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RADIATIVE DECAYS OF VECTOR MESONS IN THE NONLOCAL QUARK MODEL

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Радиационные распады векторных мезонов в нелокальной модели кварков

Электромагнитные взаимодействия вводятся в нелокальную виртон-кварковую модель. Оказалось, что чисто фотон-фотонное взаимодействие вообще отсутствует, но фотон-адронные взаимодействия благодаря обмену виртонами-кварками существуют. Мы исследовали взаимодействия векторных и псевдоскалярных мезонов с виртон-кварковым полем, описываемые лагранжианами, которые инвариантны относительно SU(3) -группы и градиентных преобразований с простейшими связями. Подсчитаны ширины радиационных распадов векторных мезонов. Результаты находятся в хорошем согласии с экспериментальными данными.

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Radiative Decays of Vector Mesons in the Nonlocal Quark Model

Electromagnetic interactions are introduced into nonlocal virton-quark model. It turned out that pure photon-photon interactions are absent at all, but photon-hadron interactions owing to virton-quark exchange do exist. We investigate interactions of vector and pseudoscalar mesons with virton-quark field described by Lagrangians, which are invariant with respect to SU(3) group and gauge transformations with the simplest couplings. The rates of radiative decays of vector mesons are calculated. The results are in good agreement with experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1977

1. Introduction

The nonlocal quark model was proposed in paper /1/. In this model quarks do not exist as usual physical particles but they do exist in the virtual state only. These nonexisting particles were called "virtons". The physical assumption consists in that hadrons described by the standard quantum-field equations do not interact with each other directly but through an intermediate virton-quark field. In this model with the simplest choice of the interaction Lagrangians, mass corrections of pseudoscalar and vector mesons and rates of weak and two-particle strong meson decays were calculated /2,3/. The good agreement with experimental data was achieved.

It turned out that coupling constants in this model for strong interactions of mesons are less than unity and the perturbation theory is applicable. It should be noted that physical amplitudes in the nonlocal theory increase in each perturbation order with growing energy. It means that this model in perturbation theory is applicable to hadron physics of low energies only. If we want to consider high energy phenomena, we have to go beyond perturbation methods.

In this paper we will consider the electromagnetic interactions in this nonlocal quark model. First of all, we have to find good calculation formulas since the introduction of the electromagnetic field in a nonlocal theory is not trivial. Then we consider the photon-photon interaction and the interaction between photons and other physical particles.

This problem was investigated in /4/. The authors considered the nonregularized theory and have found that in a particular case of the nonlocal propagator the photon-photon interactions due to virton exchange are absent . On the ground of this result they conclude that the interactions in this model are always absent.

Strictly speaking, their investigation is not correct because they considered the nonregularized theory. Their conclusion that any interaction is absent in this model is wrong if hadrons are considered as usual particles. If we want to consider hadrons as bound states of virtons, we have to look for bound states in full S-matrix for an appropriate virton-virton interaction. At present this question is at all unsolved, and it is difficult to conjecture what answer will be.

It is important to stress that the gauge invariance and regularization are the principal points of the introduction of electromagnetic field in any nonlocal model. Without regularization we cannot construct the nonlocal theory, therefore any exploited regularization should not destroy the gauge invariance if we want to get physically meaningful results.

In this paper we satisfy these two requirements. It turned out that all matrix elements describing the pure photon-photon

interaction due to virton-quark loops disappear on removing regularization. But the photon-hadron interactions due to virton-quark loops are not trivial in the same limit. Thus we can describe electromagnetic interactions of hadrons in this model.

We consider radiative decays of vector mesons in this virton-quark model and our results are in satisfactory agreement with experimental data.

2. The interaction of virtons with the electromagnetic field

The classical Lagrangian of the free virton field has the following form (see /1/):

$$\mathcal{L}_0(x) = \bar{q}(x)\, Z(\hat{p})\, q(x)\,, \tag{2.1}$$

where $\hat{p} = i\hat{\partial}$ and

$$Z(\hat{p}) = -M \exp\left\{-\ell\hat{p} - \frac{\ell^2}{4}p^2\right\}. \tag{2.2}$$

In order to obtain the Lagrangian which describes the interaction of the electromagnetic field with the virton field $q(x)$ we must replace $\partial/\partial x_\mu$ by

$$i\frac{\partial}{\partial x_\mu} \longrightarrow i\frac{\partial}{\partial x_\mu} + e_q A_\mu(x).$$

Then the gauge-invariant Lagrangian will be of the form

$$\mathcal{L}_{em}(x) = \bar{q}(x) Z(\hat{p} + e_q \hat{A}(x)) q(x) - \\ - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} . \tag{2.3}$$

The quantization of the virton field was performed in /1/. Instead of the free Lagrangian (2.1) we introduce the regularized Lagrangian

$$\mathcal{L}_0(x) \to \mathcal{L}_0^{\delta}(x) = \bar{q}^{\delta}(x) Z^{\delta}(\hat{p}) q^{\delta}(x) = \\ = \sum_{j=1}^{\infty} (-)^j \bar{q}_j^{\delta}(x) (\hat{p} - M_j(\delta)) q_j^{\delta}(x) , \tag{2.4}$$

where

$$q^{\delta}(x) = \sum_{j=1}^{\infty} (-)^j \sqrt{A_j(\delta)}\, q_j^{\delta}(x) . \tag{2.5}$$

The constants $M_j(\delta)$ and $A_j(\delta)$ are connected in an appropriate manner with the function Z in (2.2) so that

$$M_j(\delta) = \left(\frac{j^{\sigma}}{\delta} - \frac{2\ell}{L} \right) \frac{1}{L} \quad (\sigma > 0;\ j = 1, 2, \ldots),$$

$$\sum_{j=1}^{\infty} \frac{(-)^j A_j(\delta)}{M_j(\delta) - \hat{p} - i\varepsilon} = G_c^{\delta}(\hat{p}) \xrightarrow[\delta \to 0]{} G_c(\hat{p}) = \frac{1}{M} e^{\ell \hat{p} + \frac{L^2}{4} p^2} . \tag{2.6}$$

The fields $q_j^{\delta}(x)$ are quantized as the Dirac fields with indefinite metrics /1/.

