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Гладкие квазипотенциалы и описание рассеяния адронов при высоких энергиях

Исходя из высокоэнергетического представления для релятивистской амплитуды, по экспериментальным данным для упругого pp-рассеяния при энергиях 3,1 и 26,6 ГэВ в с.ц.м. восстановлен квазипотенциал. В основе рассмотрения лежат квазипотенциальное уравнение в терминах быстрот и связанное с ним нелинейное интегро-дифференциальное уравнение фазового типа. На рисунках приведены мнимые части квазипотенциала $\mathrm{Im}\,V(r)$ и релятивистского фурье-образа амплитуды $\mathrm{Im}\,A(\chi_2, r)$. Для сравнения даны кривые, отвечающие стандартной параметризации экспериментальных данных в области дифракционного пика.

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The Smooth Quasipotentials and the Scattering of High Energy Hadrons

Proceeding from the high energy representation for relativistic amplitude, the quasipotential is reconstruct using the experimental data for elastic pp-scattering at energy values 3.1 GeV and 26.6 GeV in c.m.s. The consideration is based on the QPE in terms of rapidities and connected with it nonlinear integral-differential phase type equation.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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The experimental data on high energy hadron scattering are well described using the local smooth quasipotential [1-3]. The quasipotential equations (QPE) [4,5], making the basis of this description, comprise the simplicity and physical transparency of quantum mechanics with consistent quantum-field consideration of the properties of relativistic particle interaction. Let us emphasize that an analogy of QPE with Schrödinger and Lippman-Schwinger equations makes it possible phenomenologically to take into account the interaction, in the framework of these equations, without using complex calculation procedure, necessary in a strict field - theoretical approach.

Proceeding from the analysis of experimental data and simple physical assumptions about the nature of high energy hadron interaction, the reconstruction of a smooth potential is of great interest (compare with review [1]).

The aim of this paper is to reconstruct an approximate form of the quasipotential for the interaction of two protons at high energies. Our consideration is based on the QPE in terms of rapidities [6,7] and, connected with it, nonlinear integral-differential phase type equation for the relativistic amplitude of scattering [8]. Note, that the expression for the scattering phase used in [1] was obtained on the basis of the nonrelativistic phase function method.

1. The Representation for the Scattering Amplitude

In ref. [8] the following equation for the relativistic scattering function $A(\chi_q, \vec{n}_p, r, \vec{n}_q)$

$$\frac{d}{dr} A(\chi_q, \vec{n}_p, r, \vec{n}_q) = -\frac{r^2}{4\pi} \int d\vec{n}_r V(\vec{r}, E_q) \times$$
$$\times \{[\xi^*(\chi_q, r; \vec{n}_p \vec{n}_r) + \frac{\operatorname{sh}\chi_q}{4\pi} \int A(\chi_q, \vec{n}_p, r, \vec{n}_1) \times$$
$$\times E^{(2)*}(\chi_q, r, \vec{n}_r \vec{n}_1) d\vec{n}_1][\xi(\chi_q, r; \vec{n}_q \vec{n}_r) + \tag{1.1}$$
$$+ \frac{\operatorname{sh}\chi_q}{4\pi} \int A(\chi_q, \vec{n}_q, r, \vec{n}_2) E^{(1)}(\chi_q, r; \vec{n}_r \vec{n}_2)]\}$$

was obtained.

The relativistic plane wave ξ has the form:

$$\xi(\chi_q, r, \vec{n}_q \vec{n}_r) = (\operatorname{ch}\chi_q - \vec{n}_q \vec{n}_r \operatorname{sh}\chi_q)^{-1-ir}, \tag{1.2}$$

where $\vec{q} = \vec{n} \operatorname{sh}\chi_q$ is the momentum of a particle in c.m.s., $\vec{r} = r\vec{n}_r$ is the relativistic radius-vector. The case of equal masses is considered.

The quantities $E^{(1,2)}(\chi_q, r, \vec{n}_q \vec{n}_r)$ were defined in ref. [8]. Here we only give the relations for them which one will need further:

$$\int \xi^*(\chi_q, r; \vec{n}_q \vec{n}_r) E^{(1,2)}(\chi_q, r; \vec{n}_p \vec{n}_r) d\vec{n}_r =$$
$$= 4\pi \frac{e^{\pm ir\chi_{pq}}}{r \operatorname{sh}\chi_{pq}}, \chi_p > \chi_q, \tag{1.3a}$$

$$\operatorname{ch}\chi_{pq} = \operatorname{ch}\chi_q \operatorname{ch}\chi_p - (\vec{n}_p \vec{n}_q) \operatorname{sh}\chi_p \operatorname{sh}\chi_q, \tag{1.3b}$$

$$E^{(1,2)}(\chi_q, r; \vec{n}_q \vec{n}_r) \xrightarrow[r\chi_q \to \infty]{} \frac{4\pi}{r \operatorname{sh}\chi_q} e^{\pm ir\chi_q} \delta(1 \mp \vec{n}_q \vec{n}_r).$$

