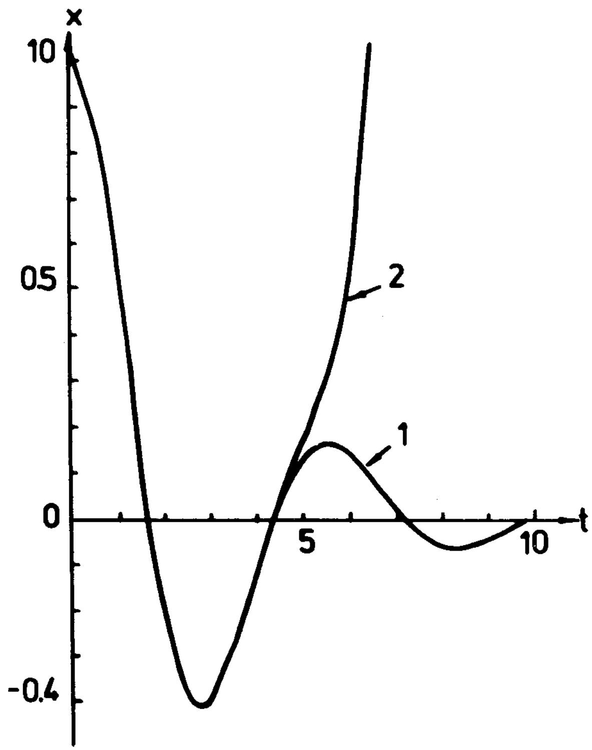

Fig. 2. The same as in fig. 1 but for somewhat different initial conditions. The curve 1 corresponds to the highly specific initial conditions (see eq. (3.4)). For any other choice of them particle goes away from the force center, although it may oscillate near it for a very long (though finite) time. The curve 2 illustrates this.

Now we consider the motion in plane. For nonzero values of coefficients A_{1x}, A_{1y} the particle goes to infinity at once or after several circulations around the origin with decreasing radius (fig. 3). For $A_{1x}=0, A_{1y}=0$ the charge particle is spiralling

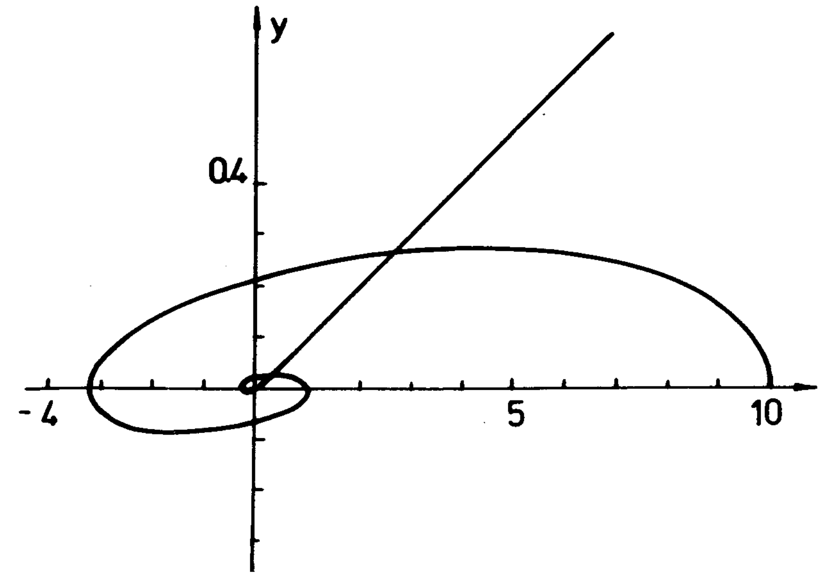

Fig. 3. Two-dimensional motion of the charged radiating particle in the field of central attractive force $\vec{F} = -k\vec{r}$. The particle once being catched inside the radiationless ellipse, initially approaches the force center along the shrinked spiral. At some moment the character of motion is changed and particle goes to infinity.

around the center with decreasing in time radius. So, the situation mentioned in section 1 is realised: the particle being catched inside the radiationless ellipse, approaches the origin spiralling, and then goes to infinity. It is highly probable that solution found by P.Clavier for the Coulomb case corresponds to the oscillator solution with $A_1=0$ and does not present the analytical continuation of the numerical solution obtained by G.Plass. So we believe that the following situation is valid for the charge particle motion in an arbitrary central potential: for an arbitrary $\vec{r}_0, \vec{v}_0, \vec{w}_0$ a particle goes away to infinity; for very specific choice of the initial conditions particle remains in a finite region space near the origin.

