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Уравнение движения точечного источника скалярного поля в общей теории относительности

В работе выведено уравнение движения точечного источника (заряда) скалярного поля в общей теории относительности. Рассматриваются два варианта уравнения скалярного поля - с традиционной и конформно-инвариантной кинематической частью. Сравниваются два подхода к выводу уравнения движения излучающего заряда: общерелятивистское развитие подхода Дирака и непосредственное выделение расходимостей. Оказывается, что первый из них не дает приемлемого результата в случае конформно-инвариантной кинематической части, а второй приводит к уравнению движения заряда, не зависящему от выбора уравнения поля. Как и в случае электрического заряда, на скалярный заряд действует сила, зависящая от кривизны пространства-времени в точке нахождения заряда, что противоречит некоторым формулировкам принципа эквивалентности.

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The Equation of Motion of Point Scalar Field Source in General Relativity

A general relativistic equation of motion of a point source of a scalar field is obtained. The equation has the same form for two different forms of the field equation, one with the traditional kinematical part and the second with the conformal-invariant one. As in the case of electric charge [2,10] the source is subjected to a force that depends explicitly on the space-time curvature in the point of localization of the source. Disagreement of this fact with the principle of equivalence is discussed.

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1. FORMAL PRELUDES

$V_{1,3}$ is a Riemannian space with a metric tensor $g_{\mu\nu}(x)$, $x \in V_{1,3}$, of signature +2 (space-time); $\det(g_{\mu\nu}) \equiv g$.

Tensor indices $\mu, \nu, ... = 0,1,2,3$ refer to an arbitrary point $x \in V_{1,3}$, while $\alpha, \beta, ..., \lambda = 0,1,2,3$ refer to a point on the world-line $z^\alpha = z^\alpha(\tau)$ of the point source (charge), τ being its proper time. A primed index refers to the argument $z' = z(\tau')$, e.g., $\Omega^{\alpha'} \equiv \Omega^\alpha(z')$.

The covariant derivative with respect to τ is denoted by a dot over a function or as $D/d\tau$. Note that

$$\dot{z}^2 = g_{\alpha\beta}\dot{z}^\alpha \dot{z}^\beta = -c^2 ,$$

c being the light velocity. The covariant differentiation with respect to x^μ or z^α is denoted by a dot in the lower indices in front of the corresponding indices which may be both of the lower and upper cases. As an example we write out the condition that determines the curvature tensor $R^\mu{}_{\nu\sigma\rho}(x)$:

$$A^\mu{}_{.\nu\rho} - A^\mu{}_{.\rho\nu} = R^\mu{}_{\nu\sigma\rho} A^\sigma ,$$

$A^\mu(x)$ being an arbitrary vector field,

$$R_{\mu\nu} = g^{\sigma\rho} R_{\mu\sigma\nu\rho}$$

is the Ricci tensor.

$\sigma(x,z)$ is the half of the geodesic interval between points x and z ("world function"); it is determined by the equations

$$\sigma_{\cdot\mu}\sigma_{\cdot}^{\ \mu} = \sigma_{\cdot\alpha}\sigma_{\cdot}^{\ \alpha} = 2\sigma$$

and the condition $\lim_{x\to z}\sigma=0$. For instance, in the Minkowsky space

$$2\sigma = -(x^{o}-z^{o})^{2} + (x^{1}-z^{1})^{2} + (x^{2}-z^{2})^{2} + (x^{3}-z^{3})^{2}.$$

$\bar{g}_{\mu\alpha}(x,z)$ is the two-point tensor of parallel transport. It is defined by the condition

$$\bar{\lambda}_{\mu}(x) = \bar{g}_{\mu\alpha}(x,z)\lambda^{\alpha}(z),$$

where $\bar{\lambda}_{\mu}(x)$ is a resultant of the parallel transport of $\lambda_{\alpha}(z)$ along the geodesic segment connecting x and z. Definition and properties of $\sigma(x,z)$, $\bar{g}_{\mu\alpha}(x,z)$ and other two-point functions are perfectly represented in the first part of paper /1/ by DeWitt and Brehme. Our notation is in complete accordance with theirs.

2. FORMULATION OF THE PROBLEM

Our purpose is to deduce the general relativistic equation of motion of the point particle radiating the scalar field which takes into account the reaction of the radiation on the particle (the radiation damping). Such a source of the scalar field may be called a scalar charge only conditionally because there is no conservation law analogous to that of electric charge.

