СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2 - 11160

B. Pontecorvo

LEPTON CHARGES AND LEPTON MIXING

1978

E2 - 11160

B. Pontecorvo

LEPTON CHARGES AND LEPTON MIXING

A rapporteur talk presented at the Conference on High Energy Physics, Budapest, July, 1977.

Понтекорво Б. E2-11160

Лептонные числа и лептонное смешивание

Дается обзор теоретических и экспериментальных исследований по лептонным числам и лептонному смешиванию известных автору к моменту проведения Международной конференции по физике высоких энергий (Будапешт, июнь 1977).

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1978

Pontecorvo B. E2-11160

Lepton Charges and Lepton Mixing

A review is given of theoretical and experimental investigations of lepton charges and lepton mixing known to the author at the time of the Budapest Conference, July 1977. The following points are being discussed. Experimental limits on probabilities of processes violating the conservation of lepton numbers, neutrino stability; lepton mixing with two types of neutrinos: Majorana and Dirac neutrinos, the $\mu\to e\gamma$ decay, neutrino oscillations; lepton mixing when there are $N \gtrsim 2$ neutrino tapes; the Brookhaven solar neutrino experiment and the "solar neutrino puzzle"; heavy lepton mixing: the $\mu\to e\gamma$ decay in a special model, parity nonconservation in heavy atoms, the $\mu\to 3e$ decay, relation between neutrino oscillations and the $\mu\to e\gamma$ decay. An extensive list of theoretical papers wherein there are obtained "large" probabilities of processes like $\mu\to e\gamma$ is presented, the papers being classified in 4 groups.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1978

I. INTRODUCTION

When the question arose at the end of 1976 about giving a talk on lepton charges and on its contradictory notion, lepton mixing, at the Budapest EPS Conference, the situation was quite different from what it is now. The number of relevant papers, experimental as well as theoretical, was then very limited. We (wrongly) believed that theoretical work on possible neutrino oscillations and rare decay processes such as $\mu\to e\gamma$, etc., was conducted mainly at Dubna. However, during the first part of this year there was a tremendous boom on lepton mixing. Neutrino oscillations and processes like $\mu\to e\gamma$, etc., became quite fashionable (things which might not exist can be fashionalbe alright!). Thus, I am entirely unable now to give a full account of the very large number of theoretical papers on lepton mixing; I am too old for that, not to say that many of these papers are a too highbrow stuff for me. The interest towards the hypothesis of lepton mixing is due to the fact that mixing arises quite naturally in modern theory. Most of the recent theoretical papers on lepton charge nonconservatiion, however, were triggered by unfounded rumours that at SIN in Zurich the decay $\mu\to e\gamma$ had been observed with a branching ratio

$$R_{\mu\to e\gamma} = \frac{\Gamma_{\mu^+\to e^+\gamma}}{\Gamma_{\mu^+\to e^+\nu_e\bar{\nu}_\mu}} \sim 10^{-9} .$$

As it happens, rumours may be useful. Our theoretical work in Dubna has a different origin and I am going to say a few words about it. On one hand, in our Institute S.Korenchenko and his colleagues were searching experimentally for processes like $\mu \to 3e$, $\mu \to e\gamma$, etc., and had designed the powerful facility ARES, with the help of which it will be possible to observe the $\mu \to e\gamma$ decay, if $R_{\mu \to e\gamma} \gtrsim 10^{-11}$. On the other hand, we were engaged in theoretical work on neutrino oscillations and within a scheme with two neutrinos only (and no other neutral leptons) it became clear that even in the presence of observable oscillations $\nu_e \rightleftarrows \nu_\mu$, the $\mu \to e\gamma$ rate, which was calculated exactly at Dubna on the basis of the Weinberg-Salam theory with lepton mixing, must be fantastically small. At first impression, then, the experimental investigations mentioned above would have little sense. However, we rejected dogmatic conclusions and tried to provide theoretical support for experimentalists. The question was: under what circumstances can the $\mu \to e\gamma$ decay rate, calculated in a unified gauge theory, be sufficiently large for the observation of the process? As it will be shown later, the existence of sufficiently heavy leptons and lepton mixing do the job. For heavy neutral leptons, the formula for the $\mu \to e\gamma$ rate happened to be ready from the case of two neutrinos, after neutrino masses are substituted by heavy neutral lepton masses.

Coming back to the present paper, which is more biased towards experiment than theory and should be considered an invited paper much more than a rapporteur talk, its content is as follows. In the second paragraph there are summarized the more recent and relevant experimental limits on possible lepton charge non-conservation, obtained by measuring probabilities of various processes. In the third and fourth paragraphs the status of the lepton mixing theory in the case when the only neutral leptons are neutrinos is reviewed, the main

points being the $\mu \to e\gamma$ decay and neutrino oscillations. The "solar neutrino puzzle" is discussed in the fifth paragraph. In the sixth paragraph the Dubna theoretical work on heavy lepton mixing, mentioned above, is exposed. A model of the $\mu \to e\gamma$ and $\mu \to 3e$ decays is given as an example of drastic effects of heavy lepton mixing, and the relation between processes like $\mu \to e\gamma$, etc., and neutrino oscillations is considered. Recent papers on lepton nonconservation effects are then classified in groups, the related literature being presented extensively, if not fully.

II. EXPERIMENTAL LIMITS ON PROBABILITIES OF PROCESSES VIOLATING THE CONSERVATION OF LEPTON NUMBERS

1. Electron and Muon Lepton Numbers

The lepton current in the standard weak interaction theory is given by the expression

$$J_\alpha^\ell = (\bar{\nu}_{e_L} \gamma_\alpha e_L) + (\bar{\nu}_{\mu L} \gamma_\alpha \mu_L), \qquad (1)$$

where $e_L = \frac{1+\gamma_5}{2} e$, etc., are the left-handed components of the electron field, etc., $\nu_e (\nu_\mu)$ are the electron (muon) neutrino field operators. In agreement with experimental data, presented in Table I, expression (1) clearly does conserve the additive electron lepton number (L_e) and the muon lepton number (L_μ), that is $\Sigma L_e =$ const., $\Sigma L_\mu =$ const. (ν_e and e^- have $L_e = 1$ and $L_\mu = 0$, ν_μ and μ^- have $L_\mu = 1$ and $L_e = 0$, antileptons have opposite values of lepton numbers). There are a few types of experiments in which lepton number conservation can be tested. In every experiment an attempt is made to discover a process in which lepton numbers are not conserv-