If we introduce the electromagnetic interaction, the gauge invariance must be kept at all stages of calculations. It means that the electromagnetic field should be introduced in such a way that the regularized Lagrangian must be gauge invariant too, i.e.,

$$\mathcal{L}_0^{\delta}(x) \longrightarrow \bar{q}^{\delta}(x) Z^{\delta}(\hat{p} + e_q \hat{A}(x)) q^{\delta}(x) . \tag{2.7}$$

Now let us introduce the system of fields

$$q_j^{\delta}(x) = \sqrt{A_j(\delta)}\, \frac{Z^{\delta}(\hat{p} + e_q \hat{A}(x))}{\hat{p} + e_q \hat{A}(x) - M_j(\delta)}\, q^{\delta}(x) \quad (j = 1, 2, \ldots)$$

and

$$q^{\delta}(x) = \sum_{j=1}^{\infty} (-)^j \sqrt{A_j(\delta)}\, q_j^{\delta}(x) , \tag{2.8}$$

so that under the gauge transformation

$$A_\mu \to A_\mu + \partial_\mu f \ , \quad q^{\delta} \to q^{\delta} e^{i e_q f}$$

the fields q_j^{δ} are transformed in the same way

$$q_j^{\delta}(x) \to q_j^{\delta}(x) e^{i e_q f(x)} .$$

The Lagrangian (2.7) can be represented in the form

$$\mathcal{L}^{\delta}_{em}(x) = \sum_{j=1}^{\infty} (-)^j \bar{q}^{\delta}_j(x)(\hat{p} + e_q \hat{A}(x) - M_j(\delta)) q^{\delta}_j(x) =$$
$$= \mathcal{L}^{\delta}_o(x) + \mathcal{L}^{\delta}_{Iem}(x) , \qquad (2.9)$$

where
$$\mathcal{L}^{\delta}_{Iem}(x) = e_q J^{\delta}_{\mu}(x) A_{\mu}(x),$$
$$J^{\delta}_{\mu}(x) = \sum_{j=1}^{\infty} (-)^j \bar{q}^{\delta}_j(x) \gamma_{\mu} q^{\delta}_j(x). \qquad (2.10)$$

Further we will use this interaction Lagrangian.

The regularized S-matrix is defined in the usual way .

$$S^{\delta} = T \exp\left\{ ie_q \int dx J^{\delta}_{\mu}(x) A_{\mu}(x) \right\}.$$

Any physical matrix elements are given in the limit $\delta \to 0$. As the virton field $q^{\delta}(x)$ disappears in this limit (see /1/), we have to investigate Feynman diagrams containing virton loops only.

Let us consider the interaction between photons owing to the exchange of virtons.

First we consider the diagram of vacuum polarization (Fig.1).

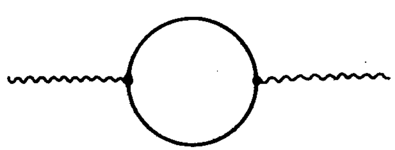

Fig.1

The S-matrix term which corresponds to this diagram may be represented in the form

$$-i A_{\mu}(x) \Pi^{\delta}_{\mu\nu}(x-y) A_{\nu}(y),$$

where

$$\Pi^{\delta}_{\mu\nu}(x-y) = -i e_q^2 \, Sp \sum_{j=1}^{\infty} \gamma_{\mu} S_j(x-y) \gamma_{\nu} S_j(y-x),$$

$$\langle 0 | T(q^{\delta}_j(x) \bar{q}^{\delta}_{j'}(x')) | 0 \rangle = (-)^j \delta_{jj'} S_j(x-x') =$$
$$= (-)^j \delta_{jj'} \frac{1}{(2\pi)^4 i} \int \frac{dp \, e^{-ip(x-x')}}{M_j(\delta) - \hat{p} - i\varepsilon} .$$

The vacuum polarization contains the ultraviolet divergences. In order to remove them, we will use the gauge invariant Pauli-Villars regularization procedure with additional conditions (see /5/). Then we obtain

$$\tilde{\Pi}^{\delta}_{\mu\nu}(p) = \int dx \, e^{-ipx} \Pi^{\delta}_{\mu\nu}(x) = (g_{\mu\nu} p^2 - p_{\mu} p_{\nu}) \Pi^{\delta}(p^2),$$

$$\Pi^{\delta}(p^2) = \frac{e_q^2}{12\pi^2} \sum_{j=1}^{\infty} \frac{p^2}{4M_j^2(\delta)} \int_0^1 \frac{du \sqrt{1-u} \, (1 + \frac{1}{2}u)}{1 - \frac{p^2}{4M_j^2(\delta)} u - i\varepsilon} .$$

This series converges well because for $\delta \to 0$ $M_j(\delta) \simeq \frac{j^{\sigma}}{\delta} \frac{1}{L}$ $(\sigma > 1)$ and we get (when $\delta \to 0$).

$$\Pi^{\delta}(p^2) \simeq \frac{e_q^2}{60\pi^2}\, p^2 L^2 \sum_{j=1}^{\infty} \frac{\delta^{2}}{j^{2\delta}} . \tag{2.11}$$

The function $\Pi^{\delta}(p^2)$ tends to zero as δ^2 because the series in (2.11) converges.

The virton loops containing more than two photon lines (Fig.2) can be represented in the form

$$\Pi^{\delta}_{\mu_1,\ldots,\mu_{2n}}(p_1,\ldots,p_{2n}) \sim$$

$$\sim \sum_{j=1}^{\infty} \int dk \sum_{(1,\ldots,2n)} Sp\Big\{\gamma_{\mu_{2n}} \tilde{S}^{\delta}_j\Big(k+\sum_{i=1}^{2n-1} p_i\Big)\gamma_{\mu_{2n-1}} \cdots \gamma_{\mu_2} \tilde{S}^{\delta}_j(k+p_1)\gamma_{\mu_1} \tilde{S}^{\delta}(k)\Big\}. \tag{2.12}$$

Here $\sum_{(1,\ldots,2n)}$ means the sum over all permutations of photon vertices $\gamma_{\mu_1},\ldots,\gamma_{\mu_{2n}}$.

