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DECAY MODES OF Υ (9.4), Υ'(10) AND THEIR PSEUDOSCALAR PARTNERS η_q, η_q'**

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Схемы распада $\Upsilon(9,4)$ и $\Upsilon'(10)$ и их псевдоскалярных партнеров

Лептонные и адронные ширины новых частиц $\Upsilon(9,4)$ и $\Upsilon'(10)$ вычислены в предположении, что квадрат волновой функции $|\Psi(0)|^2$ имеет ту же зависимость от массы, что и для известных векторных мезонов. Рассмотрены также распады псевдоскалярных партнеров $\Upsilon(9,4)$ и $\Upsilon'(10)$.

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Bertlmann R.A. **E2 - 11029**

Decay Modes of $\Upsilon(9,4)$, $\Upsilon'(10)$ and Their Pseudoscalar Partners

Motivated by the rather small widths of the new particles $\Upsilon(9.4)$ and $\Upsilon'(10)$ we calculate their leptonic and hadronic decay modes by assuming for the wave function squared $|\Psi(0)|^2$ the same mass dependence as for the known vector mesons. Decays of the pseudoscalar partners of $\Upsilon(9.4)$ and $\Upsilon'(10)$ are also considered.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

Very recently at Fermilab [1,2] were discovered at least two rather narrow resonances in the $\mu^+\mu^-$-mass spectrum of the reaction $p+(Cu,Pt)\to\mu^+\mu^-+X$. The widths of these particles $\Upsilon(9.4)$ and $\Upsilon'(10)$ are less than the mass resolution. These particles also have to be discovered and their properties investigated at the new big e^+e^--machines PETRA at DESY and PEP at SLAC which will start to work in the next years.

Up to this time theorists will have the chance to think about the implications of these new resonances to e^+e^--processes. This is certainly an important and very interesting problem. As the particles $\Upsilon(9.4)$ and $\Upsilon'(10)$ appear to be quite narrow we take guided by the sensational discoveries of the Ψ-family the rather enthusiastic point of view to assume them to be bound states of a quark-antiquark pair of a new flavour. We develop in analogy to Charmonium a new Quarkonium picture. The bound states are described by the wave functions of the quarks and can be calculated once a potential is given. Our proceeding will be more phenomenological. As already noticed by J.D.Jackson [3] the wave function squared for the known vector mesons $\rho, \omega, \phi, J/\Psi$ show all the same mass dependence (see Fig. 1) which is a very remarkable fact.