T a b l e I

Ratio W_I/W_{II} of the probabilities of the forbidden (I) and allowed (II) processes

Process forbidden by lepton conservation (I)	Observed process allowed by lepton conservation (II)	W_I/W_{II} *	Confidence Limits	Ref.
a) Neutrinoless double beta decay $^{82}Se \to {}^{82}Kr + e^- + e^-$	Double beta decay $^{82}Se \to {}^{82}Kr + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$	< 0.09	68%	/1,2/
b) $\nu_\mu + N \to \mu^+ + \dots$	$\nu_\mu + N \to \mu^- + \dots$	$< 5\times10^{-3}$	95%	/3/
c) $\mu^+ \to e^+ + \gamma$	$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$	$< 2.2\times10^{-8}$	90%	/4/
c) $\mu^+ \to e^+ + e^+ + e^-$	$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$	$< 1.9\times10^{-9}$	90%	/5/
c) $\mu^- + Cu \to e^- + \dots$	$\mu^- + Cu \to \nu_\mu + \dots$	$< 1.6\times10^{-8}$	90%	/6/
c) $\nu_\mu + N \to e^- + \dots$	$\nu_\mu + N \to \mu^- + \dots$	$< 3\times10^{-3}$	95%	/7/
d) $\mu^- + Cu \to e^+ + \dots$	$\mu^- + Cu \to \nu_\mu + \dots$	$< 2.6\times10^{-8}$	90%	/6/

* In the literature often a parameter α is used in order to characterize qualitatively the maximum relative amplitude of the process which is forbidden by lepton conservation. Roughly speaking, $\alpha \approx (W_I/W_{II})^{1/2}$ for all cases in Table II, except for the neutrinoless double beta decay of ^{82}Se. In this case $\alpha \approx (W_I/W_{II})^{1/2}\cdot 10^{-3}$, the factor 10^{-3} taking into account the fact that the phase space for the neutrinoless double beta decay is $\sim 10^6$ times larger than for the "ordinary" double beta decay.

ed. In Table I there is listed a number of processes which were looked for, but not discovered, and there is given the corresponding upper limit of the ratio W_I/W_{II} of the probabilities of a forbidden process (I) and of a corresponding but allowed process (II). In T a b l e I only the most recent data are presented. They are relevant to the question of: a) possible nonconservation of the electron lepton number, b) possible nonconservation of the muon lepton number, c) possible nonconservation of both lepton numbers, their sum being conserved and d) possible nonconservation of both lepton numbers.

2. Neutrino Stability

I shall present now the results of an analysis, which is a by product of high energy neutrino experiments. Gamma-rays from the hypothetical decay of neutrinos were searched for in the inclusive reaction:

$$\text{Neutrino} \to \gamma + X, \qquad (2)$$

where X means anything, either known or unknown. The idea of the experiment is that photon induced e^+e^- pairs pointing to the neutrino beam direction in large bubble chambers would be a good signature for the decay. Knowing the neutrino track length, which in high energy neutrino experiment is of the order of a few light years and the photon detection efficiency (a few percent), one can measure the lower limit of the laboratory life-time of neutrinos and hence, knowing the neutrino beam momentum distribution, the lower limit of the proper neutrino lifetime as a function of the neutrino rest mass m. An analysis of CERN Gargamelle chamber /7/ and ANL 12-foot bubble chamber /8/ experiments, in which very few, if any, pairs were observed, gives the following results (the centre of mass lifetime is in seconds, the neutrino mass in eV, the indices ν_μ, $\bar{\nu}_\mu$, $\bar{\nu}_e$ refer only to the phenomenological description of the beam used in the experiment):

$\tau_{\nu_\mu} > 7 \cdot 10^{-3}$ m
$\tau_{\bar{\nu}_\mu} > 5 \cdot 10^{-2}$ m (Gargamelle)
$\tau_{\nu_\mu} > 6 \cdot 10^{-3}$ m (ANL).

An experiment with reactor antineutrinos /9/ makes use of a large (1400 litres) scintillator as a detector of photons (with energy in the 0.1 - 0.5 MeV range) emitted in reaction (2). The result is:

$\tau_{\bar{\nu}_e} > 600$ m (Reactor).

The question about the neutrino being unstable (and not necessarily according to reaction (2)) was discussed theoretically in various occasions /10-13/, in particular, as a possible solution of the so-called "solar neutrino puzzle" /13/ (see below). Now it is convenient to run a little ahead. If more definite assumptions are made, i.e., that there are only two types of neutrinos and that neutrino oscillations $\nu_e \rightleftarrows \nu_\mu$ /14/ do take place, the phenomenological reaction (2) must be written in the form

$$\nu_1 \to \nu_2 + \gamma \quad , \tag{3}$$

where ν_1 and ν_2 are the particles with definite masses required by neutrino mixing, the ν_1 mass m_1 being larger than the ν_2 mass m_2. In such a case the width of process (3) can be calculated exactly /15/ according to the neutrino mixing theory. In the simple case, where $m_1 \gg m_2$, it is given by the expression

$$\left(\frac{1}{\tau}\right)_{\nu_1 \to \nu_2 + \gamma} = \frac{9}{16}\,\frac{G^2}{128\pi^4}\,\alpha\, m_1^5 \sin^2\theta \cos^2\theta \left(\frac{m_\mu^2}{m_W^2}\right)^2 , \tag{4}$$

where m_μ, m_W are, respectively, the muon and the charged intermediate boson masses, θ is the mixing angle, G is the Fermi constant. From the above expression it is seen that the lifetime in the lab. system of neutrino having any momentum of practical interest is much larger than the age of the Universe.

III. LEPTON MIXING WITH TWO TYPES OF NEUTRINOS

1. Introduction

Hereafter neutral leptons with masses, say, smaller than the electron mass, will be called neutrinos, whereas other hypothetical neutral leptons will be simply called heavy neutral leptons.