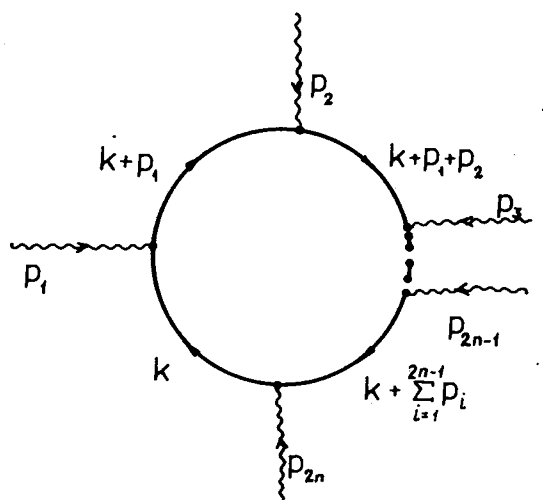

Fig. 2

The integral in (2.12) does not contain ultraviolet divergences and the series over j converges well. In the limit $\delta \to 0$ because of $M_j(\delta) \to \infty$ we get

$$\lim_{\delta \to 0} \Pi^{\delta}_{\mu_1,\ldots,\mu_{2n}}(p_1,\ldots,p_{2n}) = 0.$$

Thus any matrix elements of the S-matrix describing the photon-photon interaction by means of the virton loops are zero.

3. The interaction of virtons with photons and other physical particles

Let us introduce into consideration other physical particles (for example, π -mesons, ρ -mesons and so on). We will assume that fields of physical particles are the usual quantized fields satisfying appropriate equations. Our hypothesis consists in that the physical fields do not interact with each other directly but by means of the exchange of virtons. In this case it turned out that the S-matrix elements describing the interaction between photons and physical particles (π -mesons, for example) are not zero in the limit $\delta \to 0$.

As an example, let us assume that the interaction between mesons and virtons is described by a Lagrangian of the type

$$\mathcal{L}_I(x) = g\, \pi(x)\big(\bar{q}(x)\Gamma q(x)\big), \tag{3.1}$$

where Γ is a Dirac matrix. Now we consider a part of the virton loop containing n photon lines between two meson vertices as it is shown in Fig.3. This term can be represented in the following form:

$$T^{\delta}_{\mu_1,\dots,\mu_n}(k_1,\dots,k_n,q) =$$

$$= \sum_{(1,\dots,n)} \sum_{j=1}^{\infty} (-)^j \sqrt{A_j(\delta)}\, \Gamma \frac{(-)^j}{M_j-\hat{q}_{n+1}} (-)^j \gamma_{\mu_n} \cdot \ldots \cdot (-)^j \gamma_{\mu_1} \frac{(-)^j}{M_j-\hat{q}_1} \Gamma (-)^j \sqrt{A_j(\delta)} =$$

$$= \sum_{(1,\dots,n)} \sum_{j=1}^{\infty} (-)^j A_j(\delta)\, \Gamma \frac{1}{M_j-\hat{q}_{n+1}} \gamma_{\mu_n} \frac{1}{M_j-\hat{q}_n} \gamma_{\mu_{n-1}} \cdots \gamma_{\mu_1} \frac{1}{M_j-\hat{q}_1} \Gamma . \qquad (3.2)$$

Here $q_k = q + \sum_{i=1}^{k-1} k_i$.

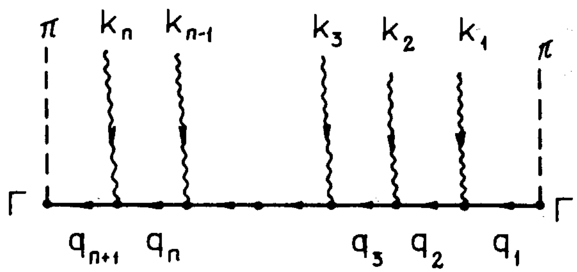

Fig.3

Let us introduce into consideration the d_μ -operator following paper /6/. This operator is defined as follows

$$n_\mu d_\mu(k) F(\hat{q}) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} e^{(k\frac{\partial}{\partial q})} \left[F(\hat{q} + \varepsilon \hat{n} e^{-(k\frac{\partial}{\partial q})}) - F(\hat{q}) \right], \qquad (3.3)$$

where n_μ is a 4-vector, $F(\hat{q})$ is a function depending on $\hat{q}$. For the Dirac propagator this formula gives

$$d_\mu(k) \frac{1}{M-\hat{q}} = \frac{1}{M-\hat{q}-\hat{k}} \gamma_\mu \frac{1}{M-\hat{q}} ,$$

$$d_{\mu_2}(k_2) d_{\mu_1}(k_1) \frac{1}{M-\hat{q}} = \frac{1}{M-\hat{q}-\hat{k}_1-\hat{k}_2} \gamma_{\mu_1} \frac{1}{M-\hat{q}-\hat{k}_2} \gamma_{\mu_2} \frac{1}{M-\hat{q}} +$$

$$+ \frac{1}{M-\hat{q}-\hat{k}_1-\hat{k}_2} \gamma_{\mu_2} \frac{1}{M-\hat{q}-\hat{k}_1} \gamma_{\mu_1} \frac{1}{M-\hat{q}} . \qquad (3.4)$$

Applying these formulas to (3.2) we obtain

$$T^{\delta}_{\mu_1,\dots,\mu_n}(k_1,\dots,k_n,q) = d_{\mu_n}(k_n) \cdot \ldots \cdot d_{\mu_1}(k_1)\, \Gamma \sum_{j=1}^{\infty} \frac{(-)^j A_j(\delta)}{M_j - \hat{q} - i\varepsilon} \Gamma =$$

$$= d_{\mu_n}(k_n) \cdot \ldots \cdot d_{\mu_1}(k_1)\, \Gamma G^{\delta}_c(\hat{q}) \Gamma . \qquad (3.5)$$

For example, let us consider the Feynman diagrams shown in Fig.4.(a,b,c). We get

$$M^{\delta}_{(a)} = \int dq\, Sp\{ d_\mu(k) G^{\delta}_c(\hat{q}) \Gamma \},$$

$$M^{\delta}_{(b)} = \int dq\, Sp\{d_{\mu_2}(k_2) d_{\mu_1}(k_1) G^{\delta}_c(\hat{q})\Gamma\},$$

$$M^{\delta}_{(c)} = \int dq\, Sp\{[d_{\mu}(k) G^{\delta}_c(\hat{q})]\Gamma G^{\delta}_c(\hat{q}+\hat{p})\Gamma\}. \tag{3.6}$$