The general relativistic generalization of the Lorentz-Dirac equation for point electric charge e with (physical) rest mass M was obtained by Hobbs /2/ in the following form

$$M\ddot{z}^{\alpha} = ec^{-1}F^{(in)\alpha}{}_{\beta}(z)\dot{z}^{\beta} + 2e^{2}(\Gamma^{\alpha} - P^{\alpha}) + \\ + e^{2}c^{-1}\dot{z}^{\beta}\int_{-\infty}^{\tau} f^{\alpha}{}_{\beta\gamma'}(z,z')\dot{z}^{\gamma'}\,d\tau', \qquad (1)$$

where $F^{(in)}_{\mu\nu}(x)$ is the tensor of an external electromagnetic field; $f^{\mu}{}_{\nu\rho'}(x,x')$ is a some two-point tensor, which is a second rank antisymmetric tensor function of x (indices μ and ν) obeying the free Maxwell equation and is a vector function of x',

$$\Gamma^{\alpha} = \frac{1}{3c^{3}}(\dddot{z}^{\alpha} - c^{-2}\ddot{z}^{2}\dot{z}^{\alpha}) \qquad (2)$$

is Abragam's vector and

$$P^{\alpha} = \frac{1}{6c}[R^{\alpha\beta}(z)\dot{z}_{\beta} + c^{-2}\dot{z}^{\alpha}R_{\beta\gamma}(z)\dot{z}^{\beta}\dot{z}^{\gamma}]. \qquad (3)$$

Actually Hoobs corrected an error in calculation of De Witt and Brehme /1/, who had obtained equation (1) without the term proportional to P_{α}. They noted with satisfaction that the equation does not show any dependence on curvature at the charge location and interpreted this fact as an agreement with the principle of equivalence. Indeed, if their result were correct, the equation of motion in a locally inertial reference frame (i.e., in normal coordinates with origin at $z(\tau)$) would differ from the corresponding special relativistic one only by the last "tail" term. One might conclude from such a difference only that <u>somewhere</u> on the world-line of the charge the curvature is not zero. On the contrary, at least in conformal-flat $V_{1,3}$, where $f^{\mu}{}_{\nu\rho'}(x,x')\equiv 0$, the vector P_{α} provides a principal possibility to detect that $R_{\alpha\beta}\neq 0$, if so, <u>at a given point,</u> i.e., through observation of an arbitrary small part of world-line of <u>one</u> point charge. This means that one may determine whether a gravitational field is present at the point. This possibility of course is a local manifestation of the nonlocal system "charge+ its field" and it should be considered as an indication of necessity of a more exact formulation of the principle of equivalence rather than as a contradiction with the latter (see also a discussion in /3/).

So, the presence of the vector P_{α} in the equation of motion of an electric charge is of principal importance and it is interesting to know how equations of motion of point sources of other fields appear. The case of the scalar field is of particular interest because various authors write the kinematical part of field equation in two different forms: in the traditional minimal form

$$\Box\phi = g^{\alpha\beta}\phi_{\cdot\alpha\beta} = (-g)^{-1/2}\partial_\alpha\{(-g)^{1/2}\, g^{\alpha\beta}\partial_\beta\phi\}$$

and in conformal-invariant one [4,5]: $(\Box + \frac{R}{6})\phi$, where R is the scalar curvature. The term with R in the latter case is also considered sometimes as a breakdown of the principle of equivalence and the question arises whether equation of motion of a scalar charge depends on choice of the field equation.

The starting point of our consideration is the system of equations describing interaction of a scalar field ϕ with rest mass m and its point source (charge) with rest mass M_0:

$$(\Box + \frac{\Lambda}{6}R - m^2)\phi(x) = \lambda c \int_{-\infty}^{\infty} d\tau\, \delta^4(x,z), \tag{4}$$

$$\frac{D}{d\tau}\{[\mathfrak{M}_0 + \lambda\phi(z)]\dot{z}^\alpha\} = -\lambda c^2 \phi_{\cdot}{}^{\alpha}(z). \tag{5}$$

To make the constant of interaction λ dimensionless, we have introduced here $m = mc/\hbar$, $\mathfrak{M}_0 = M_0 c/\hbar$. The invariant δ-function $\delta^4(x,z)$ is defined by the condition

$$\int (dx)^4 \sqrt{-g(x)}\,\delta^4(x,z)\, f(x) = f(z).$$

The constant Λ in Eq. (4) is introduced as to take into account the two aforementioned forms of the scalar field equation, corresponding to the values $\Lambda = 0$ and $\Lambda = 1$.