To complete the discussion, we analyse the case of the repulsive oscillator potential, i.e.,

$$V = -\frac{kx^2}{2}.$$

In the absence of the radiation, the particle goes to infinity without complications. For example, if energy $E>0$ (and $x_0>0$), then the particle goes to the left or right infinity (depending on the sign of v_0) at once. If $E<0$ (and $x_0>0$), then particle always goes to right infinity: at once (if $v_0>0$) or after passing the turning point $x_1=\sqrt{x_0^2-\frac{k}{m}v_0^2}$(if $v_0<0$).

Now include the radiation. If $\gamma\left(\equiv k\cdot\left(\frac{e^2}{m_0c^3}\right)^2\right)$ is greater than 1/2 the solution has the form similar to (3.2):

$$x(t) = A_1\cdot\exp(-\alpha_1 t) + \exp(\alpha_2 t)\cdot(A_2\cdot\cos\omega t + A_3\cdot\sin\omega t),$$

$$\alpha_1 = \beta_1+\beta_2-\frac{1}{2}, \quad \alpha_2 = \frac{1+\beta_1+\beta_2}{2}, \quad \omega = \frac{\sqrt{3}}{2}(\beta_1-\beta_2),$$

$$\beta_{1,2} = \left[\frac{\gamma}{2}-\frac{1}{8}\pm\sqrt{\frac{\gamma}{2}\left(\frac{\gamma}{2}-\frac{1}{4}\right)}\right]^{1/3} \quad (\alpha_1, \alpha_2 > 0).$$

In this case the solution is presented as a combination of continuously decreasing exponent and of the increasing exponent with the periodical factor in front of the latter one. This means that the velocity of particle periodically vanishes and that direction of motion reverses at these moments (fig. 4).

For $0<\gamma<\frac{1}{2}$ the situation is slightly complicated. All three roots of the cubic equation are real. The solution has the form:

$$x(t) = A_1\cdot\exp(\alpha_1 t) + A_2\cdot\exp(\alpha_2 t) + A_3\cdot\exp(-\alpha_3 t),$$

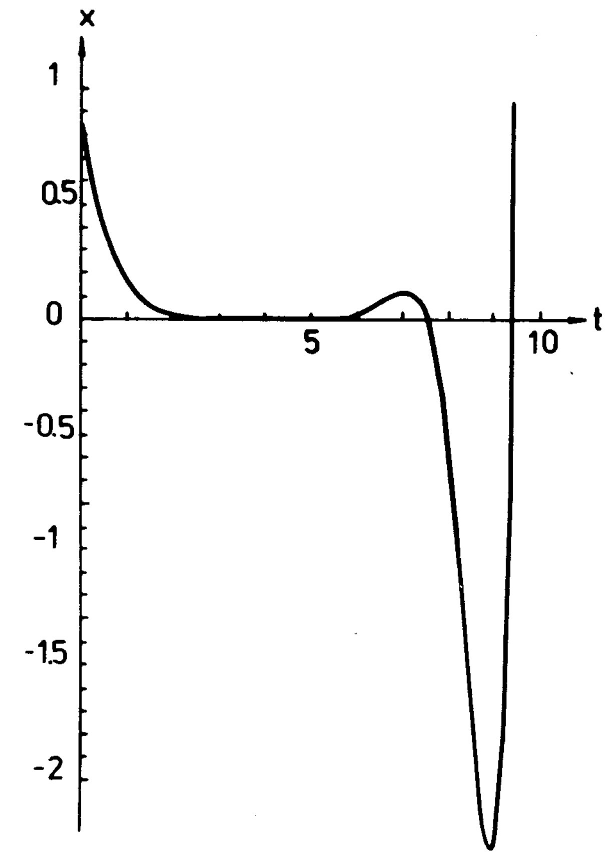

Fig. 4. One-dimensional motion of the charged radiating particle in the field of repulsive force $F=kx$ for sufficiently large values of the constant k. For the chosen initial conditions particle approaches the force center where it may stay for a very long (though finite) time. After that particle begins to oscillate around the origin with increasing amplitude.