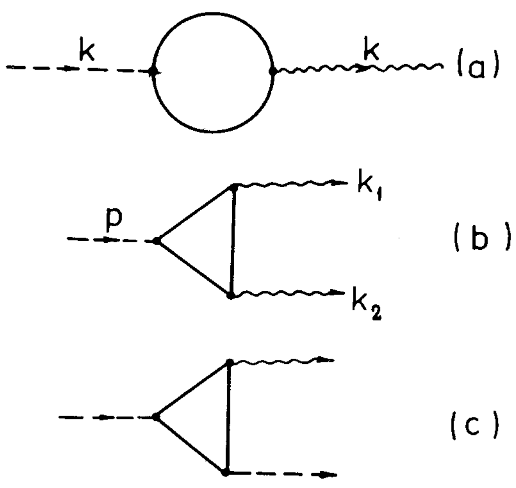

Fig.4

In order to take the limit $\delta \to 0$ we have to go to the Euclidean metric and then put $\delta = 0$. The integrals in (3.6) will converge and are not zero. For matrix elements (3.6) we obtain $(\hat{p} = i\hat{p}_E,\ \gamma^E_\mu \gamma^E_\nu + \gamma^E_\nu \gamma^E_\mu = 2\delta_{\mu\nu})$

$$M_{(a)} = i\int dq_E\, Sp\{d_\mu(k_E) G(i\hat{q}_E)\Gamma^E\},$$

$$M_{(b)} = i\int dq_E\, Sp\{d_{\mu_2}(k_{2E}) d_{\mu_1}(k_{1E}) G(i\hat{q}_E)\Gamma^E\},$$

$$M_{(c)} = i\int dq_E\, Sp\{[d_\mu(k_E) G(i\hat{q}_E)]\Gamma^E G(i\hat{q}_E + i\hat{p}_E)\Gamma^E\}. \tag{3.7}$$

Here all momenta are taken in the Euclidian region.

4. The calculation of $d_{\mu_n}(k_n) \ldots d_{\mu_1}(k_1)\, G(\hat{p})$

Now the problem is how to calculate the functions $d_{\mu_n}(k_n) \ldots d_{\mu_1}(k_1)\, G(\hat{p})$ because the direct use of the definition (3.3) is quite difficult. We proceed in the following way. The propagator $G(\hat{p})$ is the entire analytical function (2.6). We can represent it in the form

$$G(i\hat{q}_E) = \frac{1}{2\pi i}\int_C \frac{d\zeta\, G(i\zeta)}{\zeta - \hat{q}_E}, \tag{4.1}$$

where the contour C envelops the positive real axis as is shown in Fig.5 because we integrate in the Euclidian region $(0 \le q_E^2 \le \infty)$, where

$$G(i\hat{q}_E) = \frac{1}{M}\exp\{il\hat{q}_E - \frac{l^2}{4}q_E^2\}$$

and

$$G(i\zeta) = \frac{1}{M}\exp\{il\zeta - \frac{l^2}{4}\zeta^2\}. \tag{4.2}$$

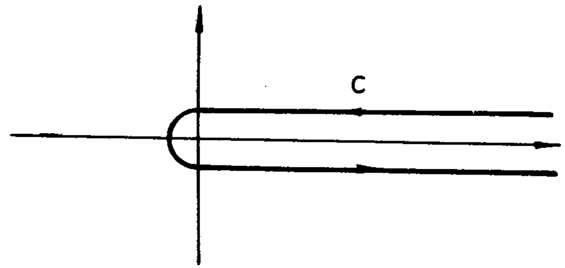

Fig.5

Making use of the representation (4.1) and formulas (3.4) we obtain

$$d_{\mu_n}(k_{nE})\cdot\ldots\cdot d_{\mu_1}(k_{1E})\,G(i\hat{q}_E)=$$

$$=\frac{1}{2\pi i}\int_C d\zeta\, G(i\zeta)\, d_{\mu_n}(k_{nE})\cdot\ldots\cdot d_{\mu_1}(k_{1E})\frac{1}{\zeta-\hat{q}_E}\,. \qquad (4.3)$$

This representation is very convenient for different calculations.

For example, simple calculations produce easily:

$$d_\mu(k_E)G(i\hat{q}_E)=\frac{1}{2\pi i}\int_C d\zeta\, G(i\zeta)\frac{1}{\zeta-\hat{q}_E-\hat{k}_E}\gamma_\mu^E\frac{1}{\zeta-\hat{q}_E}=$$

$$=\frac{1}{2\pi i}\int_C\frac{d\zeta\, G(i\zeta)[\zeta+\hat{q}_E+\hat{k}_E]\gamma_\mu^E[\zeta+\hat{q}_E]}{[\zeta^2-(q_E+k_E)^2][\zeta^2-q_E^2]}=$$

$$=i\gamma_\mu^E G_2\big((q_E+k_E)^2\big)+\frac{2q_{\mu E}+\hat{k}_E\gamma_\mu^E}{(q_E+k_E)^2-q_E^2}\Big\{G_1\big((q_E+k_E)^2\big)-G_1(q_E^2)+i\hat{q}_E\big[G_2\big((q_E+k_E)^2\big)-G_2(q_E^2)\big]\Big\}, \qquad (4.4)$$

where

$$G(i\zeta)=G_1(\zeta^2)+i\zeta G_2(\zeta^2)\,.$$

Let us calculate the function $M_{(a)}$ in (3.7) for $\Gamma=\gamma_\nu$. We have

$$M_{\mu\nu}^{(a)}=i\int dq_E\, Sp\{d_\mu(k_E)G(i\hat{q}_E)\gamma_\nu^E\}=$$

$$=i\int dq_E\frac{1}{2\pi i}\int_C d\zeta\, G(i\zeta)\, Sp\Big\{\gamma_\nu^E\frac{1}{\zeta-\hat{q}_E-\hat{k}_E}\gamma_\mu^E\frac{1}{\zeta-\hat{q}_E}\Big\}$$

Using the Feynman α -parametrization, one can get after standard transformations

$$M_{\mu\nu}^{(a)}=i\pi^2\int_0^\infty du\, 2\int_0^1 d\alpha\,\frac{1}{2\pi i}\int_C\frac{d\zeta\, G(i\zeta)}{[\zeta^2-u-\alpha(1-\alpha)K_E^2]^2}\times$$

$$\times\Big\{\delta_{\mu\nu}[2\zeta^2-2\alpha(1-\alpha)K_E^2-u]+4\alpha(1-\alpha)[\delta_{\mu\nu}K_E^2-K_{\mu E}K_{\nu E}]\Big\}.$$