System (4),(5) is of a formal sense since ϕ and $\phi_{\cdot\alpha}$ include selfinteraction of the charge and take infinite values on its world-line. It is, however, remarkable that the infinities can be separated in the form "const $\cdot \ddot{z}^\alpha(\tau)$" and can be included into the observed mass of charge $\mathfrak{M}$ so that $\mathfrak{M}_0$ acquires the meaning of a bare mass. Such a renormalization of mass requires intermediate regularization of the field $\phi(z)$ which actually reduces to consideration of the field not at the point of location of the charge but "beside" it. The regularization and its removal may be performed in various way and, since in general relativity there is no invariance condition, the independence of final result of regularization is not obvious. We consider two procedures here, the first being the general relativistic development [1,2] of Dirac's approach[6] and the second being that has been used in the book [7] by Sokolov and Ternov.

3. DIRAC'S APPROACH

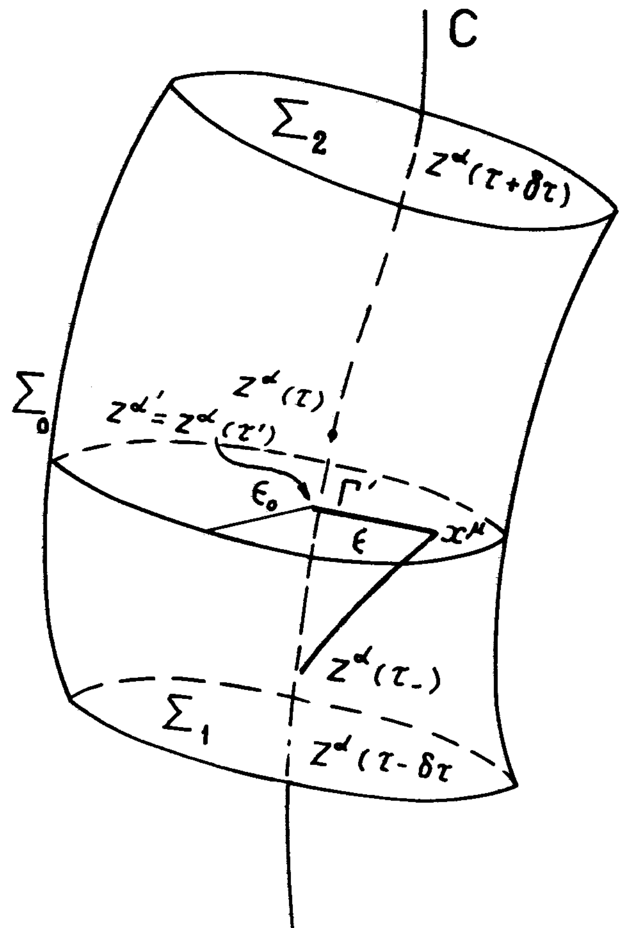
The approach is essentially as follows. The variable point $z^\alpha(\tau)$ of charge world-line is enveloped by a closed hypersurface Σ which is a cylinder, its height and radius being $2\delta\tau$ and ϵ_0, respectively. The latter is measured along a space-like geodesic normal to the world-line. One applies the Gauss theorem to the contraction of the symmetric energy-momentum tensor $T_{\mu\nu}$ with vectors $\xi_i{}^\mu(x)$ $(i = 1,2,3,4)$ of arbitrary field of reperes, i.e.,

$$\int_\Sigma d\Sigma^\mu(x) T_{\mu\nu}(x)\,\xi_i{}^\nu(x) = \int_{V_4} dV_4 (T_{\mu\nu}\xi_i{}^\nu)_{\cdot}{}^{\mu} = \int_{V_4} dV_4\, T_{\mu\nu}\xi_i{}^{\nu}{}_{\cdot}{}^{\mu}. \tag{6}$$

Here $T_{\mu\nu\cdot}{}^{\mu} = 0$ in virtue of Eqs.(4),(5).

If $\xi^\nu(x)$ is a Killing vector, then $T_{\mu\nu}\xi^{\nu}{}_{\cdot}{}^{\mu} = 0$.