where

$$a_1 = \frac{1}{2} + \cos\frac{\phi}{3}, \quad a_2 = \frac{1}{2} - \frac{1}{2}\left(\cos\frac{\phi}{3} - \sqrt{3}\,\sin\frac{\phi}{3}\right),$$

$$a_3 = -\frac{1}{2} + \frac{1}{2}\left(\cos\frac{\phi}{3} + \sqrt{3}\,\sin\frac{\phi}{3}\right), \quad \mathrm{tg}\phi = \frac{\sqrt{\frac{\gamma}{2}\left(\frac{1}{4} - \frac{\gamma}{2}\right)}}{\frac{1}{8} - \frac{\gamma}{2}}.$$

The solution now is a combination of two increasing exponents and one decreasing. Coefficients A_1, A_2, A_3 are determined by the initial condition. If the coefficient in front of the major exponent is large enough, then the particle goes to infinity at one. For smaller values the particle initially goes to one side, then stops and goes to infinity (fig. 5).

These difficulties of the point electron theory which are the same both for the singular and non-singular potentials give a credit to the consideration of an extended model of the electron.

4. G.Herglotz[9] for the case of the uniformly charged electron obtained following equation which takes into account the selfinteraction of the electron:

$$m_{mech}\cdot\frac{d^2x_i}{dt^2} + \frac{24e^2}{ac^2}\sum_{n=0}^{\infty}\frac{(-2a)^n}{(n+2)(n+3)(n+5)\cdot n!}\,\frac{1}{c^n}\,\frac{d^{n+2}x_i}{dt^{n+2}} = F_i, \tag{4.1}$$

where F_i is the external force, m_{mech} is the mass of the nonelectromagnetic origin. The first two terms of the sum in (4.1) are giving the electron acceleration (with coefficient equal to 4/3 of the electromagnetic mass) and the usual Lorentz selfaction of the electron. For the oscillator external force $F_i = k\cdot x_i$ and (4.1) reduces to the linear equation. As earlier, try to find the solution in an exponential form:

$$x = A\cdot\exp(-\omega t). \tag{4.2}$$

Then we have the following characteristic equation for ω:

$$m_{mech}\cdot\omega^2 + \frac{24e^2}{ac^2}\,\omega^2\sum_{n=0}^{\infty}\left(\frac{2\omega a}{c}\right)^n\frac{1}{(n+2)(n+3)(n+5)\,n!} + k = 0. \tag{4.3}$$

The summation in (4.3) may be performed in a finite form[9,10] Putting

$$\Lambda^2\sum\Lambda^n\frac{1}{(n+2)(n+3)(n+5)\,n!} = \phi(\Lambda) \qquad \left(\Lambda = \frac{2a\omega}{c}\right)$$

one has

$$\phi(\Lambda) = \left(\frac{1}{\Lambda} - \frac{4}{\Lambda^2} + \frac{4}{\Lambda^3}\right)e^{\omega} + \frac{1}{3} + \frac{1}{\Lambda} - \frac{4}{\Lambda^3}. \tag{4.4}$$

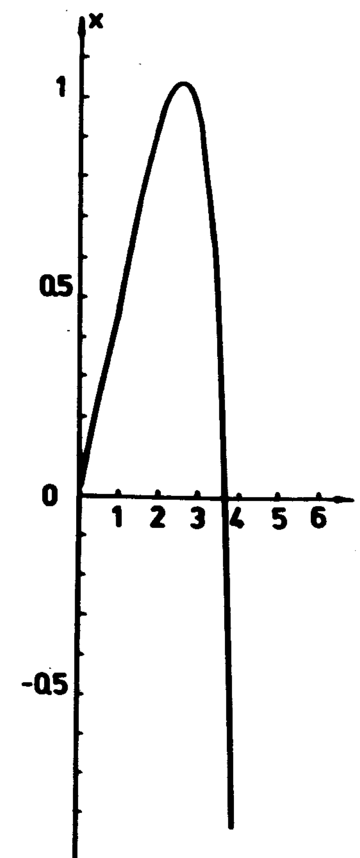

Fig. 5. The same as in fig. 4 but for relatively small values of the constant k. The particle goes away from the repulsive center, stops, goes back to the origin, passes it and finally goes to infinity.