It can be easily verified that the term with $\delta_{\mu\nu}$ is zero. Really

$$\int_0^\infty du\,\frac{1}{2\pi i}\int_C d\zeta\, G(i\zeta)\,\frac{u[2\zeta^2-2\alpha(1-\alpha)K_E^2-u]}{[\zeta^2-u-\alpha(1-\alpha)K_E^2]^2}=$$

$$=\int_0^\infty du\,\frac{1}{2\pi i}\int_C d\zeta\, G(i\zeta)\Big\{-1+\frac{(\zeta^2-\alpha(1-\alpha)K_E^2)^2}{[u-(\zeta^2-\alpha(1-\alpha)K_E^2)]^2}\Big\}=$$

$$= \frac{1}{2\pi i} \int_C d\zeta\, G(i\zeta) \int_0^\infty \frac{du\ (\zeta^2 - \alpha(1-\alpha)K_E^2)^2}{[u - (\zeta^2 - \alpha(1-\alpha)K_E^2)]^2} =$$

$$= \frac{1}{2\pi i} \int_C d\zeta\, G(i\zeta) [\zeta^2 - \alpha(1-\alpha)K_E^2] = 0 .$$

For the remaining term one can obtain finally

$$M^{(a)}_{\mu\nu} = -8\pi^2 (k_\mu k_\nu - \delta_{\mu\nu} k^2) \int_0^1 d\alpha\, \alpha(1-\alpha) \int_0^\infty du\, G_2(u + \alpha(1-\alpha)K_E^2) .$$

Now let us calculate the function $M_{(b)}$ in (3.7) for $\Gamma = \gamma_5$. This matrix element is connected with the decay $\pi^0 \to \gamma\gamma$. Making use of the representation (4.3), we obtain

$$M^{(b)}_{\mu\nu} = i \int dq_E\, Sp\{\gamma_5^E d_\mu(k_{2E}) d_\nu(k_{1E}) G(i\hat{q}_E)\} =$$

$$= i \int dq_E \frac{1}{2\pi i} \int_C d\zeta\, G(i\zeta)\, Sp\Big\{\gamma_5^E \frac{1}{\zeta - \hat{q}_E - \hat{k}_{1E} - \hat{k}_{2E}} \gamma_\mu^E \frac{1}{\zeta - \hat{q}_E - \hat{k}_{1E}} \gamma_\nu^E \frac{1}{\zeta - \hat{q}_E} +$$

$$+ \gamma_5^E \frac{1}{\zeta - \hat{q}_E - \hat{k}_{1E} - \hat{k}_{2E}} \gamma_\nu^E \frac{1}{\zeta - \hat{q}_E - \hat{k}_{2E}} \gamma_\mu^E \frac{1}{\zeta - \hat{q}_E} \Big\} =$$

$$= -i\, 16\pi^2 \varepsilon_{\mu\nu\alpha\beta} K_{1\alpha}^E K_{2\beta}^E \int_0^\infty du\, u \iiint_0^1 d\alpha_1 d\alpha_2 d\alpha_3\, \delta(1 - \alpha_1 - \alpha_2 - \alpha_3) \times$$

$$\times \frac{1}{2\pi i} \int_C d\zeta\, \zeta\, G(i\zeta) [\zeta^2 - u - K_{1E}^2 \alpha_2\alpha_3 - K_{2E}^2 \alpha_1\alpha_3 - p_E^2 \alpha_1\alpha_2]^{-3} =$$

$$= -i\, 8\pi^2 \varepsilon_{\mu\nu\alpha\beta} K_{1\alpha}^E K_{2\beta}^E \iiint_0^1 d\alpha_1 d\alpha_2 d\alpha_3\, \delta(1 - \alpha_1 - \alpha_2 - \alpha_3) \times$$

$$\times G_1(K_{1E}^2 \alpha_2\alpha_3 + K_{2E}^2 \alpha_1\alpha_3 + p_E^2 \alpha_1\alpha_2) ; \quad (p_E^2 = (k_{1E} + k_{2E})^2) .$$

If $K_1^2 = K_2^2 = 0$, then this matrix element can be written in the form

$$M^{(b)}_{\mu\nu} = -i\, 2\pi^2 \varepsilon_{\mu\nu\alpha\beta} K_{1\alpha}^E K_{2\beta}^E \int_0^1 dt\, G_1\Big(\frac{p_E^2}{4} t\Big) \ln \frac{1 + \sqrt{1-t}}{1 - \sqrt{1-t}} .$$

The last matrix element $M_{(c)}$ in (3.7) can be calculated by using the explicit form for $d_\mu(K_E) G(i\hat{q}_E)$ (4.4). The transformations are standard and we will not list them here.

5. Radiative and lepton decays of vector mesons

The conventional approaches to consideration of the radiative decays of vector mesons can be divided conditionally into phenomenological and dynamical types. Phenomenological considerations make use of the SU(3)-symmetry only. But in order to calculate decays of the type $V \to P\gamma$, $V \to \ell^+\ell^-, \ldots$, some additional assumptions (like the vector-dominance hypothesis) are needed to describe decays which are not connected by the SU(3)-symmetry.

In dynamic considerations there was made an attempt to describe hadrons as bound states of quarks. But until present time good relativistic theory of bound states is not yet formulated and any approach(for example, the Bethe-Salpeter equation)contains many reasonable and nonreasonable assumptions.

Now let us list some papers concerning consideration of the radiative decays of vector mesons.

In the "naive" nonrelativistic quark model /7/ these decays are considered as M1-transitions. Their rates can be calculated if the operator of the quark magnetic moment is known. The results of this work are given in Table 1, column I.

Table1.

	exper. (kev)	I	II	III	IV	V	VI	VII
$\omega\to\pi\gamma$	870±61	1.170	1100	730	900	330	350	898
$\omega\to\eta\gamma$	<50	6.4	13	13	13.8	–	1.9	4.8
$\rho\to\pi\gamma$	35±10	120	115	67	85.4	38	36	95
$\rho\to\eta\gamma$	<160	44	90	–	81.5	37	13.5	38
$K^{*0}\to K^0\gamma$	75±35	280	270	–	204	141	75	158
$K^{*+}\to K^+$	<80	70	65	–	49.5	–	15	15
$\varphi\to\pi^0\gamma$	5.9±2.1	0	0	11	2.8	–	0	0
$\varphi\to\eta\gamma$	65±15	304	110	61	91	–	77	136

In review /8/ the electromagnetic decay rates of vector mesons were considered in a phenomenological way under the following assumptions:

a) the vertices $VP\gamma$ are connected according to the SU(3)-symmetry,

b) the "ideal" $\varphi-\omega$ mixing,

c) the linear mass formula for the pseudoscalar mesons,

d) the rate $\Gamma(\omega\to\pi\gamma)=1100\,Kev$ is taken in the fit.