$$|\Psi(0)|^2 \sim M^n \qquad \text{with } n=1.89\pm 0.15. \qquad (1.1)$$

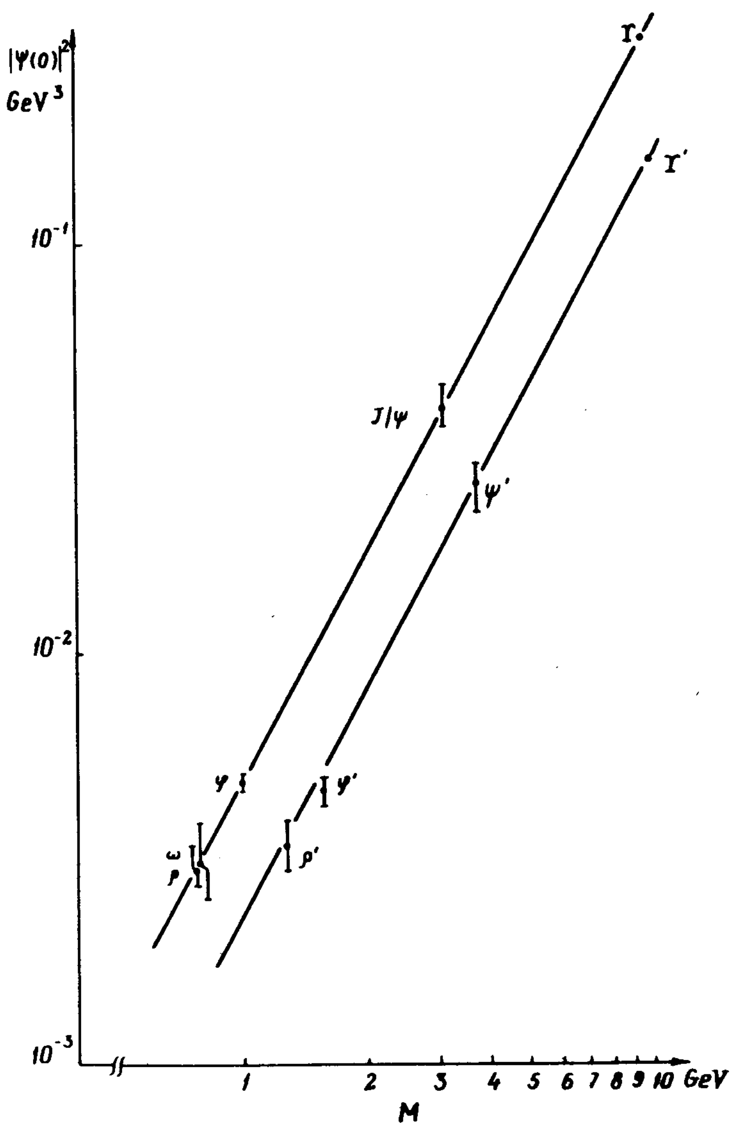

Fig. 1. The wave function squared $|\Psi(0)|^2$ as a function of the mass M is drawn for several vector mesons. The straight line corresponds to a slope 1.89 ± 0.15.

Further we assume that the newly discovered upsilon $\Upsilon(9.4)$ is the 1^3S_1 bound state and that the wave function squared $|\Psi(0)|^2$ has the same mass dependence as the already known vector mesons. Physically this means that the potential responsible for the binding of the new quarks is of the same nature as that of the old quarks. Under these quite reasonable assumptions it follows for the $\Upsilon(9.4)$ (see Fig. 1) that

$$|\Psi(0)|^2 \approx 0.32 \quad \mathrm{GeV}^3 . \tag{1.2}$$

With this value for the wave function squared we calculate in the next section the leptonic and hadronic widths, the electromagnetic corrections, which will turn out to be very important, the total width, and the area under the resonance. In Sec. 3 we proceed along the same lines with the $\Upsilon'(10)$ considered to be the first radial excitation 2^3S_1 of the $\Upsilon(9.4)$ and in Sec. 4 we calculate the electromagnetic and hadronic decay of the pseudoscalar partners η_q as the $1\ ^1S_0$ bound state and η'_q as the first radial excited bound state $2\ ^1S_0$. Finally in Sec. 5 we make some remarks about the photon-vector meson coupling constants.

2. WIDTHS OF $\Upsilon(9.4)$

2.1. Leptonic Decay Width

If Υ is a bound state of $q\bar{q}$ decaying electromagnetically to a lepton pair then we obtain for the leptonic width /3,4,5/

$$\Gamma(\Upsilon \to \ell\ell) = 16\pi \frac{\alpha^2 Q^2}{M^2} |\Psi(0)|^2 , \tag{2.1}$$

where Q is the quark charge in units of e, and M is the mass of Υ.

Using value (1.2) in formula (2.1) we get

$$\Gamma(Y \to \ell\bar{\ell}) = 9.7\, Q^2\, \mathrm{keV}. \qquad (2.2)$$

We immediately recognize that a measurement of the leptonic width implies a measurement of the charges of the involved quarks. We shall come back to this point later. As we yet don't know the new quark charge we take along the two values $Q = \frac{2}{3}$, $-\frac{1}{3}$ leading to

$$\Gamma(Y \to \ell\bar{\ell}) = \begin{cases} 4.3 & \mathrm{keV} \quad \text{f. } Q = \frac{2}{3} \\ 1.1 & \phantom{\mathrm{keV}} \quad \text{f. } Q = -\frac{1}{3} \end{cases} . \qquad (2.3)$$

These values can be compared with the ones of Ref.[6], where a logarithmic potential for the $q\bar{q}$ binding was used.

$$\Gamma(Y \to \ell\ell) = \begin{cases} 2.8 & \mathrm{keV} \quad \text{f. } Q = \frac{2}{3} \\ 0.7 & \phantom{\mathrm{keV}} \quad \text{f. } Q = -\frac{1}{3} \end{cases} . \qquad (2.4)$$

Their result is about a factor 0.6 smaller than ours, but coincides with the value quoted in Ref.[14].

