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# ON PATTERN RECOGNITION OF FIREBALLS AND ISOBARS IN THE LOBACHEVSKY VELOCITY SPACE

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О распознавании образов файрболов и изобар
в пространстве скоростей Лобачевского

Показано, что многомерный статистический анализ струй частиц высокой энергии означает классификацию инварнантных изображений их кинематики по типам (с образованием файрболов и изобар или без того или другого) и распознавание статистических образов файрболов и изобар в пространстве скоростей Лобачевского (инварнантном фазовом пространстве физики высоких энергий). В основу такой классификации положено понятие сходства (близости) одиотнпных изображений. Два известных способа формализации этого понятия обобщены применительно к неевклидовой метрике пространства скоростей.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

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Bubelev E.G.

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On Pattern Recognition of Fireballs and Isobars
in the Lobachevsky Velocity Space

It is shown that a multidimensional statistical analysis of high energy jets means the classification of invariant pictures for their kinematics into types (with the production of fireballs and isobars and without one or the other) and recognition of statistical patterns for fireballs and isobars in the Lobachevsky velocity space (invariant phase space in high energy physics). The concept of likeness (closeness) of single-type pictures is taken as the basis for such a classification. Two known methods of formalizing this concept are generalized in application to the noneuclidean metric of the velocity space.

The investigation has been performed at the Laboratory of High Energies, JINR.

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## 1. INTRODUCTION

Main qualitative results of research in multiparticle hadron collisions at TeV energies are compatible with the fruitful heuristic hypotheses of fireball and isobar production (see review [1]). For a detailed multidimensional analysis of such collisions we used long ago a descriptive representation of hadron collision kinematics which is Lorentz-invariant and convenient at any number, energy and nature of detected particles. It is based on the Chernikov geometric formulation of relativistic kinematics in patterns of the Lobachevsky velocity space (LVS) [2].

## 2. LOBACHEVSKY VELOCITY SPACE

Some elements of this formulation are being used unmanifestly by many physicists. This is rapidity $\rho$ which is a distance in the LVS (in units of the velocity of light $c$ ) between the frame of reference and the rest frame of a jet particle with momentum $p$ and rest mass $m$ [2,3]

$$\rho = \operatorname{arcosh}(E/m) = \operatorname{arsinh}(p/m) = \operatorname{artanh}\beta = \tfrac{1}{2}\log[(E+p)/(E-p)]$$
$$(c \equiv 1). \tag{1}$$

Its orthogonal projection $\rho^{\|}$ onto the jet axis in the LVS (which connects initial colliding particles) is the known longitudinal rapidity $y$ [3]

$$\rho^{\|} = \operatorname{arcosh}[E/(E^2 - p^{\|2})^{1/2}] = \tfrac{1}{2}\log[(E+p^{\|})/(E-p^{\|})] = y. \qquad (2)$$

The well-known logarithmic half-angle scale $\eta(\theta)$ is the famous reverse Lobachevsky function $\Pi^{-1}(\theta)$ [3] which can be considered as a longitudinal quasi-rapidity for a jet particle in the "photon approximation" $m/p^{\perp} \to 0$

$$\eta(\theta) = -\log\tan(\theta/2) = \Pi^{-1}(\theta) = y + O(m/p^{\perp}). \qquad (3)$$

And finally, the volume element of the LVS multiplied by $m^3$ is equal to the Lorentz-invariant volume element of the momentum phase space [2a]

$$m^3 d\rho^3 = m^3 \mathrm{sh}^2\rho\, d\rho \cdot \sin\theta\, d\theta\, d\phi = dp^3 (m/E). \qquad (4)$$

Motions in the LVS (translations and rotations) are equivalent to the Lorentz group [2]. Therefore the LVS can be considered as an invariant phase space (IPS)* in high energy physics. Points of the LVS represent the so-called "world velocities" [2] or, simplier, rest frames of particles with rest masses $m>0$ and also Lorentz reference frames. Its infinitely distant points represent photon "world velocities".

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*More precisely, the IPS for $n$ particles is the $n$-multiple direct product of the LVS multiplied by itself.

## 3. PICTURES, PATTERNS AND HYPOTHESES

"World velocities" $\{a_\nu\}$ of two colliding and n detected jet particles, which are loaded with their rest masses according to Chernikov[2], form a (n+2)-point picture of jet kinematics in the LVS. This picture is called a <u>discrete kinematic figure (DKF).</u> It is Lorentz-invariant and contains the whole (3n + 1) -dimensional information on jet kinematics[3c].

Therefore DKFs are convenient for a multidimensional statistical classification of jets into types (classes) according to their kinematics for any energy and number of particles. They essentially facilitate the formulation of hypotheses on jet structure including the production of fireballs, isobars and other clusters[3]. Such hypotheses mean a qualitative clustering of jet particles and corresponding DKF points into like groups - clusters repeating in single-type jets. Points of the LVS, which are the centers of masses for these clusters, are loaded with cluster masses.