The results of these calculations are given in Table 1, column II.

In paper /9/ the electromagnetic decay rates of neutral vector and pseudoscalar mesons are calculated in a simple model combining vector dominance, SU(3) symmetry and the $\omega-\varphi$ and $\eta-\eta'$ mixing hypotheses. The width of the decay $\pi^0\to\gamma\gamma$ is used in the fit. The results conserning the electromagnetic decays of the vector mesons are given in Table 1, colomn III.

In paper /10/ the predictions of the exact SU(3)-symmetry are investigated in detail on the basis of reliable data on resonance decays. In particular, the rate $\Gamma(\omega\to\pi\gamma)=900\,Kev$ was taken as the fit for the investigation of decays $V\to P\gamma$. The results are given in Table 1, column IV.

In paper /11/ the magnetic dipole decays of vector mesons are considered in the framework of a quark model with a relativistic harmonic oscillator. The results are given in Table 1, column V.

All considered approaches predict the width of the decay $K^{*0}\to K^0\gamma$ three times larger than the experimental value. In paper /12/ the attempt was done to remove this disagreement by different violations of the SU(3)-symmetry. The author concluded that any simple SU(3)- breaking mechanism in the framework of the vector-dominance model cannot fit all the data with "reasonable" parameters.

The lepton decays of the vector mesons can be explained (with an accuracy of 10%) by the vector-dominance model (see, for example, /8, 13,14/ and further references).

Here in the framework of the developed electrodynamics of virtons we calculate the rates of decays $V \to P\gamma$ and $V \to \ell^+\ell^-$. Our results are given in Table 1, columns VI and VII and in Table 2. The general features of our model are described in the introduction and in refs. /1,2,3/. We will consider two models: with uncolored and colored quarks.

Table 2.

	exper (kev)	A	B
$\rho^0 \to e^+e^-$	6.44 ± 0.89	1.3	6.3
$\omega \to e^+e^-$	0.76 ± 0.47	0.15	0.72
$\varphi \to e^+e^-$	1.31 ± 0.15	0.42	1.22
$\pi^0 \to \gamma\gamma$	$(7.92 \pm 0.42) \cdot 10^{-3}$	$0.4 \cdot 10^{-3}$	$3.9 \cdot 10^{-3}$

The interaction Lagrangian of the vector and pseudoscalar mesons with quark-virtons is chosen in the following simple form.

1. Uncolored quarks

$$\mathcal{L}_I = g V_\mu^{SK} (\bar{q}^S \gamma_\mu q^K) + i h M^{SK} (\bar{q}^S \gamma_5 q^K), \tag{5.1}$$

2. Colored quarks

$$\mathcal{L}_I^{(c)} = g_c V_\mu^{SK} (\bar{q}_a^S \gamma_\mu q_a^K) + i h_c M^{SK} (\bar{q}_a^S \gamma_5 q_a^K), \tag{5.2}$$

where $g_c = \frac{g}{3^{1/3}}$, $h_c = \frac{h}{3^{1/3}}$, $L_c = 3^{2/3} L$ and $\xi_c = \xi$, $\eta_c = 1,4$.

Under this choice of parameters, in the model with colored quarks, mass corrections, strong meson decay widths are unchanged. Here M^{SK} is the octet matrix of the pseudoscalar mesons, V_μ^{SK} is the nonet matrix of the vector mesons (we consider the "ideal" $\varphi - \omega$ mixing). Further, K, S =1,2,3 are the SU(3) indices and a =1,2,3 the "colored" index. The other parameters of this model were obtained /2,3/ by fitting the mass corrections to the pseudoscalar and vector mesons and the rates of weak and strong meson decays. The numerical values of the parameters L and

$$\lambda_h = \frac{1}{16\pi^2} \frac{h^2}{(ML)^2}, \quad \lambda_g = \frac{1}{16\pi^2} \cdot \frac{g^2}{(ML)^2}$$

are shown in Fig.6 (a,b) as functions of parameter $\xi = \frac{2\ell}{L}$.

It was shown in /2,3/ that a satisfactory fit of experimental data can be performed in the region $0.3 \lesssim \xi \lesssim 3$.

We suppose that our virton-quarks have the fractional charges ($e_1 = e_p = \frac{2}{3}e$, $e_2 = e_3 = e_n = e_\lambda = -\frac{1}{3}e$) which are independent of the color index.

The electromagnetic interaction of the quark-virtons is described by the Lagrangian

$$\mathcal{L}_{Iem} = \sum_{S=1}^{3} \left(\bar{q}_a^S \{ Z_S(\hat{p} + e_S \hat{A}) - Z_S(\hat{p}) \} q_a^S \right) \tag{5.3}$$