In some way the idea that leptons may mix has been present in the mind of experimentators for a long time, I would say, since they started to look into the question about the possibility of the $\mu \to e\gamma$ decay (1947) and later on of neutrino oscillations (1957). Lepton mixing, independently of considerations about analogies between lepton and quarks, was introduced in ref. /16/ in order to describe quantitatively the hypothetical phenomenon of neutrino oscillations /17/ : in this theory the particles described by stationary states are neutrinos Majorana.

A formulation of lepton mixing and hadron mixing with 4 leptons and 4 hadrons was given in ref. /10/ at about the same time as, and independently of, the introduction of hadron mixing a la Gell-Mann-Levy-Cabibbo /18,19/. In ref. /12,14/ the analogy between leptons and quarks, more specifically the idea that for leptons there should exist a mixing mechanism with the magical qualities of the well-known GIM mechanism for quarks /20/ was the strongest motivation for the introduction of lepton mixing. Such a mixing implies the existence of neutrino oscillations /14,21/. Work on this hypothetical phenomenon has been closely connected with the unified theory of the weak and electromagnetic interactions of Weinberg and Salam.

2. Lepton Mixing: Two Alternatives

It is assumed that the ν_e and ν_μ field operators in expression (1) are orthogonal superpositions of fields of neutrinos with definite and finite masses m_1 and m_2 ($m_1 \neq m_2$). There are two possibilities.

1) Alternative M (Majorana):

$$\nu_e = \chi_1 \cos\theta + \chi_2 \sin\theta$$
$$\nu_\mu = -\chi_1 \sin\theta + \chi_2 \cos\theta \quad , \qquad (5)$$

where χ_1 and χ_2 are the fields of Majorana neutrinos m_1 and m_2, and θ is the mixing angle. Such a scheme[16] is attractive in its "economy": there are in all four neutrino states, those which are well known. According to this scheme, however, neutrinos occupy a special position among fundamental fermions, the analogy between leptons and quarks being destroyed. The scheme implies the nonconservation of L_e and L_μ and the existence of all the processes listed in the first column of Table I. Making use of a different definition[22] of the lepton number, one can describe this scheme as one in which there is only one lepton number, having opposite signs for e^- and μ^-, not exactly conserved.

2) Alternative D (Dirac):

$$\nu_e = \nu_1 \cos\theta + \nu_2 \sin\theta$$
$$\nu_\mu = -\nu_1 \sin\theta + \nu_2 \cos\theta \quad , \qquad (6)$$

where ν_1 and ν_2 are the fields of Dirac neutrinos with masses m_1 and m_2 (note that for the neutrino masses and the mixing angle θ the same notations are used for Majorana and Dirac fields).

Let us keep in mind the Cabibbo-Glashow-Illiopulos-Maiani quark mixing

$$d' = d\cos\theta_c + s\sin\theta_c$$
$$s' = -d\sin\theta_c + s\cos\theta_c \, , \qquad (7)$$

where θ_c is the Cabibbo angle. We see that in the scheme discussed now[12,14] there is a full analogy between leptons and quarks: the number of leptons is equal to the number of quarks, the weak lepton and quark currents have the same form, both the lepton and the quark fields are mixed. From the point of view of quantum chromodynamics the main difference between leptons and quarks is that the latter are coloured. But the weak interaction is colour blind, so maybe it is natural to expect that lepton fields should mix as quark fields do. In this sense the alternative D is preferable to alternative M, although is less "economical" (there are 4 states for every type of neutrinos, just as for any other fundamental fermion).

The main physical difference between schemes M and D is that in D neutrinoless double beta decay is obviously forbidden, whereas in M it has a finite probability (which, however, can be shown[23] to be extremely small). As far as oscillations are concerned, they are a corollary of lepton mixing in both scheme M and D and are described by identical expressions.

In the theory D, L_e and L_μ are not conserved, whereas the sum $L_e + L_\mu$ is conserved. Clearly processes such as $\mu \to e\gamma$, $\mu \to 3e$, $\mu + N \to e + N$, etc., are possible, although they are expected to have very small probabilities, for the same reason which accounts for the small probabilities of hadron processes due to neutral currents with a change of strangeness. In fact, both types of processes are induced by asymmetrical neutral currents, and their amplitudes are equal to zero in first order, because of cancellations due to orthogonal mixing (expressions (6) and (7)); the asymmetrical neutral current effects arise in higher order perturbation theory.

3. The Process $\mu \to e\gamma$

The probability of this process can be calculated on the basis of the renormalizable theory of Weinberg and Salam. The main contribution is given by the diagrams of Fig. 1.

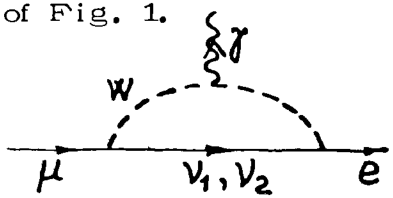

Fig. 1

The correct expression for the $\mu\to e\gamma$ probability in the mixing scheme D was first obtained in ref. /15/ and was then confirmed by several authors /24/. For the ratio $R_{\mu\to e\gamma}$ between the probability of the $\mu\to e\gamma$ decay and the probability of the $\mu^+\to e^+ + \nu_e + \bar{\nu}_\mu$ decay the following expression holds

$$R_{\mu\to e\gamma} = \frac{3}{32}\frac{\alpha}{\pi}\left(\frac{m_1^2 - m_2^2}{M_W^2}\right)^2 \sin^2\theta\cos^2\theta , \qquad (8)$$

where M_W is the W-boson mass ($M_W \geq 37$ GeV). Even if one tries hard to make $R_{\mu\to e\gamma}$ as large as possible, for example, if one takes for m_1 and m_2 the measured upper limits of the ν_μ and ν_e masses (0.65 MeV /25/ and 35 eV /26/, respectively) and assumes maximum mixing, one gets

$$R_{\mu\to e\gamma} \lesssim 10^{-26} , \qquad (9)$$

which is by 18 orders of magnitude smaller than the experimental upper limit (Table I). In fact, one should use for $|m_1^2 - m_2^2|$ the upper limit $\leq 1\ (\mathrm{eV})^2$, which is obtained by analysing experimental data in terms of neutrino oscillations, and then one gets a limit for $R_{\mu\to e\gamma}$ smaller than the value (9) by many orders of magnitude. The reason why $R_{\mu\to e\gamma}$ is unmeasurably small (just as are the relative probabilities of similar processes like $\mu\to 3e$, etc.), in spite of lepton mixing, is that in the process amplitudes there appears a factor $|m_1^2 - m_2^2|/M_W^2$, which is extremely small, if the masses of the neutral leptons are small. One could say that although the very notion of lepton numbers is lost when neutrino mixing is introduced, nevertheless, an effective conservation of lepton numbers does take place because of the smallness of neutrino masses. We must keep in mind this lesson when we ask ourselves under which conditions, then, can the process $\mu\to e\gamma$ "become measurable". As far as there is mixing of neutrinos only, we see that lepton mixing can be tested experimentally only by studying neutrino oscillations.