2.2. Direct Hadronic Decay Width

In calculating the direct hadronic decay it is assumed that the $q\bar{q}$ pair annihilates into 3 coloured massless gluons which are quasi-free and turn into normal hadrons with essentially unit probability. The relevant formula is [7]

$$\Gamma_{dir}(Y \to \mathrm{had}) = \frac{160(\pi^2 - 9)}{81} \frac{\alpha_s^3}{M^2} |\Psi(0)|^2 . \qquad (2.5)$$

Here α_s is the so-called running strong coupling constant leading to asymptotic freedom in QCD [8]. Following Ref.[8] the α_s will have a logarithmic s-dependence:

$$\alpha_s(s) = \frac{\alpha_s(m_0^2)}{1 + (11 - \frac{2}{3}N)\frac{\alpha_s(m_0^2)}{4\pi} \log \frac{s}{m_0^2}} , \qquad (2.6)$$

where m_0 is a free scale parameter, and N is the number of quark flavours, in our case we take $N = 5$. From the direct hadronic decay of the J/Ψ, $\Gamma_{dir}(J/\Psi \to \mathrm{had}) = 47.4$ keV[9], results

$$\alpha_s(s = (3.1)^2) = 0.19 . \qquad (2.7)$$

By setting $m_0 = 1$ GeV[13] we can calculate $\alpha_s(m_0^2)$ and from (2.6) the strong coupling constant for $Y(9.4)$,

$$\alpha_s(s = (9.4)^2) = 0.15 . \qquad (2.8)$$

Using this value in formula (2.5) the direct hadronic decay width of Y will be

$$\Gamma_{dir}(Y \to \mathrm{had}) = 21.4 \ \mathrm{keV}. \qquad (2.9)$$

We notice the decrease of about a factor 2 compared to the corresponding J/Ψ width. This decrease is mainly due to the smaller value of α_s, since $|\Psi(0)|^2/M^2 \approx \mathrm{const}$ (see eq. (1.1)).

In passing we note that if $\alpha_s(s)$ is applied to the ϕ-decay, we rather take the strange-quark masses ($m_s \approx 0.13$ GeV) in formula (2.6) than the ϕ mass. Then the coupling constant comes out to be

$$\alpha_s(s = (0.26)^2) = 0.45 \qquad (2.10)$$

leading to

$$\Gamma(\phi \to 3\pi) \approx \Gamma(\phi \to \text{non strange had}) = 0.67 \ \mathrm{MeV}, \qquad (2.11)$$

in remarkable agreement with experiment.

2.3. Width of the Electromagnetic Decay into Hadrons

In the resonance region of the process $e^+e^- \to$ $\to$ hadrons the hadrons will be produced by (see Fig. 2)

i) direct coupling to the resonance,
ii) one-photon coupling to the resonance,
iii) non-resonant coupling.

This last contribution iii) is rather small compared to i) and ii) and is negligible for our purpose.

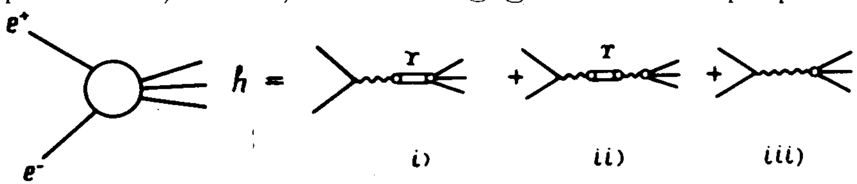

Fig. 2. Graphs for hadron production by: i) direct coupling to the resonance, ii) one-photon coupling to the resonance, iii) non-resonant coupling.

The Υ decay into hadrons via one photon represents the second order electromagnetic correction and its width can easily be derived by looking at Fig. 3 from which obviously follows

$$\Gamma(\Upsilon \to \gamma \to \text{had}) = R\big|_{\text{off resonance}} \cdot \Gamma(\Upsilon \to \mu\bar{\mu}) . \tag{2.12}$$

Here we have to take $R = \sigma_{had}/\sigma_{\mu\mu}$ off resonance but before threshold of the new quarks, since we are dealing with "ordinary" hadrons.

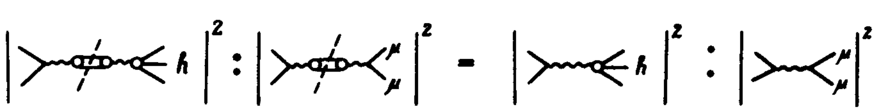

Fig. 3. Ratio of hadronic to muonic amplitudes squared.

By using R=10/3, the Quark Model value, this electromagnetic decay of the Υ will be

$$\Gamma(\Upsilon \to \gamma \to \text{had}) = \begin{cases} 14.4 & \text{f. } Q = \frac{2}{3} \\ & \text{keV} \\ 3.6 & \text{f. } Q = -\frac{1}{3} \end{cases} . \tag{2.13}$$

We see that the second order electromagnetic corrections become very important for the Υ(9.4) (even more than for J/Ψ) and must not be neglected.