In the Minkowsky space there is a natural repere field $\xi_i{}^\nu(x)$ which is composed of four Killing vectors that generates the translations. Then the condition for the integral over Σ in Eq.(6) to vanish gives rise to the Lorentz-Dirac equation. In general relativity there is no such natural repere field, and the only physically justified way to introduce a repere field is to relate it with the charge world-line $z^\alpha(\tau)$. . Hobbs [2] accomplished this by the parallel transport of an orthonormal repere specified at $z^{a}(\tau)$ to x along the geodesic. De Witt and Brehme [1] used actually parallel transport of a coordinate repere (four vectors tangent to coordinate lines at $z^{a}(\tau)$). Our way is somewhat intermediate between these. We choose the hypersurface Σ as it has been described (Fig.) and introduce any conti-



nous field $\xi_i^{\,a} = \xi_i^{\,a}(\tau)$ along the world-line $z^a = z^a(\tau)$ (denote it by C) of charge. Then the repere $\xi_i^{\,\mu}(x)$ at an arbitrary point x in the neighbourhood of C is obtained by parallel transport along the geodesic perpendicular Γ' from x to C, i.e.,

$$\xi_i^{\,\mu}(x) = \bar{g}^{\mu}{}_{a'}(x, z')\,\xi_i^{\,a'}(\tau'),$$

$z' \equiv z(\tau')$ is the end of Γ' on C.

The energy-momentum tensor $T_{\mu\nu}$ is obtained by variation of the action for system (4),(5) and has the form

$$T_{\mu\nu}(x;\Lambda) = \overset{(p)}{T}_{\mu\nu}(x) + \overset{(f)}{T}_{\cdot\mu\nu}(x;\Lambda), \tag{7}$$

$$\overset{(p)}{T}_{\mu\nu} = c^{-1}(\mathfrak{M}_0 + \lambda\phi(x))\int_{-\infty}^{\infty} d\tau''\delta^4(x,z'')\bar{g}_{\mu\alpha''}(x,z'')\bar{g}_{\nu\beta''}(x,z'')\dot{z}^{\alpha''}\dot{z}^{\beta''}, \tag{8}$$

$$\overset{(f)}{T}_{\mu\nu} = \phi_{\cdot\mu}\phi_{\cdot\nu} - \tfrac{1}{2} g_{\mu\nu}[\phi_{\cdot\rho}\phi_{\cdot}^{\ \rho} + (m^2 - \Lambda\frac{R}{6})\phi^2] - \frac{\Lambda}{6}[R_{\mu\nu}\phi^2 + \phi^2_{\cdot\mu\nu} - g_{\mu\nu}\Box\phi^2]. \tag{9}$$

Let us substitute into $T_{\mu\nu}(x;\Lambda)$

$$\phi(x) = \phi^{in}(x) + \phi^{-}(x),$$

where the $\phi^{in}(x)$ is a free external field and $\phi^{-}(x)$ is the retarded field of the charge [1] :

$$\phi^{-}(x) = -\frac{\lambda c}{4\pi}\left\{\frac{\Delta^{1/2}(x,z)}{\dot{\sigma}(x,z)} - \int_{-\infty}^{\tau} d\tau''v(x,z'')\right\}_{\tau=\tau_-}. \tag{10}$$

Here

$$\Delta(x,z) = -\det(\sigma_{\cdot\mu a})(-g(x))^{1/2}(-g(z))^{1/2}, \tag{11}$$

$v(x,z)$ is a two-point solution of Eq.(4) with the vanishing right-hand side which is regular on the light-cone and satisfy the condition [1,8]

$$\lim_{x\to y} v(x,y) = \frac{1-\Lambda}{12}R(y) - \tfrac{1}{2}m^2 \tag{12}$$

and $\tau_-(x)$ is the proper time of intersection of the charge world-line and the past light-cone of the point x (Fig.).

Evaluate now the limits of $\epsilon_0 \to 0$ of integrals in the main equality (6). We start with the right-hand side and omit the repere index i for simplicity :

$$J_V = \int_{V_4} dV_4\, T^{\mu\nu}(x;\Lambda)\{\bar{g}_{\mu\alpha'\cdot\nu}\,\xi^{\alpha'} + (\bar{g}_{\mu\alpha'\cdot\beta'}\xi^{\alpha'}\dot{z}^{\beta'} + \bar{g}_{\mu\alpha'}\dot{\xi}^{\alpha'})\tau'_{\cdot\nu}\}. \tag{13}$$