4. Neutrino Oscillations

In agreement with existing data (Table I) it has been generally assumed that the electron and the muon lepton numbers are strictly conserved. Of course, if this is the case, neutrino oscillations cannot take place. Oscillations may arise if, in addition to the usual weak interaction, an interaction which does not conserve lepton numbers is also taking place. In such a case the neutrino masses are different from zero, and as it was already mentioned, the state vectors of the ordinary electron and muon neutrinos ν_e and ν_μ are superpositions of the state vectors of neutrinos ν_1 and ν_2 with definite masses m_1 and m_2. If a beam of, say, muon neutrinos is produced in a weak process, at a certain distance from the production place the beam will be a coherent superposition of ν_μ and ν_e (that is there arise oscillations $\nu_\mu \rightleftarrows \nu_e$).

Such a situation is analogous to the one we are familiar with in the case of neutral kaon oscillations.

Now we shall be laconic, as neutrino oscillations have been covered recently in ref. /27/. In both schemes M and D possible oscillations $\nu_e \rightleftarrows \nu_\mu$, $\bar{\nu}_e \rightleftarrows \bar{\nu}_\mu$

are described by identical expressions, in which two parameters are present, the mixing angle θ and the difference $M^2 = |m_1^2 - m_2^2|$ of the neutrino masses squared:

$$I_{\nu_\ell}(R,p) = [1 - \frac{1}{2}\sin^2 2\theta\,(1 - \cos 2\pi \frac{R}{L})]\, I^\circ_{\nu_\ell}(R,p) \qquad (10)$$
$$\ell = e, \mu$$

$$I_{\nu_{\ell'}}(R,p) = [\frac{1}{2}\sin^2 2\theta\,(1 - \cos 2\pi \frac{R}{L})]\, I^\circ_{\nu_\ell}(R,p). \qquad (11)$$
$$\ell \neq \ell',\ \ell, \ell' = e, \mu$$

Here $I_{\nu_\ell}(R,p)$, $I_{\nu_{\ell'}}(R,p)$ are the intensities of ν_ℓ, $\nu_{\ell'}$ neutrinos, respectively, with momentum p at a distance R from a source of ν_ℓ neutrinos, $I^\circ_{\nu_\ell}(R,p)$ is the intensity of ν_ℓ neutrinos which would be expected in the absence of oscillations and

$$L = \frac{4\pi p}{|m_2^2 - m_1^2|} = \frac{4\pi p}{M^2} \qquad (12)$$

is the oscillation length (a useful formula for which is $L = 2.5 \frac{p/\mathrm{MeV}}{M^2/(\mathrm{eV})^2}$ meters). The observation of effects due to neutrino mixing (which we shall call also oscillation effects) includes one or both of the following two aspects: i) to observe the cosine term in the neutrino intensity and ii) to establish that either the constant term in formula (10) is different from 1 or the constant term in formula (11) is different from zero.

In order to observe the cosine term, one must require that it would not vanish on averaging over the distance from the neutrino source to the detector (which means averaging over the source and detector dimensions and over the time of measurement, if the average distance source-detector is not constant) and over the neutrino spectrum (see formula (12)). In particular, a necessary (but not sufficient) condition for the observation of the cosine term in the neutrino intensity is that the neutrino source dimensions were smaller than the oscillation length and that the uncertainty in the time of neutrino emission were smaller than the oscillation period.

Further any oscillation effect might be observable only if

$$L \leq R. \qquad (13)$$

From (12) and (13) one can see that a necessary (but not sufficient) condition for observing oscillation effects is that

$$|m_2^2 - m_1^2| \gtrsim 4\pi p/R. \qquad (14)$$

An analysis in term of neutrino oscillations of reactor and CERN experiments[7,28] which, incidentally, were not designed with the purpose to observe oscillations, shows that $|m_1^2 - m_2^2| \leq 1\,(\mathrm{eV})^2$ (if the mixing is maximum). The ultimate sensitivities which can be expected according to (14) at various neutrino "facilities" are conditioned by the minimum value of $4\pi p/R$ obtainable at each facility as follows:

$10^{-2}(\mathrm{eV})^2$	(reactor, meson factories, high energy accelerators)
$10^{-3}(\mathrm{eV})^2$	(cosmic neutrinos)
$10^{-12}(\mathrm{eV})^2$	(solar neutrinos)

At reactor facilities there are three proposals to investigate oscillations: at the Moscow Kurchatov Institute of Atomic Energy[29], at the Grenoble Institute Laue-Langevin[30] and at the Irvine University of California[31].

An oscillation experiment is soon to be performed at the Brookhaven accelerator[32]. The originality of the proposal is that the 30 GeV proton accelerator will be used as a meson factory (at a proton energy of only ~ 800 MeV and at a beam intensity of ~10^{14} protons per second).

We would like to mention also the ref.[33] in which it was proposed to place the neutrino detector in Canada at a distance of ~ 1000 km from the Fermilab accelerator. Without doubt this type of experiment is not only of interest in connection with the oscillation problem, but also because it gives the possibility of measuring directly a distance between two points on the Earth separated by enormous quantities of matter *.