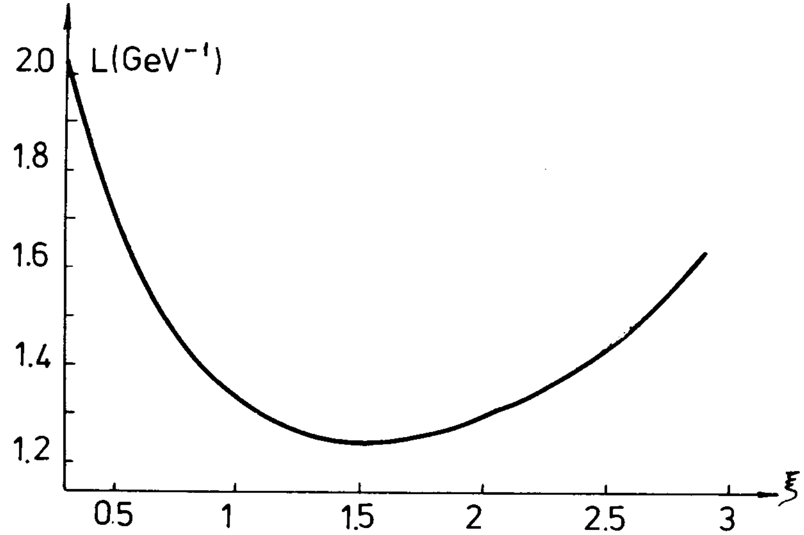

Fig.6a

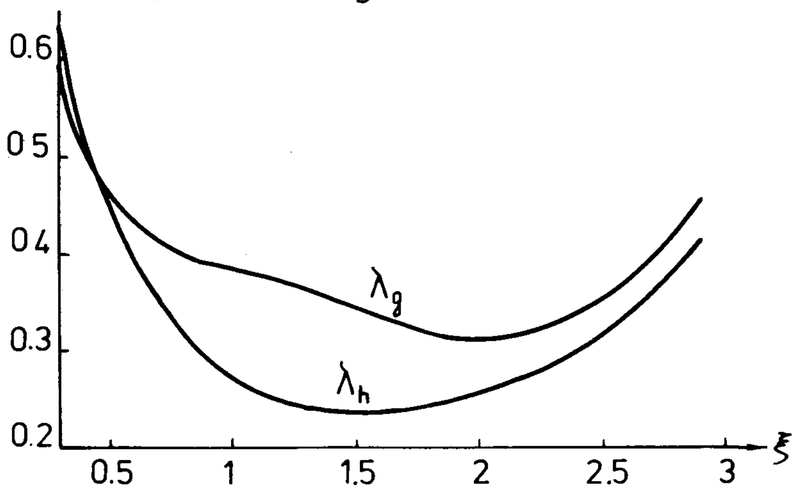

Fig.6b

or in the regularized form

$$\mathcal{L}^{\delta}_{Iem} = \sum_{s=1}^{3} e_s \left\{ \sum_{j=1}^{\infty} (-)^j \left(\bar{q}^s_{ja} \gamma_\mu q^s_{ja}\right) \right\} A_\mu . \tag{5.4}$$

Now we can calculate the matrix elements for the radiative decays of the vector mesons. Methods of calculation were developed in refs. /1-3/ and in sections 3 and 4. Here we present the results only.

I. The decay $V \to P\gamma$

This decay is described by the Feynman diagram shown in Fig. 7. The matrix element corresponding to this diagram is written for uncolored quarks in the form

$$T^{KS}_{\mu\nu}(p,q_1) = \frac{egh}{(2\pi)^4}\left(\frac{2}{L}\right)^3 \int dk_E \, Sp\left\{\gamma^E_\mu G_K(i\hat{k}_E - i\hat{P}_E)\gamma^E_5 \times d_\nu(-Q_{1E}) G_S(i\hat{k}_E)\right\}.$$

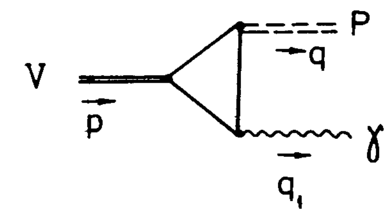

Fig.7

Here $P=\frac{L}{2}p$, $Q_1=\frac{L}{2}q_1$.
Since $\mu^2=\frac{1}{4}(M_V L)^2<1$ we will neglect this parameter.
For the width of this decay we obtain for the uncolored quarks

$$\Gamma(V\to P\gamma)=\frac{\alpha}{24}\, g^2_{VP\gamma}\, M_V^3\left(1-\frac{M_P^2}{M_V^2}\right)^3,$$

$$g^2_{VP\gamma}=256\,\lambda_g\lambda_h L^2\cdot R^2_{VP}(\xi)\cdot\beta_{VP}.$$

Here β_{VP} are the SU(3) indices:

$$\beta_{\rho\pi}=\frac{1}{9},\ \beta_{\rho\eta}=\frac{1}{3},\ \beta_{\omega\pi}=1,\ \beta_{\omega\eta}=\frac{1}{27},$$
$$\beta_{\varphi\eta}=\frac{8}{27},\ \beta_{\varphi\pi}=0,\ \beta_{K^{*+}K^{+}}=\frac{1}{9},\ \beta_{K^{*0}K^{0}}=\frac{4}{9}.$$

The functions $R_{VP}(\xi)$ are defined as follows:

$$R_{\rho\pi}=R_{\rho\eta}=R_{\omega\pi}=R_{\omega\eta}=\int_0^\infty du\,u\,e^{-2u^2}\sin\xi u\left[u\cos\xi u+\frac{1}{2}\xi\sin\xi u\right],$$
$$R_{\varphi\eta}=\int_0^\infty du\,u\,e^{-2,2u^2}\sin\xi u\left[1,1u\cos\xi u+\frac{1}{2}\xi\sin\xi u\right],$$
$$R_{K^{*+}K^{+}}=\int_0^\infty du\,u\,e^{-2,1u^2}\left[0,49\,\xi u\sin 2\xi u+0,5\,\xi\sin^2\xi u-0,075\,\xi u^2\right],$$
$$R_{K^{*0}K^{0}}=\int_0^\infty du\,u\,e^{-2,1u^2}\sin\xi u\left[1,05u\cos\xi u+0,5\,\xi\sin\xi u\right].$$

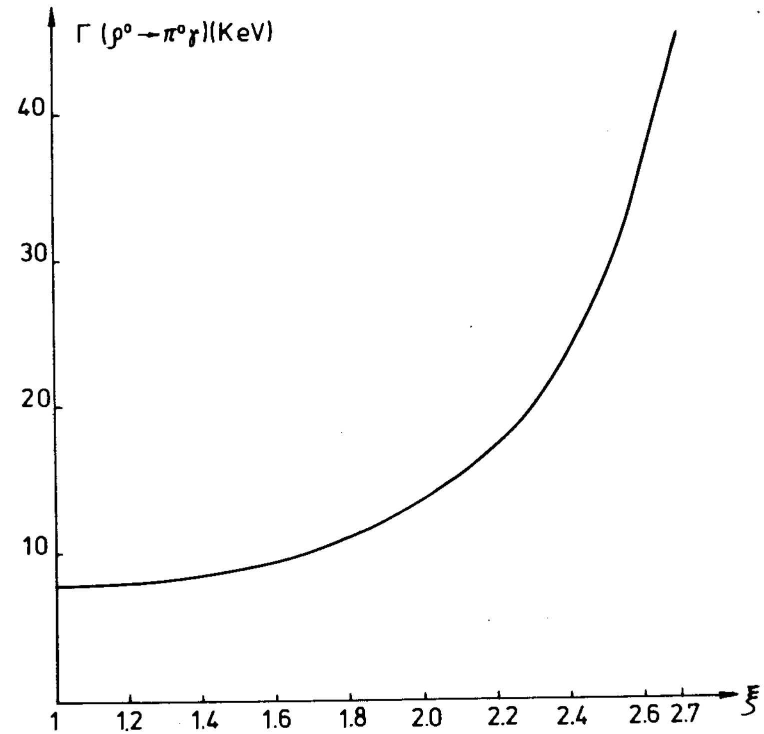

Fig 8.