Then permissible values of ω could be found from the following transcendental equation

$$\frac{m_{mech}\cdot c^2\cdot\Lambda^2}{4a^2}+\frac{6e^2}{a^3}\phi(\Lambda)+k=0. \qquad (4.5)$$

The properties of the function $\phi(x)$ were studied in the mentioned above paper of G.Herglotz who considered the free motion of the electron. He also assumed that the electron has the pure electromagnetic mass. In this case $m_{mech}=0$ and (4.5) reduces to:

$$\phi(\Lambda)=0.$$

Herglotz has been able to show that $\phi(\Lambda)$ has zeros only with a positive real part that corresponds to solution of (4.2) with decreasing in time amplitude. The case of the free motion with $m_{mech}\neq 0$ was studied by H.Steinwadel /10/ and K.Wildermith /11/. They prooved that selfaccelerating nonphysical solutions exist only in the case $m_{mech}<0$. Turning back to the motion of the extended electron in the attractive oscillator field one notes that due to the positivity of the oscillator constant k all the conclusions of /9-11/ for free motion are valid also for the treated case. So the complete equation (4.3) has an infinite number of physical solutions though its shortened version (i.e., nonrelativistic Lorentz-Dirac equation) has only nonphysical ones. Here we want to pay time to the pioneering works of M.Markov /12/ and D.Bohm /13/ who nearly 30 years ago stated that the correct equation, should possess only physical solutions.

Except these solutions corresponding to the oscillations with decreasing amplitude there are solutions corresponding to the motion along the stationary orbits. In this case $\mathrm{Re}\,\omega=0$. Putting $\omega=iy$ in (4.3) we have the following equations for real and imaginary parts:

$$\operatorname{tg} y = \frac{4y}{4-y^2}, \qquad (4.6a)$$

$$k=\frac{m_{mech}\cdot c^2\cdot y^2}{4a^2}-\frac{2e^2}{a^3}. \qquad (4.6b)$$

Equation (4.6a) defines the discrete spectrum of frequencies ω_n. Equation (4.6b) tells us that the stationary motion is possible only for the discrete values of the oscillator constant k:

$$k_n=\frac{m_{mech}\cdot c^2\cdot\omega^2}{4a^2}-\frac{2e^2}{a^3}. \qquad (4.7)$$

Equation (4.6a) has infinite number of zeros. For large values of y they are forming the equidistant spectrum

$$y_n\approx n\cdot\pi \qquad \text{for} \qquad n\gg 1. \qquad (4.8)$$

For such n the second term of the right-hand side of (4.7) can be neglected and we have:

$$k_n\approx\frac{m_{mech}\cdot c^2\cdot n^2\cdot\pi^2}{a^2}. \qquad (4.9)$$

Despite of the fact that energy losses its sense (as the motion integral) for the motion defined by eq. (4.1) (the same is true for the Lorentz-Dirac equation) the energy has well defined discrete values for y_n, k_n given by the (4.8), (4.9):

$$E_n=\frac{mx^2}{2}+\frac{kx^2}{2}\sim\frac{m_{mech}\cdot c^2\cdot n^2\cdot\pi^2}{4a^2}.$$

So, under definite conditions radiating charged particle can move along the stationary orbits. This means that irreversible energy losses are not reduced to

$$-\frac{2}{3}\frac{e^2}{c^3}w^2$$

as for Lorentz-Dirac equation, but they are represented as a more complicated expression. For the y_n, k_n satisfying (4.8), (4.9) these losses are negligible. Due to the finite dimensions of the electron the radiated energy has a chance to be absorbed before it leaves the oscillator region.

5. Here we want briefly to review the present situation with the Lorentz-Dirac type equation. There are many attempts to substitute another equation for the Lorentz-Dirac equation. As an example, mention Mo-Papas equation [14] in which the radiative effects are treated phenomenologically through the term containing the products of the four-acceleration vector and the external force one, the Bonnor [15] equation, where the loss of the radiation is due to the reduction of the proper mass of the particle, the Herrera equation [16], which contains terms quadratic in external forces, etc. It is possible to write many similar equations, but they must have as a limiting case the Lorentz-Dirac equation. The interesting attempts in this direction are the papers [7-19]. In ref. [17] it was suggested to substitute the finite-difference equation for the Lorentz-Dirac equation: for the motion of the free particle this equation contains, as in our case, radiationless motion. This idea was developed further in refs. [18,19]. To this end we mention the series of papers of E.Monit and D.Sharp [20-22] who also considered the extended model of the electron. These authors have shown, that for the forces which do not depend upon coordinates the selfaccelerated motions are absent. So, the content of the present paper is in a line with refs. [17-22] but emphasize is made on the physical presentation using exactly soluble model.

We are very thankful to Prof. N.A.Chermikov and Drs. E.A.Tagirov, S.Iljin for the useful discussions.

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Received by Publishing Department on December 21, 1977.