At the underground Neutrino Observatory of the Institute for Nuclear Research of the Academy of Sciences, an experiment is being prepared[34] in which there will be detected high energy muon neutrinos emitted by mesons, which are produced in collisions of cosmic ray protons with nitrogen and oxygen nuclei in the atmosphere. High energy muons produced by ν_μ's interacting with nuclei in the Earth will be detected by 4 hodoscope plane systems (every one of which has an area of 1500 m^2) of organic scintillators. The scintillator systems are in coincidence, the logic giving information on the muon trajectory and also establishing whether the detected muon has come either from "above" or from "below" (in the last case it is produced by a muon neutrino impinging upon the Earth opposite face and passing through the Earth). The average neutrino momentum in such experiments is 5-10 GeV, and the distance from the neutrino source to the detector is $R \simeq 10^4$ km for neutrinos coming from the Earth opposite face. Thus, the sensitivity of those experiments for testing neutrino mixing according to expressions (10) is intermediate between that of experiments wherein artificial (reactor, accelerator) neutrinos are used and that of the investigations wherein solar neutrinos are used. However, the statistical accuracy which can be attained ($\lesssim$ 100 events/year) is quite low.

* See Note I at the end of the paper.

If $|m_1^2 - m_2^2| < 10^{-3}$ $(eV)^2$ the only hope to observe neutrino oscillations is the development of Solar neutrino astronomy[17]. The Brookhaven Solar neutrino experiment will be discussed in a separate paragraph.

In conclusion, it may be of interest to illustrate the fantastic sensitivity of oscillation experiments for testing the law of lepton number conservation on a simple example. Suppose $\theta = \pi/4$ and $|m_1^2 - m_2^2| = 10^{-2}$ $(eV)^2$: whereas oscillation experiments proving the mixing hypothesis are perfectly feasible, the value of $R_{\mu\to e\gamma}$ expected from formula (8) is 10^{-51} !

IV. LEPTON MIXING WHEN THERE ARE $N \geq 2$ NEUTRINO TYPES

The mixing of $N>2$ neutrinos has been studied in refs.[35,36,37] and here I shall only state the main results. 1) In neutrino experiments, let us say Solar neutrino experiments, the measured average intensity of electron neutrinos $\bar{I}_{\nu_\ell}$ may turn out to be considerably smaller than 1/2 of the intensity $\bar{I}^\circ_{\nu_e}$ expected in the absence of oscillations, the minimum value being $(1/N)\bar{I}^\circ_{\nu_e}$ and even less in certain exotic schemes. 2) The expected $\mu \to e\gamma$ decay probability remains extremely small.

A summary of various neutrino mixing schemes and types of oscillations in ν_e beams is presented in Table II. This Table was prepared for an invited paper at the Tbilisi 1976 Conference but somehow it disappeared from the published version of the paper. In the Table $(\delta_{\nu_e;\nu_e})_{min}$ is the minimum value of the ratio of the average intensity $\bar{I}_{\nu_e}$ to the intensity $\bar{I}^\circ_{\nu_e}$ expected without neutrino mixing.

Table II

Schemes of Neutrino Mixing and Oscillations in ν_e Beams

Number of neutrino types	Number of neutrino states	Bare neutrino masses	Particles with definite masses	Oscillations in ν_e beams	$(\delta_{\nu_e;\nu_\mu})_{min}$	Ref.
1. Two(ν_e, ν_μ)	4	0	2 Majorana neutrinos	$\nu_e \leftrightarrows \nu_\mu$	1/2	16
2. "–"	8	$\neq 0$	2 Dirac neutrinos	$\nu_e \rightleftarrows \nu_\mu$	1/2	14
3. "–"	8	$\neq 0$	4 Majorana neutrinos	$\nu_e \leftrightarrows \nu_\mu$ $\nu_e \leftrightarrows \bar{\nu}_{eL}$ $\nu_e \rightleftarrows \bar{\nu}_{\mu L}$	1/4	36
1a. N>2 $(\nu_e, \nu_\mu, \nu_M \ldots)$	2N	0	N Majorana neutrinos	$\nu_e \rightleftarrows \nu_\mu$ $\nu_e \leftrightarrows \nu_M$	1/N	35
2a. "–"	4N	$\neq 0$	N Dirac neutrinos	$\nu_e \rightleftarrows \nu_\mu$ $\nu_e \rightleftarrows \nu_M$	1/N	35–37
3a. "–"	4N	$\neq 0$	2N Majorana neutrinos	$\nu_e \rightleftarrows \nu_\mu$, $\nu_e \rightleftarrows \nu_M$ $\nu_e \rightleftarrows \bar{\nu}_{eL}$, $\nu_e \leftrightarrows \bar{\nu}_{\mu L}$ $\nu_e \leftrightarrows \bar{\nu}_{ML}$....	1/2N	36

V. THE BROOKHAVEN SOLAR NEUTRINO EXPERIMENT AND THE "SOLAR NEUTRINO PUZZLE"

In ref. [38] there are given the results of measurements performed from April 1970 till February 1976 with the aim of detecting solar neutrinos. The ^{37}Ar production rate by solar neutrinos in a huge detector of C_2Cl_4, based on the reaction [39] $\nu_e + {}^{37}Cl \to {}^{37}Ar + e^-$, was found to be (1.3 ± 0.4) SNU (1 SNU = 10^{-36} events/sec. ^{37}Cl atom*). The expected rate, according to the standard solar model, is equal to (6 ± 2) SNU (see ref. [40] and related reference therein). This "deficiency" is called in the literature the "solar neutrino puzzle". If there are oscillations, the solar neutrino signal should be suppressed, because a fraction of neutrinos (the ν_μ fraction in the case of two neutrino types) is sterile [17]. If $L(\bar{p}) \ll R$, the oscillating term in (10) vanishes on averaging, so that the observed intensity $\bar{I}_{\nu_e}$ may be found in the interval from $1/2\, I^\circ (\theta = \pi/4)$ to I° ($\theta = 0$, no mixing). If the number of neutrino types is $N \geq 2$, the averaged intensity may be found in the interval from $1/N I^\circ_{\nu_e}$ to $I^\circ_{\nu_e}$ and in some exotic schemes even from $(1/2N) I^\circ_{\nu_e}$ to $I^\circ_{\nu_e}$ (see Table II).

Even in the case of only two types of neutrinos the observed signal from solar neutrinos may be considerably smaller than the expected one, if the oscillation length is about equal to the Sun-Earth distance. The cosine term in (10) **may** then survive after averaging over the momentum of neutrinos, so that we have another possibility [16] of getting quite a low neutrino signal (this possibility, of course, is rather accidental).