The scattering function A coincides with the amplitude of the scattering on the cut-off potential

$$V(\vec{r}', r) = V(\vec{r}')\theta(r - r'), \tag{1.4}$$

which is, generally speaking, spherically unsymmetric:

$$A(\chi_q, \vec{n}_p, r, \vec{n}_q) = -\frac{1}{4\pi} \int \xi^*(\chi_q, r'; \vec{n}_p \vec{n}_{r'}) \times$$
$$\times V(\vec{r}', E_q)\theta(r - r')\psi_q(\vec{r}', r) d\vec{r}'. \tag{1.5}$$

Here $\psi_q(\vec{r}', r)$ is the wave function corresponding to the potential (1.4). From (1.5) the boundary condition for the equation (1.1) follows:

$$A(\chi_q, \vec{n}_p; 0, \vec{n}_q) = 0. \tag{1.6}$$

The amplitude of the scattering on the whole potential $V(\vec{r}')$ is obtained in the limit $r \to \infty$:

$$A(\vec{p}, \vec{q}) = A(\chi_q, \vec{n}_p; \infty, \vec{n}_q). \tag{1.7}$$

For spherically symmetric potential, taking into account (1.3a), we rewrite equation (1.1) in the form:

$$\frac{d}{dr}A(\chi_q, r; \vec{n}_p\vec{n}_q) = -\frac{rV(r,E_q)}{\mathrm{sh}\,\chi_{pq}}\sin r\chi_{pq} -$$

$$-\frac{rV(r,E_q)\,\mathrm{sh}\,\chi_q}{4\pi}\Big[\int A(\chi_q, r; \vec{n}_p\vec{n})\frac{e^{ir\chi_{qn}}}{\mathrm{sh}\,\chi_{qn}}d\vec{n} +$$

$$+\int A(\chi_q, r; \vec{n}_q\vec{n})\frac{e^{ir\chi_{pn}}}{\mathrm{sh}\,\chi_{pn}}d\vec{n}\Big] - \frac{r^2\mathrm{sh}^2\chi_q}{(4\pi)^3}V(r,E_q)\times \qquad (1.8)$$

$$\times\int A(\chi_q, r; \vec{n}_p\vec{n}_1)A(\chi_q, r; \vec{n}_q\vec{n}_2)E^{(2)*}(\chi_q, r; \vec{n}_r\vec{n}_1)\times$$

$$\times E^{(1)}(\chi_q, r; \vec{n}_r\vec{n}_2)\,d\vec{n}_r\,d\vec{n}_1\,d\vec{n}_2,$$

where

$$\mathrm{ch}\,\chi_{pn} = \mathrm{ch}^2\chi_q - (\vec{n}_p\vec{n})\,\mathrm{sh}^2\chi_q,$$

$$\mathrm{ch}\,\chi_{qn} = \mathrm{ch}^2\chi_q - (\vec{n}_q\vec{n})\,\mathrm{sh}^2\chi_q. \qquad (1.9)$$

We also take into account the fact, that the amplitude of the scattering on spherically symmetric potential depends only on the angle between $\vec{n}_p$ and $\vec{n}_q$ (the scattering angle).

The equations (1.1) and (1.8) are exact. However, since they are cubersome and due to the presence of the quantity E in integrand, the analysis of these exact equations is complicated. Let us, therefore, search for an approximate equation corresponding to

high energies (comp. /9/). Using the asymptotic form for the quantity E (1.3b), and integrating in quadratic in A term, we pass to the expression:

$$-\frac{V(r,E_q)}{4\pi}e^{2ir\chi_q}\int A(\chi_q, r; -\vec{n}_p\vec{n})A(\chi_q, r; \vec{n}\,\vec{n}_q)d\vec{n}. \qquad (1.10)$$

It is easy to verify that at high energies this term is small in comparison with the rest terms of equation.