The behaviour of the rate $\Gamma(\rho^0\to\pi^0\gamma)$ as a function of ξ is shown in Fig. 8.

For the colored quarks we obtain the rates

$$\Gamma_c(V\to P\gamma)=9\,\Gamma(V\to P\gamma).$$

The numerical values of the rates are given in Table 1, column VI is for the uncolored quarks when $\xi=2,6$ and column VII is for the colored quarks when $\xi=1,7$.

2. The decay $V \to e^+e^-$

This decay is described by the Feynman diagram shown in Fig. 9. The matrix element corresponding to this diagram is of the form as for the uncolored quarks

$$T_\mu^{\,s} = -\frac{e^2 g}{(2\pi)^4}\left(\frac{2}{L}\right)^3 I_s(Q^2)\,\frac{1}{M_V^2}\,\gamma_\mu \ .$$

Here

$$I_s(Q^2) = \frac{1}{3}\int dk_E\, Sp\left\{\gamma_\mu^E\, d_\mu(-Q_E)\, G_s(i\hat{k}_E)\right\},$$

$$Q^2 = -Q_E^2 = \frac{1}{4}(M_V L)^2 .$$

The width $\Gamma(V\to e^+e^-)$ for the uncolored quarks is

$$\Gamma(V\to e^+e^-) = \frac{\alpha^2}{3} M_V \left(\frac{f_V^2}{4\pi}\right)^{-1} .$$

Here

$$\left(\frac{f_V^2}{4\pi}\right)^{-1} = \lambda_g\,\frac{64}{9\pi}\,R_V^2(\xi)\cdot\beta_V \ ;$$

$$\beta_V:\ \beta_\rho = \frac{1}{2},\ \beta_\omega = \frac{1}{18},\ \beta_\varphi = \frac{1}{9}\ ;$$

$$R_\rho(\xi) = R_\omega(\xi) = \int_0^\infty du\, \sin\xi u\, e^{-u^2},$$

$$R_\varphi(\xi) = \int_0^\infty du\, \sin\xi u\, e^{-1{,}1u^2} .$$

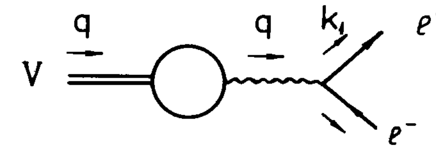

Fig.9

The behaviour of the rate $\Gamma(\rho^0\to e^+e^-)$ as a function of ξ is given in Fig. 10.

For the colored quarks we obtain

$$\Gamma_c(V\to e^+e^-) = 3^{4/3}\,\Gamma(V\to e^+e^-) .$$

The numerical values of these rates are given in Table 2, column A is for the uncolored quarks when ξ =2,6 and column B is for the colored quarks when ξ =1,7. In the second case the agreement with experimental data is quite good.

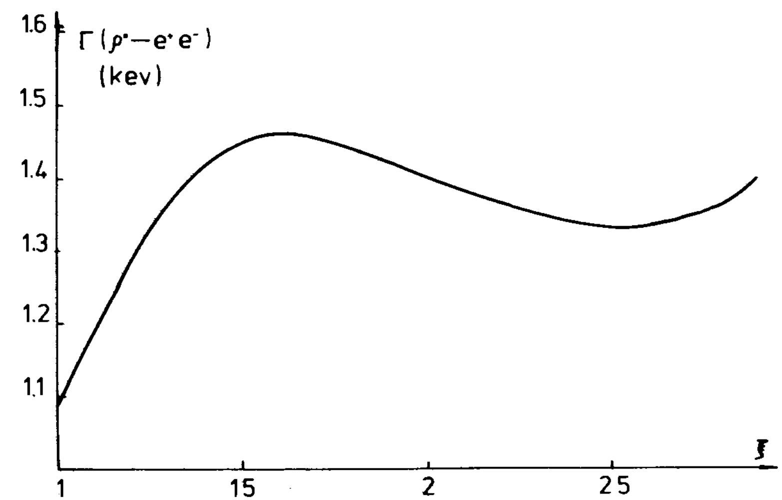

Fig.10

3. The decay $\pi^0 \to \gamma\gamma$

Finally we want to consider the decay $\pi^0 \to \gamma\gamma$ in our model. This matrix element was calculated in section 4. The rate $\Gamma(\pi^0 \to \gamma\gamma)$ can be written in the form for the uncolored quarks

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{\pi}{4}\alpha^2 m_\pi^3 g^2_{\pi^0\gamma\gamma},$$

$$g^2_{\pi^0\gamma\gamma} = \frac{\lambda_h}{18\pi^2} L^2$$

and for the colored quarks

$$\Gamma_c(\pi^0 \to \gamma\gamma) = 3^{8/3}\,\Gamma(\pi^0 \to \gamma\gamma).$$

The numerical value of this rate is given in Table 2.

Thus our results are in satisfactory agreement with the experimental data. It should be noted that the model with colored quarks looks more preferable since it gives good agreement with experiment.

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References:

1. A.Z.Dubničkovà, G.V.Efimov, JINR, E2-10371, Dubna, 1977.
2. A.Z.Dubničkovà, G.V.Efinov, M.A.Ivanov, JINR, P2-10734, Dubna, 1977.

3. G.V.Efimov, M.A.Ivanov, JINR, P2-10740, Dubna, 1977.
4. W.Becker, D.Grosser, Tübingen, 1977.
5. G.V.Efimov, Annals of Physics, 71,466, 1971.
6. N.M.Kroll, Nuovo Cimento, 45A, 65, 1966.
7. C.Bechi, G.Morpurgo, Phys.Rev., B140, 687, 1965.
8. G.Morpurgo. Lectures given at the E.Majorana 1971 Summer School. Properties of the fundamental interactions, Volume 9, Part B, Editor A.Zichichi.Editrice compositori Bologna, 1973.
9. A.Kotlewski, W.Lee, M.Suzuki, J.Thaler, Phys.Rev., D8, 348,1973.
10. P.Rotelli, M.Scadron, Nuovo Cimento, 15A, 648, 1973.
11. A.B.Govorkov, S.B.Drenska, JINR, E2-10078, Dubna, 1976.
12. R.Thews, Phys.Rev., D14, 3021, 1976.
13. T.Kitazoe, T.Teshima, Nuovo Cimento, 57A, 497, 1968.
14. R.P.Feynman. Photon-Hadron Interactions. W.A.Benjamin, Inc., Reading, Massachusetts, 1972.

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