Thus, there is nothing surprising if the solar neutrino signal turns out to be definitely smaller than the expected one, the only requirement being that the mixing angle should not be small.

---

*See Note II at the end of the paper.

If really the solar neutrino flux is definitely smaller than the calculated one and if the related calculations are reliable, the solution of the "puzzle" in terms of lepton mixing seems to be much more natural than any other solution put forward until now. Many such suggestions are listed in ref. /40/, where one may find the corresponding references. They include the assumption that neutrino decay on their way from the Sun to the Earth and the following exotic astrophysical suggestions: the Sun energy is not generated in thermo-nuclear reactions; there is a black hole inside the Sun; the Sun is not in a state of equilibrium and its apparent luminosity, due to the very slow process of diffusion of photons from the central part to the surface, is much higher than its "internal luminosity", about which information is almost instantaneously obtained in neutrino experiments; the Sun in the past has substantially increased its mass from outside, so that its internal and external regions have an entirely different composition, a circumstance which would make quite wrong the result of calculations based on homogeneous models, etc.

Thus, if we really believe that there is a solar neutrino deficiency, we have in our hands an explanation reasonable, not exotic, attractive from the point of view of today elementary particle physics, and explanation which was not invited "ad hoc" to solve the "solar neutrino puzzle": lepton mixing.

Let us note in conclusion that the solution of the "solar neutrino puzzle" in terms of lepton mixing would imply for the simplest case of two neutrino types that: 1) the neutrino mixing is substantial (θ is not far away from $\pi/4$); 2) the oscillation length is smaller than the Sun-Earth distance, from which it follows that $|m_1^2 - m_2^2| \geq 10^{-11}\mathrm{eV}^2$ /41/.

VI. HEAVY LEPTON MIXING

1. The $\mu \to e\gamma$ Decay in a Special Model

In the sections 1-4 of the present paragraph only the Dubna work is reported. As already stated in the Introduction, an inspection of formula (8), valid when only neutrinos are mixed /15/ suggested that processes of the type $\mu \to e\gamma$, $\mu \to 3e$, etc., might well be observable if there exist heavy leptons and if there is lepton mixing. As an example let us consider the* scheme /42/ in which together with the left-handed doublets

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \tag{15}$$

[ν_e, ν_μ are given by expressions (6)], there are right-handed doublets

$$\begin{pmatrix} N_e \\ e \end{pmatrix}_R , \quad \begin{pmatrix} N_\mu \\ \mu \end{pmatrix}_R . \tag{16}$$

Here

$$\begin{aligned} N_e &= N_1 \cos\theta' + N_2 \sin\theta' , \\ N_\mu &= -N_1 \sin\theta' + N_2 \cos\theta' , \end{aligned} \tag{17}$$

where N_1 and N_2 are the field operators of heavy leptons with masses M_1, M_2 ($M_1, M_2 > M_k$) and θ' is the mixing angle.

* At present there have been published many models with heavy leptons and mixing according to which the probability of the $\mu \to e\gamma$ decay and similar processes turns out to be "large" (the literature is presented in the last Section). It is interesting that an identical model, the one which is exposed here, has been considered in the first two papers /42,43/, published at the same time and independently in different continents.

In addition to the diagrams for the $\mu\to e\gamma$ decay of Fig. 1, there arise new diagrams (Fig. 2), the contribution of which is dominant

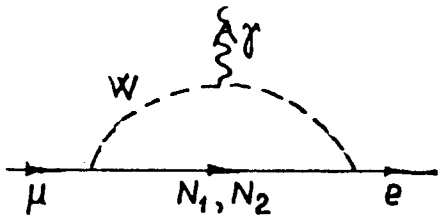

Fig. 2

For the ratio $R_{\mu\to e\gamma}$ we have

$$R_{\mu\to e\gamma} = \frac{3}{32}\,\frac{\alpha}{\pi}\left(\frac{M_1^2-M_2^2}{M_W^2}\right)^2 \sin^2\theta' \cos^2\theta' . \tag{18}$$

In Table III values of $R_{\mu\to e\gamma}$ for $\theta'=\pi/4$ are tabulated.

Table III

$\lvert M_1^2 - M_2^2 \rvert^{1/2}$ (GeV)	$R_{\mu\to e\gamma}$
1	$4.2\cdot 10^{-12}$
2	$6.7\cdot 10^{-11}$
3	$3.4\cdot 10^{-10}$
4	$1.1\cdot 10^{-9}$

This Table is an illustration of the general statement which can be made by considering various models with lepton mixing: if there exist neutral leptons with masses of a few GeV and if their fields are mixed, the $\mu\to e\gamma$ decay probability may be relatively close to its measured upper limit (see Table I). At present I am aware of work at SIN, TRIUMPH and LAMPF with NaI scintillators, in which the decay $\mu\to e\gamma$ might be observed if $R_{\mu\to e\gamma} > 10^{-10}$. The facility ARES [44] mentioned in the Introduction,

is a cylindrical magnetic spectrometer with proportional chambers (16000 wires) and should be capable of revealing the $\mu\to e\gamma$ decay if $R_{\mu\to e\gamma} \gtrsim 10^{-11}$. Incidentally ARES is a rather universal facility and with its help it is possible to investigate the $\mu\to 3e$, $\mu^- + N \to e^- + \ldots$, $\mu^- + N \to e^+ + \ldots$ processes as well.

2. Parity Nonconservation in Heavy Atoms

Let us note the vector character of the neutral current of charged leptons in a theory with left-handed and right-handed doublets (15) and (16). The hadron neutral current, on the other hand, has a vector as well as an axial component. Thus, in the model considered, P odd effects should be strongly suppressed in heavy atoms [45], although the weak interaction of charged leptons and hadrons does violate parity.

3. The $\mu\to 3e$ Decay

In the scheme with doublets (15, 16) the decay $\mu\to 3e$ was investigated theoretically [46] and the ratio $R_{\mu\to 3e}$ of the probability of such decay to that of the $\mu^+\to e^+\nu_e\bar{\nu}_\mu$ decay was calculated. The results are given in Table IV, where it was assumed that $\theta'=\pi/4$, M_W = 60 GeV, $\sin^2\theta_W=1/3$ and $M_2 \gg M_1$.