For a definite type reaction the latter points are the centers of domains in the LVS permitted for cluster decay products by the 4-momentum conservation law. These domains are called <u>continuous kinematic figures</u> (CKFs) and are spherical layers for isobars and ellipsoidal domains for fireballs. According to Chernikov[2a], CKFs are alotted by the Lorentz-invariant $3\ell$ -dimensional probability distribution density for $\ell$ decay cluster products

$$d^{3\ell}\sigma(\vec{p}_1,\dots,\vec{p}_\ell)/\prod_{\nu=1}^{\ell} dp^3_\nu = d^{3\ell}(\vec{p}_1,\dots,\vec{p}_\ell)/\prod_{\nu=1}^{\ell}(dp^3/Em^2)_\nu \quad (5)$$

Such multidimensional "probability clouds" in the LVS are invariant generalized statistical patterns (GPs) of the discussed clusters. In principle, they contain the whole information on the production and decay dynamics of these clusters which should be extracted from experimental data.

## 4. PATTERN RECOGNITION CONCEPTS

A multidimensional statistical analysis of jets contains two mutually connected problems, namely, classification of jets into types (classes) of invariant pictures for their kinematics - DKFs and determination from data on jet kinematics parameters of GPs for fireballs, isobars and other clusters as structure elements of GPs for jet types (classes). Concepts taking place in such an analysis exactly correspond to those of the general problem of statistical pattern recognition /4/.

The LVS is a space of invariant kinematic variables.

A DKF in the LVS for an individual jet is a mutually single-valued Lorentz-invariant geometric picture of its kinematics.

A multitude of all possible DKFs is a multidimensional abstract space of pictures.

A class is submultitude of single-type (i.e., like) DKFs, i.e., a compact submultitude of DKFs in the space of pictures.

The concept of DKFs likeness (size closeness) is the basis for classification of DKFs in the LVS as it reflects the feature of class compactness in the space of pictures.

A generalized pattern (GP) of a class is a CKF for a given type of hadron collisions alloted according to [2a] by a multidimensional probability density which is an invariant differential cross section (or multidimensional structure function) for this type of collisions.

It is convenient to consider the DKF for the most probable event of the $k$-th type (class) described in the LVS by the $3n$-component state-vector $\{\vec{\rho}^{\circ}\}_k = \{\vec{\rho}^{\circ}_1, \ldots, \vec{\rho}^{\circ}_n\}_k$ as a standard of GP for this DKF type (class) and vector $\{\vec{\rho}^{\circ}\}_k$ as a standard vector.

## 5. METHODS

It is necessary to generalize two methods of formalizing the likeness (closeness) concept known in pattern recognition in application to the noneuclidean metric of the LVS.

At the stage of preliminary classification of jets into unknown types (classes), it is convenient to characterize the likeness (closeness in IPS) of single-type DKFs by generalized distance (GD) $\rho(A^i, A^j)$. It is evaluated in the generalized hyperbolic metric of the LVS between two comparable DKFs $A^{i(j)} = \{\vec{\rho}_\mu\}^{i(j)}$ $(1 \le \mu \le n+2)$ for the $i$-th and $j$-th jets by the formula

$$\cosh \rho(A^i, A^j) = \prod_{\mu=1}^{n} (\cosh \rho^{ij}_\mu)^{w_\mu} \quad \text{under condition} \quad \prod_{\mu=1}^{n} w_\mu = 1, \tag{6}$$

where

$$\cosh \rho^{ij}_\mu = \cosh \rho^i_\mu \cosh \rho^j_\mu - \sinh \rho^i_\mu \sinh \rho^j_\mu \cos(\widehat{\vec{\rho}^{\,i}_\mu \vec{\rho}^{\,j}_\mu}). \tag{7}$$

Here $\rho_\mu^{ij} = |\vec{\rho}_\mu^{\,i} - \vec{\rho}_\mu^{\,j}| = [a_\mu^i, a_\mu^j]$ is the distance between the components of the $\mu$-th pair of the nearest points $a_\mu^i$ and $a_\mu^j$ for comparable DKFs which correspond to particles with the same charges and rest masses; $\cosh\rho_\mu^{ij} = \gamma_\mu^{ij}$ being the relative Lorentz-factor between particles of the $\mu$-th pair; $w_\mu$ are the statistical weights introduced to increase the contributions to GD of particles with narrower distributions in the LVS (nucleons and K-mesons, for example). In the nonrelativistic limit $\rho \approx \beta \ll 1$ eq. (6) transforms into the usual generalized root-mean-square distance [4] $\beta(A^i, A^j)$ in a small quasi-Euclidean domain of the LVS

$$\beta^2(A^i,A^j) = \sum_{\mu=1}^{n} w_\mu (\beta_\mu^{ij})^2 = \sum_{\mu=1}^{n} w_\mu |\vec{\beta}_\mu^{\,i} - \vec{\beta}_\mu^{\,j}|^2; \ \prod_{\mu=1}^{n} w_\mu = 1. \quad (6')$$