Having differentiated the orthogonality condition for C and Γ′

$$\sigma_{\cdot\alpha'}(x,z')\dot{z}^{\alpha'} = 0$$

one obtains

$$\tau'_{\cdot\nu} = -\kappa^{-2}(x,z')D_{\nu\alpha'}(x,z')\dot{z}^{\alpha'},$$

where $D_{\mu\alpha}(x,z) = -\sigma_{\cdot\mu\alpha}(x,z)$ and

$$\kappa^2 = -\ddot{\sigma}(x,z)|_{z=z'} .$$

Now we take into account that a close neighbourhood of the curve C is under consideration only and substitute expansions in powers of geodesic distance ϵ from x to C into Eq.(13). We cite them for some quantities /1/ :

$$\bar{g}_{\mu\alpha'\cdot\beta'} = -\frac{\epsilon}{2}\bar{g}_{\mu}{}^{\delta'} R_{\beta'\gamma'\alpha'\delta'}\,\Omega^{\gamma'} + O(\epsilon^2), \tag{14}$$

where $\Omega^{\gamma'}$ is the unity tangent vector to C at the point z', i.e., $\Omega^{\gamma'}\Omega_{\gamma'} = 1$, $\Omega^{\gamma'}\dot{z}_{\gamma'} = 0$,

$$\tau'_{\cdot\nu} = -\frac{1}{\kappa^2}\bar{g}_{\nu\alpha'}\dot{z}^{\alpha'}[1 + O(\epsilon^2)], \tag{15}$$

$$\phi^-(x) = -\frac{\lambda c}{4\pi}\Big[\frac{1}{\epsilon\kappa} - \int_{-\infty}^{\tau'} d\tau^* v(z', z^*) + O(\epsilon^2)\Big], \tag{16}$$

$$\phi^-_{\cdot\mu}(x) = \frac{\lambda c}{4\pi\epsilon^2}\bar{g}_{\mu\alpha'}\Big[\frac{1}{\kappa}\Omega^{\alpha'} + \frac{\epsilon}{2\kappa^2}\ddot{z}^{\alpha'} + O(\epsilon^2)\Big], \tag{17}$$

$$\phi^-_{\cdot\mu\nu}(x) = -\frac{\lambda c}{4\pi\epsilon^3}\bar{g}_{\mu\alpha'}\bar{g}_{\nu\beta'}\Big[\frac{1}{\kappa}(3\Omega^{\alpha'}\Omega^{\beta'} - g^{\alpha'\beta'} + \frac{1}{\kappa^2}\dot{z}^{\alpha'}\dot{z}^{\beta'}) + \frac{\epsilon}{2\kappa^2}(\Omega^{\alpha'}\ddot{z}^{\beta'} - \Omega^{\beta'}\ddot{z}^{\alpha'}) + O(\epsilon^2)\Big], \tag{18}$$

Actually, τ', ϵ and $\Omega^{\gamma'}$ form the Fermi coordinates /9/ of the point x with respect to the curve C. The invariant volume in these coordinates gives

$$\int_{V_4} dV_4 = \frac{1}{c}\int_{\tau-\delta\tau}^{\tau+\delta\tau} d\tau' \int_0^{\epsilon_0} d\epsilon\,\epsilon^2 \int \frac{d\Omega\,\kappa^2}{\Lambda(x,z')}. \tag{19}$$

An integral of any term containing an odd number of the "direction cosines" $\Omega^{\alpha'}$ over $d\Omega$ vanishes. The use of this fact, of properties of $\bar{g}_{\mu\alpha}$ /1/, of expressions (14)-(18) and other necessary expansions leads to

$$J_V = \frac{1}{c}[\xi^{\alpha'}\dot{z}_{\alpha'} - \mathfrak{M}(\tau')]\Big|_{\tau'=\tau-\delta\tau}^{\tau'=\tau+\delta\tau} - \frac{1}{c}\int_{\tau-\delta\tau}^{\tau+\delta\tau} d\tau'\xi_{\alpha'}\{\ddot{z}^{\alpha'}\mathfrak{M}(\tau') + \dot{z}^{\alpha'}\frac{d}{d\tau'}[\lambda\phi^{in} + \frac{\lambda^2 c}{4\pi}\int_{-\infty}^{\tau'} d\tau'' v(z', z'')]\}, \tag{20}$$

where

$$\mathfrak{M}(\tau) = \mathfrak{M}_1 + \lambda\phi^{in} + \frac{\lambda^2 c}{4\pi}\int_{-\infty}^{\tau} d\tau'' v(z, z'') \tag{21}$$

and

$$\mathfrak{M}_1 = \mathfrak{M}_0 - \frac{\lambda^2}{4\pi}\lim_{\epsilon_0\to 0}\int_0^{\epsilon_0} d\epsilon\Big[\frac{\delta(\epsilon)}{\epsilon} - \frac{1}{2\epsilon^2}(1 - \frac{2}{3}\Lambda)\Big]. \tag{22}$$