Table IV

M_2 (GeV)	$R_{\mu\to 3e}$	$R_{\mu\to 3e}/R_{\mu\to e\gamma}$
2	$1.5\cdot 10^{-10}$	2.2
3	$5.8\cdot 10^{-10}$	1.7
4	$1.2\cdot 10^{-9}$	1.7

Note that the experimental upper limit [5] is:

$$(R_{\mu\to 3e})_{exp.} \le 1.9\times 10^{-9} .$$

4. Neutrino Oscillations and the $\mu \to e\gamma$ Decay

Below we discuss the relation between the phenomena of neutrino oscillations and of the $\mu \to e\gamma$ decay [42]. The observation of such effects would show that lepton mixing takes place indeed. In such a general sense the observation of either of these phenomena would make the existence of the other more likely, in particular, the neutrino masses would be probably finite. Apart from this general connection, it should be emphasized, however, that neutrino oscillations and processes as the $\mu \to e\gamma$ decay, etc., might well be entirely unconnected.

First, one can think of the case in which neutrino oscillations might be observable, but the process $\mu \to e\gamma$ is in fact unobservable; this is just the situation when in Nature the only neutral leptons are neutrinos.

Second, one can imagine a situation in which the $\mu \to e\gamma$ decay is perfectly observable (let us say that there are heavy neutral leptons of sufficiently large masses) but neutrino oscillations are unobservable, for example, because of a small mixing angle and/or a small difference of the neutrino masses.

Third, one can imagine a state of affairs, in which, the $\mu \to e\gamma$ decay probability is relatively high (for example, because there exist heavy charged leptons, and symmetrical neutral currents are present in the Hamiltonian), but neutrino oscillations may be completely absent (not only unobservable in practice), since the $\mu \to e\gamma$ process has no relation whatever to the oscillations.

5. Recent Literature on Nonconservation of the Muon Number

The papers known to me in which there were obtained "large" theoretical probabilities of processes like $\mu \to e\gamma$, $\mu \to 3e$, $\mu^- + N \to e^- + \ldots$, $K \to \mu e \pi$, etc., are classified roughly in a few groups, the

papers being quoted within a group in a more or less accidental order.

I. Schemes where new particles are not needed: refs. [47,48].

II. Schemes where heavy leptons are introduced.

a) Neutral heavy leptons: refs. [42,43,45,46,49–65]

b) Singly charged heavy leptons: refs. [49,57,64,66–70].

c) Doubly charged heavy leptons: refs. [57,71,72].

III. Schemes where supplementary scalar particles are introduced, first of all Higgs particles: refs. [73,74,69].

IV. Schemes where supplementary intermediate bosons are introduced: refs. [75,76].

VII. FINAL REMARKS

As it became clear in the two last years, strict conservation of lepton numbers would arise in modern theory in a rather artificial way, whereas lepton mixing is natural. Quark-lepton analogies make it aesthetically attractive. In a theory with neutrino mixing, neutrino masses are not equal to zero and neutrinos have no special place among fundamental fermions. The apparent existence of lepton number conservation simply reflects the smallness of neutrino masses. In this case the fact that no one process has been found yet in which lepton number conservation is violated, therefore, is not an argument against lepton mixing. The main consequences of lepton mixing, neutrino oscillations and processes like $\mu \to e\gamma$, etc., should, of course, be searched for even at levels only slightly better than existing experimental levels. In fact, there has been already an explosion of work, theoretical and experimental, on processes like $\mu \to e\gamma$, etc., and neutrino oscillations. Let us wait and see whether we are confronted or not with a classical example of "Much ado about nothing".

Fortunately, it is easy to see how lepton mixing might be proved: one must "simply" observe neutrino oscillations and/or a process like $\mu \to e\gamma$, etc. On the contrary, unfortunately and as always, to exclude the existence of lepton mixing would be very difficult indeed.

In conclusion I wish to thank S.M.Bilenky and S.T.Petcov for their great help and advice during the preparation of the present paper.

Note I. At the Neutrino '77 Conference (USSR) at improved version of a similar proposal came to my attention: a neutrino Cherenkov H_2O detector, having a mass of a few million tons, would be located at a distance of several thousand kilometers from the Batavia accelerator in the Ocean at a depth of a few thousand meters (University of Washington and Western Washington State College Proposal 561, P.Kotzer, S.Nedermayer).

Note II. The 1977 Brookhaven neutrino solar experiment runs reported by Prof. Rowley at the Neutrino'77 Conference, leave the situation essentially unchanged.

REFERENCES

1. Cleveland B.T. et al. Phys.Rev.,Lett., 1975, 35, p.757.
2. Kirsten T., Muller H.W. Earth Planet Sci.Lett., 1969, 6, p.271. Srinivasan B. et al. Econ.Geol., 1973, 68, p.252.
3. Borer K. et al. Proc. XIV Int. Conf. on High Energy Physics, Vienna, 1968.
4. Parker S., Anderson H.L., Rey C. Phys.Rev., 1964, 133B, p.768. Korenchenko S.M. et al. Yad.Fiz., 1971, 13, p.341.