In accordance with paper [4a], the statistical weights $w_\mu$ can be obtained under some conditions by maximizing an averaged generalized distance (AGD) between pictures of different known classes A and B, which is determined in our case as a geometric average by the expression

$$\max \overline{\ln\cosh\rho(A,B)} = \max \frac{1}{N_A N_B} \sum_i^{N_A} \sum_j^{N_B} \ln\cosh\rho(A^i, B^j). \quad (8)$$

At the stage of more precise probabilistic classification of jets with simultaneous determination of GDs parameters for jet classes and clusters, the degree of likeness (closeness in IPS) of the i-th classified jet DKF $A^i = \{\vec{\rho}\}^i$ with standard DKF $A_k^\circ = \{\vec{\rho}^{\,\circ}\}_k$ for the k-th jet type (class) is expressed by the probability $P_k(\{\vec{\rho}\}^i)$. It is written symbolically by the equation

$$P_k(\{\rho\}^i) = \prod_{\nu=1}^{n_k} \int f_k(\vec{\sigma}_\nu, \vec{\rho}^{\circ}_\nu - \vec{\rho}'_\nu) g(\Delta\vec{\rho}^{\,i}_\nu, \vec{\rho}^{\,i}_\nu - \vec{\rho}'_\nu) d\vec{\rho}'^{\,3}_\nu. \qquad (9)$$

Here $f_k(\vec{\sigma}_\nu, \vec{\rho}^{\circ}_\nu - \vec{\rho}_\nu)$, $\nu = 1 \div n_k$ are the probability distribution densities for all independent physically significant kinematic variables expressed through the corresponding rapidity vectors $\vec{\rho}_\nu$ and $g(\Delta\vec{\rho}^{\,i}_\nu, \vec{\rho}^{\,i}_\nu - \vec{\rho}_\nu)$, $\nu = 1 \div n_k$ are the experimental resolution functions with invariant error domains $\Delta\vec{\rho}^{\,i}_\nu \equiv (\Delta\rho^i_\nu)^3$ in the LVS for these vectors. The standard vector $\{\vec{\rho}^{\circ}\}_k = \{\vec{\rho}^{\circ}_1, \dots, \vec{\rho}^{\circ}_{n_k}\}$ and the multitude of dispersions $\{\vec{\sigma}\}_k = \{\sigma_1, \dots, \sigma_{3n_k}\}$ set together GP $F_k(\{\vec{\sigma}, \vec{\rho}^{\circ} - \vec{\rho}\}_{n_k})$ for the k-th type (class) of hadron collisions which contains GPs $F_\lambda(\{\vec{\sigma}, \vec{\rho}^{\circ} - \vec{\rho}\}_{\ell_\lambda})$ for fireballs, isobars or other clusters participating in collisions of this type

$$F_k(\{\vec{\sigma}, \vec{\rho}^{\circ} - \vec{\rho}\}_{n_k}) = \prod_{\nu}^{n_k} f_k(\vec{\sigma}_\nu, \vec{\rho}^{\circ}_\nu - \vec{\rho}_\nu) = \qquad (10)$$

$$= \prod_{\lambda_k} \prod_{\nu=1}^{\ell_\lambda} f_\lambda(\vec{\sigma}_\nu, \vec{\rho}^{\circ}_\nu - \vec{\rho}_\nu) = \prod_{\lambda_k} F_\lambda(\{\vec{\sigma}, \vec{\rho}^{\circ} - \vec{\rho}\}_{\ell_\lambda}).$$

Such a representation of GPs for classes as products of GPs for clusters must reduce the total number of fitted parameters for the overall multidimensional probability density

$$\sum_k F_k(\{\vec{\sigma}, \vec{\rho}^{\circ} - \vec{\rho}\}_{n_k}) = \sum_k \prod_{\lambda_k} F_\lambda(\{\vec{\sigma}, \vec{\rho}^{\circ} - \vec{\rho}\}_{\ell_\lambda}), \qquad (11)$$

describing the particle distribution in the LVS for an available sample of jets. These parameters can be fitted (whenever possible) by the maximum likelihood method or by the method of moments using the sequential approximation procedure for the probabilistic classification of jets, starting from some known parameters and results of the preliminary classification as a first approximation.

It is clear that one should try to realize this probabilistic pattern recognition of fireballs,isobars and other clusters just on condition of the completeness of experimental data on jet kinematics, i.e., on momenta, escape angles and the nature of all charged and as many as possible neutral jet particles.

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