This fact has important implications to the multipion final states. The Υ is assumed to have definite G -parity which is conserved in hadronic decay, but violated in electromagnetic decays. Take G=–1 then the even pion final states must be due to electromagnetic decays. Thus the pion cross sections on resonance behave for $Q = \frac{2}{3}$ like

$$\sigma_{odd\,\pi} : \sigma_{even\,\pi} = (1 - 1.5) : 1 , \tag{2.14}$$

and for $Q = -\frac{1}{3}$ like

$$\sigma_{odd\,\pi} : \sigma_{even\,\pi} = (4 - 6) : 1 , \tag{2.15}$$

with a bit varying values depending on the ratio R taken. At this point we want to mention that a measurement of the multipion states clearly points to the charges of the quarks involved.

2.4. Hadronic Width

By neglecting interference terms between gluon and photon exchanges in the hadronic decay the width can be written as

$$\Gamma_h = \Gamma_{dir}(\Upsilon \to \text{had}) + \Gamma(\Upsilon \to \gamma \to \text{had}) . \tag{2.16}$$

Using the values (2.9) and (2.13) we get

$$\Gamma_h = \begin{cases} 35.8 & \text{keV} \quad \text{f.} \quad Q = \frac{2}{3} \\ 25.0 & \text{keV} \quad \text{f.} \quad Q = -\frac{1}{3} \end{cases} . \tag{2.17}$$

2.5. Total Width

The total width will be the sum of the hadronic width and the widths of all leptonic decay modes where we consider the heavy lepton τ /10/ as already established. Under the assumption of lepton universality we have

$$\Gamma = \Gamma_h + 3\Gamma_{\ell\bar{\ell}} . \tag{2.18}$$

Inserting the above values (2.3) and (2.17) into (2.18) we finally arrive at the following result

$$\Gamma = \begin{cases} 48.7 & \text{keV} \quad \text{f.} \quad Q = \frac{2}{3} \\ 28.3 & \text{keV} \quad \text{f.} \quad Q = -\frac{1}{3} \end{cases} . \tag{2.19}$$

Thus the total width turns out to be very small, even smaller than the one of J/Ψ ($\Gamma(J/\Psi) = 69$ keV /9/). This is mainly due to the much smaller $\Gamma_{dir}(\Upsilon \to \text{had})$ governed by the decreasing α_s^3.

2.6. Area under the Resonance

In dealing with narrow resonances we already know that the peak cross section is highly sensitive to radiative corrections and to the energy spread of the beams whereas the area under the resonance is not /11/.

The area for a Breit-Wigner cross section σ_f^{BW} is

$$\int_{\text{resonance}} \sigma_f^{BW}(s)\,d\sqrt{s} = \frac{6\pi^2}{M^2}\,\frac{\Gamma_{ee}\Gamma_f}{\Gamma} , \tag{2.20}$$

where $\Gamma_{e\bar{e}}$ means the width to e^-e^+, and Γ_f, the width to a final state f. We calculate the area values for the hadronic final state

$$A_h = \int_{\text{resonance}} \sigma_h(s)\,d\sqrt{s} = \begin{cases} 0.83 & \mu\text{b MeV} \quad \text{f.} \quad Q = \frac{2}{3} \\ 0.25 & \mu\text{b MeV} \quad \text{f.} \quad Q = -\frac{1}{3} \end{cases} , \tag{2.21}$$

and for the muonic final state

$$A_{\mu\mu} = \int_{\text{resonance}} \sigma_{\mu\mu}(s)\,d\sqrt{s} = \begin{cases} 99.5 & \text{nb MeV} \quad \text{f.} \quad Q = \frac{2}{3} \\ 10.7 & \text{nb MeV} \quad \text{f.} \quad Q = -\frac{1}{3} \end{cases} . \tag{2.22}$$

Thus in measuring these areas one gets the first hint for the charges of the quarks involved.

3. WIDTHS OF Υ'(10)

Let's turn now to the second discovered particle Υ'(10) which we regard as the first radial excitation $2\,^3S_1$ of the ground state $1\,^3S_1$, Υ(9.4). For the wave function squared $|\Psi(0)|^2$ we make the rather plausible assumption that it behaves on M as the ground states (see Fig. 1). Now we restart the whole procedure and can predict the several decay widths which are listed together with the corresponding Υ(9.4) width values in the Table.

Table

Wave function squared $|\Psi(0)|^2$, widths, and resonance areas are tabled for Υ(9.4) and Υ'(10) by using $Q=-\frac{1}{3}$.