Indeed, in this regime the forward scattering is preferable, on the contrary the backward scattering may be neglected, so as one of the amplitudes in (1.10) is always small. Besides, for the smooth potential, the contribution of this term is additionally suppressed by the oscillating factor $e^{2ir\chi_q}$ when integrating over r . The linear in A terms of (1.8) are also simplified, since the factors $e^{ir\chi_{pn}}/\mathrm{sh}\,\chi_{pn}$ and $e^{ir\chi_{qn}}/\mathrm{sh}\,\chi_{qn}$ have the pronounced maxima at $\vec{n}\parallel\vec{p}$ and $\vec{n}\parallel\vec{q}$, resrectively. Let us, therefore, take the scattering function out the sign of integral assuming $A = A(\chi_q, r; \vec{n}_p\vec{n}_q)$. As a result, we come to the equation:

$$\frac{d}{dr}A(\chi_q, r, \vec{n}_p\vec{n}_q) = -rV(r,E_q)\frac{\sin r\chi_{pq}}{\mathrm{sh}\,\chi_{pq}} +$$

$$+\frac{iV(r,E_q)}{\mathrm{sh}\,\chi_q}(e^{2ir\chi_q} - 1)A(\chi_q, r, \vec{n}_p\vec{n}_q), \qquad (1.11)$$

which is easily solved. Neglecting the oscillating quantity $e^{2ir\chi_q}$, **in the limit** $r\to\infty$ **, we have:**

$$A^{(\ell)}(\vec{p},\vec{q}) = -i\,\mathrm{sh}\chi_q \int_0^\infty dr \frac{\partial}{\partial r}\left(\frac{r \sin r\chi_{pq}}{\mathrm{sh}\chi_{pq}}\right) \times \tag{1.12a}$$

$$\times\left[e^{-\frac{i}{\mathrm{sh}\chi_q}\int_r^\infty V(r',E_q)dr'} - 1\right] =$$

$$= -\int_0^\infty dr\cdot r\cdot \frac{\sin r\chi_{pq}}{\mathrm{sh}\chi_{pq}} V(r,E_q) e^{-\frac{i}{\mathrm{sh}\chi_q}\int_r^\infty V(r',E)dr'} = \tag{1.12b}$$

$$= -\frac{1}{4\pi}\int \xi^*(\vec{p},\vec{r}) V(r,E_q)\psi_q^{(\ell)}(\vec{r})d\vec{r}, \tag{1.12c}$$

where

$$\psi_q^{(\ell)}(\vec{r}) = \xi(\vec{q},\vec{r}) e^{-\frac{i}{\mathrm{sh}\chi_q}\int_r^\infty V(r',E)dr'} \tag{1.13}$$

is the approximate wave function equal to the plane wave $\xi(\vec{q},\vec{r})$, multiplied by the slowly varying spherically symmetric factor $\exp(-\frac{i}{\mathrm{sh}\chi_q}\int_r^\infty V(r',E)dr')$.

The expressions (1.12a) and (1.12c) are the analogues of the (nonrelativistic) eikonal representations:

$$A(\vec{p},\vec{q}) = -iq\int_0^\infty db\cdot b\cdot J_0(b|\vec{q}-\vec{p}|)\times$$

$$\times\left[\exp\left(-\frac{i}{q}\int_b^\infty \frac{dr\cdot r\cdot V(r)}{\sqrt{r^2-b^2}}\right) - 1\right] = \tag{1.14a}$$

$$= -\frac{1}{4\pi}\int d\vec{r} V(r) e^{i\vec{r}(\vec{q}-\vec{p})} e^{-\frac{i}{2q}\int_{-\infty}^{z} dz' V(z',\rho)} \tag{1.14b}$$

2. The Approximate Reconstruction of Quasipotential Proceeding from Experimental Data

The expressions (1.12b,c) allow one to consider the approximate amplitude $A^{(\ell)}(\vec{p},\vec{q})$ as relativistic Fourier transformation (Born approximation) for a modified spherically symmetric potential:

$$\tilde{A}(\chi_q,r) = V(r,E_q) e^{-\frac{i}{\mathrm{sh}\chi_q}\int_r^\infty dr' V(r',E_q)} \tag{2.1}$$

where

$$\tilde{A}(\chi_q,r) = -\frac{2}{\pi}\int_0^\infty A(\vec{p},\vec{q}) \frac{\sin r\chi_{pq}}{r} \mathrm{sh}\chi_{pq} d\chi_{pq} . \tag{2.2}$$

We consider (2.1) as an equation for V . One can easily see that it reduces to nonlinear differential Riccati-type equation, the solution of which has the form:

$$V(r,E_q) = \frac{\tilde{A}(\chi_q,r)}{1-\frac{i}{q}\int_r^\infty dr'\tilde{A}(\chi_q,r')} . \tag{2.3}$$