5. Korenchenko S.M. et al. Proc. XVIII Int. Conf. on High Energy Phys., Tbilisi, v.II, p.175 (1976).
6. Bryman D.A. et al. Phys.Rev.Lett., 1972, 28, p.1469.
7. Bellotti E. et al. Lett.Nuovo Cim., 1976, 17, p.553.
8. Barnes V.E. Phys.Lett., 1976, 65B, p.174.
9. Reines F. et al. Phys.Rev.Lett., 1974, 32, p.180.
10. Nakagawa M. et al. Prog.Theor.Phys., 1963, 30, p.727.
11. Pakvasa S., Tennakone K. Phys.Rev.Lett., 1972, 28, p.1415.
12. Eliezer S., Ross D.A. Phys.Rev., 1974, D10, p.3088.
13. Bahcall J.N., Cabibbo N., Yahil A. Phys.Rev. Lett., 1972, 28, p.316.
14. Bilenky S.M., Pontecorvo B. Phys.Lett., 1976, 61B, p.248.
15. Petcov S.T. Yad.Fiz., 1977, 25, p.641.
16. Gribov V., Pontecorvo B. Phys.Lett., 1969, 28B, p.493.
17. Pontecorvo B. JETP (Sov.Fiz.), 1967, 52, p.1717; see also JETP (Sov.Fiz.), 1957, 33, p.549; 1958, 34, p.24.
18. Gell-Mann M., Levy M. Nuovo Cim., 1960, 16, p.705.
19. Cabibbo N. Phys.Rev.Lett., 1963, 10, p.531.
20. Glashow S.L., Illiopoulos J., Maiani L.Phys.Rev., 1970, D2, p.1285.
21. Eliezer S., Swift A. Nucl.Phys., 1976, B105, p.45.
22. Zeldovich Ya.B. DAN SSSR, 1952, 86, p.505. Konopinsky E.J., Mahmoud H.Phys.Rev., 1953, 92, p.1045.
23. Schepkin M. Yad.Fiz., 1973, 18, p.153.
24. See, e.g., Cheng T.D., Li L.F. Phys.Rev.Lett., 1977, 38, p.1; Marciano W., Sanda A. Phys.Lett., 1977, 67B, p.303.
25. Clark A. et al. Phys.Rev., 1974, D9, p.533.
26. Tretiakov E.F. et al. Proc. XVII Int. Conf. on High Energy Physics, Tbilisi, 1976.

27. Bilenky S.M., Pontecorvo B. Proc. XVIII Int. Conf. on High Energy Physics, Tbilisi, 1976. See also "Lepton Mixing and Neutrino Oscillations", Submitted to "Physics Reports".
28. Nezrik F.A., Reines F. Phys.Rev., 1966, 142, p.852.
29. Borovoi A., Mikaelian A. Proc.Int.Conf. "Neutrino-77", USSR, 1977.
30. Quoted in the preprint of Egelman E. et al. Harvard University, January 17, 1977.
31. Reines F. University of Calif., Irvine UCI-10-P19-106.
32. Egelman E. et al. Preprint Harvard University, Jan. 17, 1977.
33. Mann A.K., Primakoff H. Phys.Rev., 1977, D15, p.655.
34. Chudakov A.E. Cosmnews, No. 2, 1977.
35. Pontecorvo B. JETP Lett. (Pisma JETP), 1971, 13, p.281.
36. Bilenky S.M., Pontecorvo B. Lett.Nuovo Cim., 1976, 17, p.569.
37. Fritzsch M., Minkovsky P. Phys.Lett., 1976, 62B, p.72.
38. Davis R., Evans D. Proc. Seminar on Active Processes on the Sun and the Problem of Solar Neutrino, Leningrad, Oct., 1976.
39. Pontecorvo B. Chalk River Lap.Rep., 1946, PD-205.
40. Bahcall J.N., Davis R. Science, 1976, 191, p.264.
41. Bilenky S.M., Pontecorvo B. JINR, E2-10545, Dubna, 1977.
42. Bilenky S.M., Petcov S.T., Pontecorvo B. Phys.Lett., 1977, 67B, p.309.
43. Cheng T.D., Li L.F. ref. /24/.
44. Korenchenko S.M., Micelmacher G.V., Nekrasov K.G. JINR, P13-9542, Dubna, 1976.
45. Bilenky S.M, Petcov S.T. Yad.Fiz., 1977, 25, p.1223.
46. Petcov S.T. JINR, E2-10487, Dubna, 1977.

47. Barshay S. Phys.Lett., 1975, 58B, p.86.
48. Decker R., Pestieau J. Universite de Louvain, Preprint UCL-IPT 77/04, 1977.
49. Marciano W.J., Sanda A.I. Phys.Lett., 1977, 67B, p.303.
50. Barger W., Nanopoulos D.V. University of Wisconsin. Preprint COO-583 (1977).
51. Lee B.W. et al. Phys.Rev.Lett., 1977, 38, p.937.
52. Treiman S.B., Wilczek F., Zee A. Princeton University. Preprint February, 1977.
53. Cheng T.P., Li L.F. University of Missouri. Preprint UMSL-77-2 (1977).
54. Terezawa H. Prog.Theor.Phys., 1977, 57, no.5.
55. Minkowski D. University of Bern, preprint (1977).
56. Fritzsch H. Preprint CALT-68-583 (1977).
57. Marciano W.J., Sanda A.I. Preprint Rockefeller University, COO-2232B, 122 (1977).
58. Barshay S., Leite Lopes J. Preprint of the Strasbourg University, 1977.
59. Suzuki T. et al. Hokkaido University preprint (1977).
60. Leite Lopes J., Ragiadakos C. Lett.Nuovo Cim., 1976, 16, p.261.
61. Altarelli G. et al. Preprint LPTENS 77/4 (1977).
62. Mohapatra R.N., Sidhy D.P. Preprint CCNY-HEP-77/3 (1977).
63. Pais A. Preprint COO-2232 B-118 (1977).
64. Lee B.W., Shrok R.E. Preprint Fermilab-Pub-77/21, THY (1977).
65. Nieh H.T. Preprint ITP-SB-77-6 (1977).
66. Barshay S. Phys.Lett., 1977, 66B, p.246.
67. Shabalin E.P. Preprint ITEP-9, Moscow, 1977.
68. Kim J.E. Brown University, Preprint , 1977.
69. Ma E., Paksava S. University of Hawai. Preprint UH-511-229 (1977).
70. Abud M.A., Savoy S.A. Geneve, DPT 1977-03-032 (1977).
71. Wilczek J.F., Zee A. Phys.Rev.Lett., 1977, 38, p.531.

72. Altarelli G. et al. Preprint LEPTENS 77/8(1977).
73. Bjorken J.D., Weinberg S. Phys.Rev.Lett., 1977, 38, p.622.
74. Branco G.C. University of Bonn, Preprint HE-77-5 (1977).
75. Beg M.A., Sirlin A. Phys.Rev.Lett., 1977, 38, p.1113.
76. Akama K., Chikashide J., Matsuki T. University of Tokyo, Preprint INS-288 (1977).

Received by Publishing Department
on December 13, 1977.*

* The present text is exactly that delivered in July at the Budapest Conference, without additions.