The contribution to the second term in $\mathfrak{M}_1$ is given by ϕ^- in $T^{(p)}_{\mu\nu}$ and $g_{\mu\nu}\phi^-_{.\rho}\phi^{.\rho}$ in $T^{(f)}_{\mu\nu}$. Consider the left-hand side of the main equality (6). The hypersurface Σ is composed of the three parts: the two "cups" Σ^+ and Σ^- and the wall Σ°. In the coordinates τ', ϵ, $\Omega^{\gamma'}$ we have

$$\int_{\Sigma^\pm} d\Sigma^\mu = \mp \frac{1}{c} \int_0^{\epsilon_0} d\epsilon\epsilon^2 \int \frac{d\Omega}{\Delta} D^{\mu\gamma'} \dot{z}_{\gamma'}|_{\tau'=\tau\pm\delta\tau} ,$$

$$\int_{\Sigma^\circ} d\Sigma^\mu = \frac{\epsilon_0^2}{c} \int_{\tau-\delta\tau}^{\tau+\delta\tau} d\tau' \int \frac{d\Omega}{\Delta} \kappa^2 (\bar{g}^{\mu\gamma'}\Omega_{\gamma'} + O(\epsilon_0^3)).$$

Since $\bar{D}^{\alpha'\beta'} = \bar{g}_\mu{}^{\alpha'} D^{\mu\beta'} = g^{\alpha'\beta'} + O(\epsilon^2)$ the integration over $\Sigma^\pm$ is practically reduced to the same integrals over $d\tau$ and $d\Omega$ as for integration over V. The result has the form

$$\left(\int_{\Sigma^+} d\Sigma^\mu + \int_{\Sigma^-} d\Sigma^\mu\right) T_{\mu\nu}\xi^\nu = \frac{1}{c}[\xi^{\alpha'} \dot{z}_{\alpha'} - \mathfrak{M}(\tau')]|_{\tau'=\tau-\delta\tau}^{\tau'=\tau+\delta\tau} . \quad (23)$$

Having been substituted into Eq.(6) this term cansels with the first term in J_V, Eq.(20) and the left-hand side of Eq.(6) reduce to

$$J^{(f)}_{\Sigma^\circ} = \int_{\Sigma^\circ} d\Sigma^\mu \, T^{(f)}_{\mu\nu} \xi^\nu$$

since $T^{(p)}_{\mu\nu} \equiv 0$ on Σ°.

To calculate $J^{(f)}_{\Sigma^\circ}$, we need expressions of the form of Eq.(15)-(18), but of higher powers in ϵ. However, they are too cumbersome, especially, for $\phi^-_{.\mu}$ and $\phi^-_{.\mu\nu}$, to cite them in this schematic description, in spite of just these orders contain various contractions of $R_{\alpha\beta\gamma\delta}$ and $R_{\alpha\beta}$ with Ω^α and $\dot{z}^\alpha$ that give rise finally to the term P_α in the equation of motion. Calculating the integrals over directions, we make use of the relation

$$\int d\Omega\, \Omega_\alpha \Omega_\beta = \frac{4\pi}{3}(g_{\alpha\beta} + c^{-2} \dot{z}_\alpha \dot{z}_\beta)$$

and come to the following expression

$$\begin{aligned} J^{(f)}_{\Sigma^\circ} = c \int_{\tau-\delta\tau}^{\tau+\delta\tau} d\tau' \xi^{\alpha'} \{ & \lambda\phi^{\text{in}}_{.\alpha'} + \\ & + \frac{\Lambda}{3c^2} \frac{D}{d\tau'}[\dot{z}_{\alpha'}(\lambda\phi^{\text{in}} + \frac{\lambda^2 c}{4\pi} \int_{-\infty}^{\tau'} d\tau'' v(z',z''))] + \\ & + \frac{\lambda^2}{4\pi}[\tfrac{1}{2}(1-\tfrac{2}{3}\Lambda)\frac{\ddot{z}_{\alpha'}}{c^2} \lim_{\epsilon_0\to 0} \epsilon_0^{-1} + \Gamma_{\alpha'} - P_{\alpha'} + \\ & + c\int_{-\infty}^{\tau'} d\tau'' v_{.\alpha'}(z',z'') - \frac{1}{c} v(z',z')\dot{z}_{\alpha'}]\}. \end{aligned} \quad (24)$$

