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DESCRIPTION OF THE DECAY $\eta \to \pi^+\pi^-\gamma$ IN QUANTUM CHIRAL FIELD THEORY

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Описание распада $\eta \to \pi^+\pi^-\gamma$ в квантовой киральной теории

В киральной теории в однопетлевом приближении описан распад $\eta \to \pi^+\pi^-\gamma$. Результат находится в удовлетворительном согласии с экспериментальными данными.

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Description of the Decay $\eta \to \pi^+\pi^-\gamma$ in Quantum Chiral Field Thory

The decay $\eta \to \pi^+\pi^-\gamma$ is described in chiral field theory using the one-loop approximation. The result is in satisfactory agreement with experimental data.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

Recent investigations [1-4] have shown that the tree-and one-loop approximations in quantum chiral field theory are in good agreement with the known experimental data including the description of all principal characteristics of the main pion and kaon decays. It is quite natural to attempt an analogous description of the main decays of the last member of the meson octet - the η-meson. Among these decays we consider the four most probable and well measured ones. These are the decays: $\eta \to 2\gamma$, $\eta \to 3\pi^0$, $\eta \to \pi^+\pi^-\pi^0$, and $\eta \to \pi^+\pi^-\gamma$.

The decay $\eta \to 2\gamma$ is satisfactorily described in the one-loop approximation with baryon loops (see, for example, ref. [5]). This approach is in complete agreement with the chiral theory [4]. The decays $\eta \to 3\pi$ are described in the tree approximation, if we take into account the massive terms in the Lagrangian violating the $SU(3) \times SU(3)$ symmetry [6]. The scheme of the symmetry violation has to be chosen following Gell-Mann-Oakes-Renner with an additional rotation by the Cabbibo angle around the seventh axis (see [6,7]).

The last of the above-mentioned decays $\eta\to\pi^+\pi^-\gamma$ is interesting due to the following property. Its width is nearly by one order of magnitude smaller than the $\eta\to 2\gamma$ decay width, although this process has one electromagnetic vertex more. In the last years there have been made some attempts to explain this process (see, for example, [8], where an estimate for the ratio of the widths of the processes $\eta\to 2\gamma$ and $\eta\to\pi^+\pi^-\gamma$ is received, which is in agreement with experiment for some variants of this model). The purpose of this article is to calculate the absolute value of the $\eta\to\pi^+\pi^-\gamma$ width in the framework of the standard quantum chiral theory using the one-loop approximation.

2. INTERACTION LAGRANGIAN

Let us remind of the structure of the $SU(3)\times SU(3)$ invariant Lagrangian, and separate that part of it which completely defines the process $\eta\to\pi^+\pi^-\gamma$. The ordinary form of the chiral Lagrangian is

$$\mathcal{L} = \frac{1}{2} D_\mu\Phi_i D^\mu\Phi_i + \bar{B}_i(i\gamma_\mu D^\mu - M)B_i + \\ + i\frac{g}{M}[\alpha d_{ijk} - i(1-\alpha)f_{ijk}]\bar{B}_i\gamma_\mu\gamma_5 B_j D^\mu\Phi_k, \tag{1}$$

where Φ_i and B_i are the fields of the meson and baryon octets, D_μ are the covariant derivatives

$$D_\mu B_i = \partial_\mu B_i - \frac{1}{2F_\pi^2} f_{ijm} f_{k\ell m}\partial_\mu\Phi_k\Phi_\ell B_j + O(\Phi^4)B,$$
$$D_\mu\Phi_i = \partial_\mu\Phi_i - \frac{1}{6F_\pi^2} f_{i\ell n} f_{jmn}\Phi_\ell\Phi_m\partial_\mu\Phi_j + O(\Phi^5), \tag{2}$$

g is the constant of the strong interaction ($g^2/4\pi \approx 14.7$), M is the baryon mass, α is the mixing parameter of the $SU(3)$ theory ($\alpha\approx 2/3$). F_π is the pion decay constant (F_π = 92 MeV).

It is easily seen that the meson part of the Lagrangian (1) alone cannot contribute to the $\eta\to\pi^+\pi^-\gamma$ decay amplitude. All meson vertices of (1) contain an even number of meson lines whereas the process $\eta\to\pi^+\pi^-\gamma$ has an odd number of meson ends. Therefore, this process, like the process $\eta\to 2\gamma$, can proceed only via the baryon loops.