	Υ(9.4)	Υ'(10)	
$\lvert\Psi(0)\rvert^2$	0.32	0.16	GeV^3
$\Gamma(\ell\bar{\ell})$	1.1	0.47	keV
Γ_{dir}(had)	21.4	9.1	keV
Γ_γ (had)	3.6	1.6	keV
Γ_h	25.	-	keV
Γ	28.3	-	keV
A_h	0.25	-	μb MeV
$A_{\mu\mu}$	10.7	-	nb MeV

4. WIDTHS OF η_q AND η'_q

The n $^3S_1$ vector states certainly will have their n $^1S_0$ pseudoscalar partners which we call η_q and η'_q. From Quantum Mechanics of bound systems we expect the wave function squares $|\Psi(0)|$ of the two states 1 $^1S_0$ and 1 $^3S_1$ to be close together. Indeed, as a first approximation we set them equal what also implies a degeneracy in mass.

4.1. Electromagnetic Decay of η_q

The η_q can decay to two photons analogous to positronium. The relevant formula is /3,4/

$$\Gamma(\eta_q \to \gamma\gamma) = 48\pi \frac{\alpha^2 Q^2}{M^2} |\Psi(0)|^2 \tag{4.1}$$

which leads to the values

$$\Gamma(\eta_q \to \gamma\gamma) = \begin{cases} 13.0 & \text{keV} \quad \text{f. } Q = \frac{2}{3} \\ 3.3 & \quad\quad\;\; \text{f. } Q = -\frac{1}{3} \end{cases} \; . \tag{4.2}$$

4.2. Hadronic Decay of η_q

For a $^1S_0$ bound state the $q\bar{q}$ pair annihilates into two gluons which turn into normal hadrons. Thus obtaining the formula /3,4,12/

$$\Gamma(\eta_q \to \text{had}) = \frac{32\pi}{3} \frac{\alpha_s^2}{M^2} |\Psi(0)|^2 , \tag{4.3}$$

we get the value

$$\Gamma(\eta_q \to \text{had}) = 2.8 \text{ MeV}. \tag{4.4}$$

Again this decrease compared to the calculated value of charmonium η_c ($\Gamma(\eta_c \to \text{had}) =$ 6.8 MeV) is due to the decrease of α_s.

4.3. Hadronic Decay of η'_q

As the width ratios of the excited states to the ground states should be nearly equal for the pseudoscalar and vector particles

$$\frac{\Gamma(\eta'_q \to \text{had})}{\Gamma(\eta_q \to \text{had})} \approx \frac{\Gamma(\Upsilon' \to \text{had})}{\Gamma(\Upsilon \to \text{had})} , \tag{4.5}$$

we deduce from the knowledge of the right-hand side and (4.4) the following value for η'_q

$$\Gamma(\eta'_q \to \text{had}) \approx 1.2 \text{ MeV}. \tag{4.6}$$

5. PHOTON-VECTOR MESON COUPLING CONSTANT

In Vector Dominance Model the decay of a vector meson to a lepton pair is calculated via formula /15/

$$\Gamma(V \to \ell\bar{\ell}) = \frac{\alpha^2}{12} \left(\frac{f_V^2}{4\pi}\right)^{-1} M_V \text{ with } V = \rho, \omega, \phi, J/\Psi, \Upsilon. \tag{5.1}$$

The coupling constants of the new vector mesons Υ, Υ' to the photon are easily derived from (5.1) by using our values for the leptonic widths (see the Table).

$$\frac{f_\Upsilon^2}{4\pi} = \begin{cases} 9.7 & \text{f. } Q = \frac{2}{3} \\ 38.9 & \text{f. } Q = -\frac{1}{3} \end{cases} , \tag{5.2}$$

and

$$\frac{f_{\Upsilon'}^2}{4\pi} = \begin{cases} 23.5 & \text{f. } Q = \frac{2}{3} \\ 94.2 & \text{f. } Q = -\frac{1}{3} \end{cases} . \tag{5.3}$$

By comparing formula (2.1) with (5.1) we easily obtain the relation

$$\frac{M_V}{f_V^2} \sim Q^2 \frac{|\Psi_V(0)|^2}{M_V^2} . \tag{5.4}$$

As in our model $|\Psi_V(0)|^2/M_V^2 \approx \text{const}$ (see (1.1)) we get some feeling why the symmetry relations for the coupling constants f_V^{-2} are broken rather by the masses than by the masses squared.

Applying this mass breaking also to the radial excited states we calculate the following ratio

$$\frac{M_{V'}}{f_{V'}^2} / \frac{M_V}{f_V^2} \approx 0.43 \quad \text{for } \underline{\text{all}} \quad V=\rho,\omega,\phi,J/\Psi,\Upsilon. \qquad (5.5)$$

Taking for example $\rho'(1.25)$ we obtain $f_{\rho'}^2/f_\rho^2 \approx 3.9$ in good agreement with the value ~4 deduced from photo-production experiments /16/.

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