Proceeding from the relation (2.3) let us reconstruct the quasipotential using experimental data for elastic pp-scattering. We use the next relation between the scattering amplitude and the differential cross section $\frac{d\sigma}{dt}_{pq}$:

$$A(\vec{p},\vec{q}) = i\,\mathrm{Im}A(\vec{p},\vec{q})(1-i\alpha(\chi_q)), \tag{2.4}$$

$$\mathrm{Im}\,A(\vec{p},\vec{q}) = \sqrt{\frac{d\sigma}{dt}_{pq} \cdot \frac{\mathrm{sh}^2 \chi_q}{\pi(1+\alpha^2(\chi_q))}} \,. \qquad (2.5)$$

Here, the ratio α of the real part of the amplitude to its imaginary part is assumed to be weakly dependent on the scattering angle.

The Fourier transformation (2.2) of the $A(\vec{p},\vec{q})$, given with the errors is known to be incorrect problem. Therefore, we used experimental data in the form of interpolating functions. In the region of (first) diffractive peak the fitting from the papers /10/ was used; in the region of large momentum transfers this solution was sewn with the function describing the experimental data on $\frac{d\sigma}{dt}$ /12,13/ with the averaged relative error less than 8% * . The values of total cross sections and $\alpha(\chi_q)$ were taken from refs. /10,11/. The relation (2.3) expressing the quasipotential through the scattering amplitude, is approximate, first of all, due to the lack of knowledge of amplitude $A(\vec{p},\vec{q})$ in the nonphysical range ($\cos\theta_{pq} > 1$ or $\chi_{pq} > 2\chi_q$). Really, the experimental data are available for much smaller interval of values χ_{pq}:

$$\chi_{pq} \le \chi^{max} < 2\chi_q \,. \qquad (2.6)$$

*The numerical analysis shows the slight influence on the behaviour of the imaginary part of potential of variation with the errors of the fitting parameters.

These data are sufficient for reconstructing the quasipotential in the range:

$$r \ge r_{min} = 1/\chi^{max} \,. \qquad (2.7)$$

We perform our calculations at two energy values:

$$E_1 = 3.1 \text{ GeV}^{/12/} \qquad (\chi_q = 1.79) \qquad (\text{Fig.1})$$

$$E_2 = 26.6 \text{ GeV}^{/13/} \qquad (\chi_q = 3.97) \qquad (\text{Fig.2})$$

In both cases $\chi^{max} \approx 2.25$, that gives $r_{min} = 0.1$ fm. Figures 1 and 2 show the imaginary parts of the quasipotential $\mathrm{Im}\,V(r)$ (the curves (1)) and of the relativistic Fourier transform of the amplitude $\mathrm{Im}\,\tilde{A}(\chi_q, r)$ (the curves (2)).

They also show the curves corresponding to the standard parametrization of the experimental data in the region of diffractive peak:

$$\mathrm{Im}\,A(\vec{p},\vec{q}) = g(s)e^{2B(s)t} \,, \qquad B = \left(\frac{d}{dt}\ln\frac{d\sigma}{dt}\right)_{t=0} \,,$$

$$g = e^{2B}\frac{\mathrm{sh}\chi_q}{\pi B}\sqrt{\left(\frac{d\sigma}{dt}\right)_{t=0} \cdot \frac{1}{\pi(1+\alpha^2)}}$$

$$\tilde{A}_K(\chi_q, r) = -g \; K_{ir}(2B) \qquad (\text{the curves (3)})$$

$$V_K(r, E_q) = \frac{g\,K_{ir}(2B)}{1 - \frac{g}{\mathrm{sh}\chi_q}\int_r^\infty dr' K_{ir'}(2B)} \,. \qquad (2.8)$$

(the curves (4)).