For the further calculations it is useful to transform the Lagrangian (1) so that it gets rid of the derivative term linear in the meson fields as it was done in the previous papers [1,2,4] (see, also [9]). To this end let us redefine the baryon fields [9]

$$B_i = \Psi_i - \frac{g}{M}[\alpha d_{ijk} - i(1-\alpha)f_{ijk}]\gamma_5\Psi_j\Phi_k - \\ - \frac{g^2}{2M^2}[\alpha d_{\ell im} - i(1-\alpha)f_{\ell im}][\alpha d_{jkm} - i(1-\alpha)f_{jkm}]\Psi_j\Phi_\ell\Phi_k \ldots \tag{3}$$

This transformation leaves S-matrix elements unchanged. We shall be interested only in that part of the Lagrangian which contains first and second powers in the meson fields. (The vertex describing the coupling of baryon and three mesons $(\eta\pi^+\pi^-)$ at one point is forbidden in the SU(3) theory). Therefore, after the transformation (3), we receive the following three interaction Lagrangians

$$\mathcal{L}_1 = 2g[\alpha d_{ijk} - i(1-\alpha)f_{ijk}]\bar{\Psi}_i\gamma_5\Psi_j\Phi_k, \tag{4a}$$

$$\mathcal{L}_2 = 2\frac{g^2}{M}[\alpha d_{kjm} - i(1-\alpha)f_{kjm}][\alpha d_{\ell im} - i(1-\alpha)f_{\ell im}]\times \times\bar{\Psi}_j\Psi_\ell\Phi_i\Phi_k, \tag{4b}$$

$$\mathcal{L}_3 = \frac{i}{2F_\pi^2}\bar{\Psi}_i\gamma_\mu\Psi_\ell\Phi_j\partial^\mu\Phi_k\{f_{i\ell m}f_{kjm}(g_A^2-1) + g_A^2[\frac{2}{3}\alpha^2(\delta_{ij}\delta_{k\ell}-\delta_{ik}\delta_{j\ell}) + 2\alpha(\alpha-1)f_{kjm}(f_{i\ell m}-id_{i\ell m})]\} \tag{4c}$$

In deriving the last expression we have used the Goldberger-Treiman identity $g = g_A M/F_\pi$, where $g_A = 1.25$ is the renormalization constant of the axial current.

Now it is useful to remind that in the one-loop approximation most of the meson processes get their main contributions from those parts of the chiral Lagrangian which do not contain derivatives, i.e., from (4a) and (4b) (see [4]). On the contrary the process $\gamma\to\pi^+\pi^-\gamma$ is interesting by the fact that all these contributions are equal to zero. This vanishing of the baryon loops with one electromagnetic vertex and with other vertices of the type (4a) and (4b) is a consequence of the Furry theorem. The only baryon loop diagram which remains here is the triangle diagram with one electromagnetic vertex, one vertex of the type (4a) for the coupling of the η meson, and one vertex of the type (4c) describing the production of the $\pi^+\pi^-$ pair. The corresponding Lagrangians have the form

$$\mathcal{L}^{(A)} = -eA_\mu\{\bar{p}\gamma^\mu p - \bar{\Xi}^-\gamma^\mu\Xi^- + \bar{\Sigma}^+\gamma^\mu\Sigma^+ - \bar{\Sigma}^-\gamma^\mu\Sigma^-\}, \tag{5a}$$

$$\mathcal{L}^{(\eta)} = \frac{g}{\sqrt{3}}\eta\{(3-4\alpha)\bar{p}\gamma_5 p - (3-2\alpha)\bar{\Xi}^-\gamma_5\Xi^- + 2\alpha(\bar{\Sigma}^+\gamma_5\Sigma^+ + \bar{\Sigma}^-\gamma_5\Sigma^-)\}, \tag{5b}$$

$$\mathcal{L}^{(\pi^+\pi^-)} = i\frac{(\pi^-\partial_\mu\pi^+ - \pi^+\partial_\mu\pi^-)}{(2F_\pi)^2}\{(g_A^2-1)\bar{p}\gamma^\mu p + [1-g_A^2(1-4\alpha(1-\alpha))]\bar{\Xi}^-\gamma^\mu\Xi^- + 2[g_A^2(1-2\alpha+\frac{4}{3}\alpha^2)-1](\bar{\Sigma}^+\gamma^\mu\Sigma^+ - \bar{\Sigma}^-\gamma^\mu\Sigma^-)\}. \tag{5c}$$

3. CALCULATION OF THE AMPLITUDE AND WIDTH OF THE $\eta \to \pi^+\pi^-\gamma$ DECAY

Using the Lagrangians (5a-c) the following expression for the amplitude of the $\eta \to \pi^+\pi^-\gamma$ process is easily obtained

$$T = ia \frac{\epsilon^{\mu} A_{\mu}}{\sqrt{q^{\circ} p_1^{\circ} p_2^{\circ}}} \delta^{(4)}(p - p_1 - p_2 - q) \tag{6}$$

with

$$A_{\mu} = \epsilon_{\mu\nu\rho\sigma} p_1^{\nu} p_2^{\rho} q^{\sigma} . \tag{7}$$