To obtain an equation of motion, we bring together Eqs.(20),(23) and (24) into Eq.(6) and use arbitrariness in $\xi^{\alpha'}$ and $\delta\tau$. However, Eq.(24) still shows that we have different equations for the different values of Λ. In the case $\Lambda = 1$, the equation so obtained is wittingly unappropriate because it does not coincide with Eq.(4), the terms Γ_α, P_α and "tail" term (i.e., radiation damping) being neglected. We think the reason is that the Gauss theorem may not be applied to the singular expressions that arise from the terms with second derivatives in $T_{\mu\nu}(x;\ 1)$. Strictly, the theorem may not be applied even to $T_{\mu\nu}(x;\ 0)$, but still in this case $(\Lambda=0)$ Dirac's approach gives rise to the result which is proved by other approaches. Therefore, as a final result obtained by the method of Dirac, we write down now only the equation of motion for charge of the scalar field obeying Eq.(3) with $\Lambda=0$:

$$\frac{D}{d\tau}\{[\mathfrak{M}_2 + \lambda\phi^{\text{in}} + \frac{\lambda^2 c}{4\pi}\int_{-\infty}^{\tau} d\tau' v(z,z')]\dot{z}_\alpha\} =$$
$$= -c^2[\lambda\phi^{\text{in}} + \frac{\lambda^2 c}{4\pi}\int_{-\infty}^{\tau} d\tau' v(z,z')]_{.\alpha} + \frac{\lambda^2 c^2}{4\pi}(\Gamma_\alpha - P_\alpha). \quad (25)$$

Note that derivatives of the upper limits of the integrals cancel because $\tau_{.\alpha} = -c^{-2}\dot{z}_\alpha$ according to Eq.(15). The physical (renormalized) mass $\mathfrak{M}_2$ is related to the bare mass $\mathfrak{M}_0$ as follows

$$\mathfrak{M}_2 = \mathfrak{M}_0 - \frac{\lambda^2}{4\pi} \lim_{\epsilon_0 \to 0} \left[\int_0^{\epsilon_0} d\epsilon \left(\frac{2\delta(\epsilon)}{\epsilon} - \frac{1}{2\epsilon^2} \right) - \frac{1}{2\epsilon_0} \right].$$

If one assumes

$$\int_0^{\epsilon_0} d\epsilon \frac{\delta(\epsilon)}{\epsilon} = \frac{1}{2\epsilon} \Big|_{\epsilon=0}$$

then the field contribution into the mass is a divergence of the form $-(\lambda^2/8\pi)\epsilon^{-1}|_{\epsilon=0}$.

In the next section we shall show that the equation of motion for the case $\Lambda = 1$ has the same form as Eq.(25).

4. DIRECT SEPARATION OF DIVERGENCES

Now we will separate infinities and renormalize the mass directly in the Eq.(5) following the procedure that was used for electric charge in special relativity in book [7] and was considered for general relativity in paper [10]. We formally substitute into (5)

$$\phi(z) = \phi^{in}(z) + \phi^-(z)$$

representing the retarded field $\phi^-(z)$ through the retarded Green function $G^-(x, y)$:

$$\phi^-(z) = -\lambda c \int_{-\infty}^{\infty} G^-(z, z')\, d\tau', \tag{26}$$

$$G^-(z,z') = \frac{1}{8\pi}\{\Delta^{1/2}(z,z')\delta[\sigma(z,z')] - \theta[-\sigma(z,z')]v(z,z')\} \times$$

$$\times [1 + \epsilon(\tau - \tau')],$$

where $\theta(x)$ is the unity Heavyside function and $\epsilon(x) = \theta(x) - \theta(-x)$. Obviously Eq.(26) may be expressed in the form

$$\phi^-(z) = -\frac{\lambda c}{8\pi}\int_{-\infty}^{\infty} d\tau\, [1+\epsilon(\tau-\tau')]\Delta^{1/2}(z,z')\delta[\sigma(z,z')] +$$