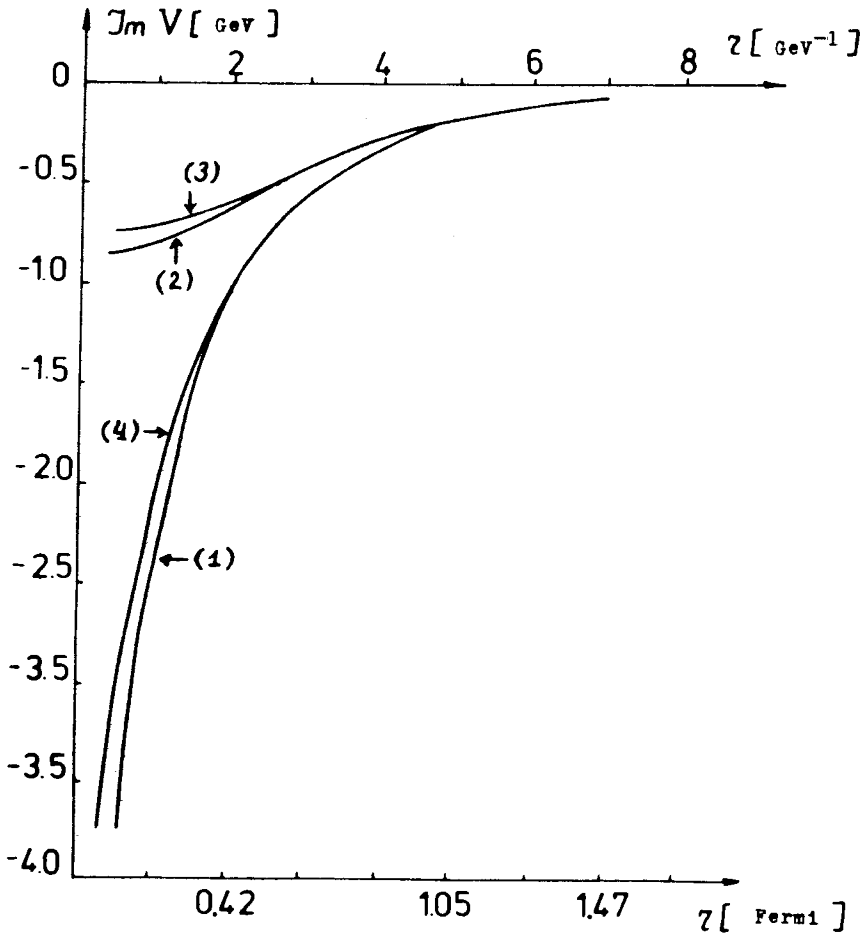

Fig. 1

In the nonrelativistic limit $\tilde{A}_K(\chi_q,r)$ turns into the Gauss function:

$$\tilde{A}_K(\chi_q,r) \to -g\sqrt{\frac{\pi}{4B}}\, e^{-2B}\, e^{-r^2/4B} . \tag{2.9}$$

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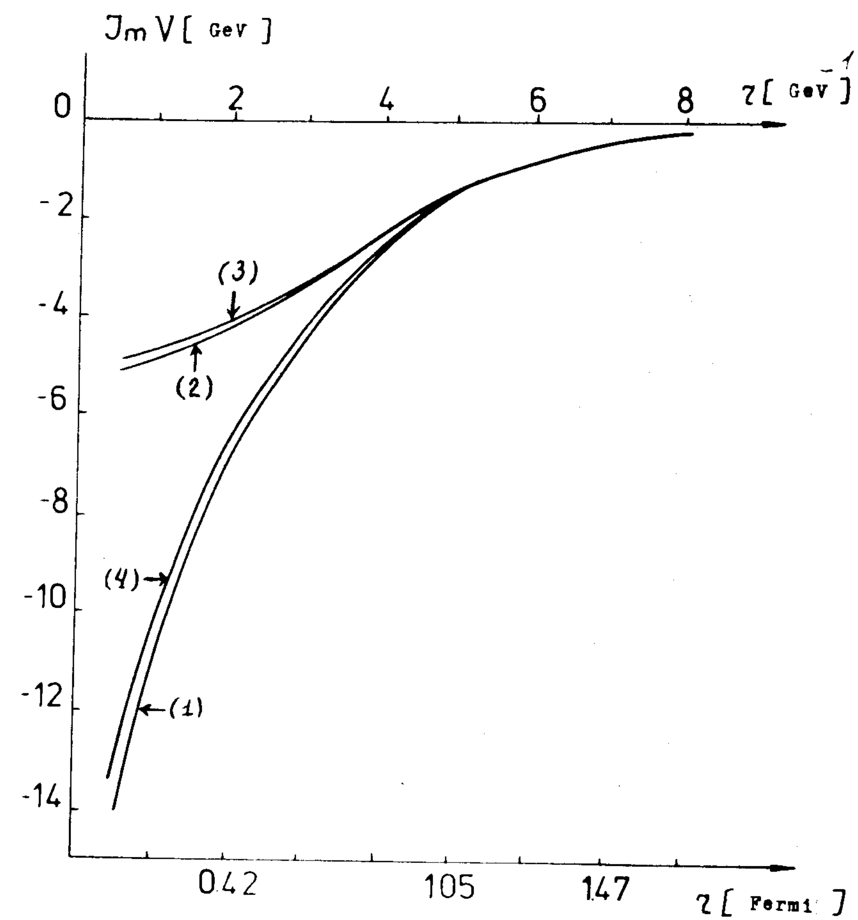

Fig. 2

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