Here $\epsilon_{\mu\nu\rho\sigma}$ is the fully antisymmetric tensor, p, q, p_1, and p_2 are the momenta of the η-meson, photon and pions, respectively. ϵ^{μ} is the photon polarization. The constant a is equal to

$$a = \frac{e g_A C}{2\pi \sqrt{3} m_{\eta} (4\pi F_{\pi})^3} , \tag{8}$$

where e is the electromagnetic charge, and C is the factor taking into account the contributions of all the baryon octet members

$$C = -6\alpha [1 - 3 g_A^2 (1 - 2\alpha + \frac{28}{27}\alpha^2)] \approx -1.64 . \tag{9}$$

For the decay width we have

$$\Gamma = \frac{a^2}{2\pi} \int \frac{d^3q \, d^3p_1 \, d^3p_2}{q^{\circ} p_1^{\circ} p_2^{\circ}} |A_{\mu}|^2 \delta^{(4)}(k - p_1 - p_2) ,$$

$$|A_{\mu}|^2 = (k^2 - 2m^2)(p_1 q)(p_2 q) - 2m^2 (p_1 q)^2 , \tag{10}$$

$$k = p - q = p_1 + p_2 , \quad m = m_{\pi} .$$

Using the formulae

$$I = \int \frac{d^3p_1 d^3p_2}{p_1^{\circ} p_2^{\circ}} \delta^{(4)}(k - p_1 - p_2) = 2\pi \sqrt{1 - \frac{4m^2}{k^2}} ,$$

$$\int \frac{d^3p_1 d^3p_2}{p_1^{\circ} p_2^{\circ}} p_1^{\alpha} p_1^{\beta} \delta^{(4)}(k - p_1 - p_2) =$$

$$= \frac{1}{3} [(m^2 - \frac{k^2}{4}) g^{\alpha\beta} + k^{\alpha} k^{\beta} (1 - \frac{m^2}{k^2})] ,$$

$$\int \frac{d^3p_1 d^3p_2}{p_1^{\circ} p_2^{\circ}} p_1^{\alpha} p_2^{\beta} \delta^{(4)}(k - p_1 - p_2) =$$

$$= \frac{1}{12} [(k^2 - 4m^2) g^{\alpha\beta} + 2k^{\alpha} k^{\beta} \frac{(2m^2 + k^2)}{k^2}] ,$$

we can bring the expression (10) to the form

$$\Gamma = \frac{a^2}{2\pi} \int \frac{d^3q}{q^{\circ}} (kq)^2 (k^2 - 4m^2) \frac{1}{6} = \frac{2}{\pi} (\frac{e g_A C}{3})^2 (\frac{m_{\pi}}{4\pi F_{\pi}})^6 m_{\eta} \mathcal{J} , \tag{11}$$

where

$$\mathcal{J} = \int_0^{3/2} dx\, x^3 (3-2x) \sqrt{\frac{(3-2x)}{2(2-x)}} \approx 0.52 \,. \qquad (12)$$

Inserting into (11) the values of the physical constants and of (9), (12), we receive

$$\Gamma_{(\eta \to \pi^+ \pi^- \gamma)} = 26 \text{ eV} \,, \qquad (13)$$

whereas the experiment gives

$$\Gamma^{exp.}_{(\eta \to \pi^+ \pi^- \gamma)} = 41 \text{ eV} \,. \qquad (14)$$

4. CONCLUSION

For comparison it is interesting to consider the results for the description of the $\eta \to 2\gamma$ process in the one-loop approximation. The amplitude of this process has the form

$$T_{(\eta \to 2\gamma)} = i \frac{e^2 g \alpha \epsilon_1^{\mu} \epsilon_2^{\nu} \epsilon_{\mu\nu\rho\sigma} \, q_1^{\rho} q_2^{\sigma}}{8\pi^2 M \sqrt{3\pi \, m_{\eta} q_1^{\circ} q_2^{\circ}}} \delta^{(4)}(p - q_1 - q_2), \qquad (15)$$

where ϵ_1^{μ} and ϵ_2^{ν} (q_1^{ρ} and q_2^{σ}) are the polarizations (momenta) of photons, α is the SU(3) mixing parameter. Then for the decay width, we obtain

$$\Gamma_{(\eta \to 2\gamma)} = 0.46 \text{ keV} \,. \qquad (16)$$

The experimental value is [10]

$$\Gamma^{exp.}_{(\eta \to 2\gamma)} = 0.33 \text{ keV} \,. \qquad (17)$$

Within the limits of accuracy of the chiral theory (20-30% in amplitude calculations) the results (13) and (16) are considered to agree with the experimental data. Therefore we can conclude that all the main decays of the pions, kaons and the η-meson are satisfactorily described in the framework of the nonlinear chiral theory without introducing any additional parameters or auxiliary intermediate particles (of the ρ-meson type).

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