$$+ \frac{\lambda c}{4\pi}\int_{-\infty}^{\tau} d\tau' v(z,z').$$

To calculate $\phi^-(z)$, $\phi^-_{.\alpha}(z)$ and $\dot{\phi}^-(z)$ we pass to the integration with respect to $t = \tau - \tau'$ and expand the integrand in powers of t using expressions [1] for $x \to x'$ of two-point functions σ, σ, $\Delta^{1/2}$, $\Delta^{1/2}_{.\beta}$:

$$\sigma = -\frac{1}{2}c^2 t^2 - \frac{1}{24}t^4 \ddot{z}^2 + O(t^5),$$

$$\sigma_{.\beta} = t\dot{z}_\beta - \frac{1}{2}t^2\ddot{z}_\beta + \frac{1}{6}t^3\dddot{z}_\beta + O(t^4),$$

$$\Lambda^{1/2} = 1 - \frac{1}{12}t^2 R_{\beta\gamma}\dot{z}^\beta\dot{z}^\gamma + O(t^3), \tag{27}$$

$$\Lambda^{1/2}_{.\beta} = -\frac{1}{6}tR_{\beta\gamma}\dot{z}^\gamma + O(t^2),$$

$$\delta(\sigma) = \frac{2}{c^2}[1 + O(t^2)]\frac{\delta(t)}{|t|}.$$

We use for calculation of the integrals which include $\delta'(\sigma)$ the formula

$$\frac{d}{d\sigma} = \left[-\frac{1}{c^2 t} + \frac{t}{6c^4}\ddot{z}^2 + O(t^2)\right]\frac{d}{dt},$$

which follows from (27) and after integration by parts we obtain the following formal expressions

$$\phi^-(z) = -\frac{\lambda}{4\pi c}\int_{-\infty}^{\infty}\frac{\delta(t)}{|t|}dt + \frac{\lambda c}{4\pi}\int_{-\infty}^{\tau} v(z,z')d\tau', \tag{28}$$

$$\dot{\phi}^-(z) = \frac{\lambda}{2\pi c}\int_{-\infty}^{\infty}\frac{\delta^2(t)}{|t|}dt + \frac{\lambda c}{4\pi}v(z,z) +$$

$$+ \frac{\lambda c}{4\pi}\dot{z}^\beta \int_{-\infty}^{\tau} v_{.\beta}(z,z')d\tau', \tag{29}$$

$$\phi^{-}_{\cdot\alpha}(z) = \frac{\lambda}{8\pi c^3}\ddot{z}_\alpha \int_{-\infty}^{\infty} \frac{\delta(t)}{|t|}dt - \frac{\lambda}{2\pi c^3}\dot{z}_\alpha \int_{-\infty}^{\infty} \frac{\delta^2(t)}{|t|}dt - $$
$$- \frac{\lambda}{4\pi}(\Gamma_\alpha - P_\alpha) - \frac{\lambda}{4\pi c}\dot{z}_\alpha v(z,z') + \frac{\lambda c}{4\pi}\int_{-\infty}^{\tau} v_{\cdot\alpha}(z,z')d\tau'. \qquad (30)$$

Substitution of Eqs. (28)-(30) with addition of corresponding derivatives of ϕ^{in} into Eq.(5) leads finally to Eq.(25), where $\mathfrak{M}_2$ is changed by

$$\mathfrak{M}_3 = \mathfrak{M}_0 - \frac{\lambda}{8\pi c}\int_{-\infty}^{\infty}\frac{\delta(t)}{|t|}dt.$$

But this equation is correct now both for $\Lambda = 1$ and for $\Lambda = 0$, and the difference between these two cases is that ϕ^{in} and $v(x,z)$ obey the different equations.

5. CONCLUSIONS

1. The use of Dirac's method to obtain an equation of motion of a point source (charge) of a scalar field gives an acceptable results only when the field obeys Eq.(4) with $\Lambda = 0$, i.e., when the kinematic part is traditional.

2. The direct separation of divergences in Eq.(5) gives an equation of the same form both for $\Lambda = 0$ and $\Lambda = 1$ in Eq.(4).

3. The equation of motion of a scalar charge contains the vector P_α explicitly depending on the Ricci tensor at the point of charge localization with the coefficient ½ in comparison with the case of electric charge.

4. The diverging contribution of scalar field in the charge mass is negative.

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