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RELATION BETWEEN LINEAR AND NONLINEAR REALIZATIONS OF SUPERSYMMETRY

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Связь линейной и нелинейной реализаций суперсимметрии

Установлена связь между суперполевым подходом к суперсимметрии и нелинейной реализацией Волкова-Акулова. Получены общие формулы перехода от одной реализации к другой. Показано, что в любой перенормируемой модели со спонтанным нарушением суперсимметрии инвариантное действие, первоначально записанное через суперполя, может быть эквивалентно представлено в терминах полей нелинейной реализации.

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Relation between Linear and Nonlinear Realizations of Supersymmetry

The intimate correspondence between the superfield approach to the supersymmetry and the Volkov-Akulov nonlinear realization is established. General formulas for transition from the linear realization to the nonlinear one and backwards are obtained.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. There exist two different ways of implementing the supersymmetry transformations. One of them is the linear realization on superfields $\Phi_k(x,\theta)$ /1/.

$$\Phi_k'(x,\theta) = \Phi_k(x - \frac{1}{2i}\epsilon\gamma\theta, \theta-\epsilon), \tag{1}$$

where k represents the Lorentz index; another is the Volkov-Akulov nonlinear realization /2/ which involves, as a basic entity, the nonlinearly transforming Goldstone spinor $\psi(x)$

$$\delta\psi(x) = \epsilon - \frac{1}{2i}\bar{\epsilon}\gamma_\mu\psi(x)\partial^\mu\psi(x) \tag{2}$$

and acts on other fields $\sigma_k(x)$ as follows:

$$\delta\sigma_k(x) = -\frac{1}{2i}\bar{\epsilon}\gamma_\mu\psi(x)\partial^\mu\sigma_k(x). \tag{3}$$

The nonlinear realization exhibits most purely the idea of spontaneous supersymmetry breaking.

Based on the analogy with internal symmetries one may expect that these two approaches are related with each other* . In the present note we find a general relation between linear and nonlinear realizations of supersymmetry and give concrete prescriptions how to pass from one realization to the other. In particular, we show how to construct linear superfields of nonlinearly transforming quantities $\psi(x)$, $\sigma_k(x)$. It is noticed, that in any renormalizable model of spontaneously broken supersymmetry one may use the parametrization of invariant action by fields $\psi(x)$, $\sigma_k(x)$ equally with the conventional parametrization by linearly transforming superfields.

2. The general theory of nonlinear realizations of internal symmetries /5/ says that any linear multiplet of a given group can be converted into the direct sum of nonlinearly transforming fields by means of the group transformation with the Goldstone field as a parameter. The analogous theorem holds in the case of supersymmetry. Let us perform, in some superfield $\Phi_k(x,\theta)$, local supertranslation with parameter $-\psi(x)$:

$$\Phi^{\sigma}_k(x,\theta) = \Phi_k\left(x + \frac{1}{2i}\bar{\psi}(x)\gamma\theta,\ \theta + \psi(x)\right). \tag{4}$$

*The possible existence of such a relation was pointed out by Ogievetsky, who in 1974 constructed certain linearly transforming functions of the Goldstone spinor $\psi(x)$ in the space-time of dimensionality 2+1 (see also Zumino /3,4/)

One may check that under transformations (1), (2) components of the "shifted" superfield $\Phi^{\sigma}_k(x,\theta)$ transform independently of each other, according to (3):

$$\delta\Phi_k(x,\theta) = -\frac{1}{2i}\bar{\epsilon}\gamma_\mu\psi(x)\partial^\mu\Phi^{\sigma}_k(x,\theta). \tag{5}$$

Thus, by changing variables, any superfield with linear transformation law (1) can always be brought into the splitting form (4) in which it is equivalently represented by a set of nonlinearly transforming components. The components of $\Phi^{\sigma}_k(x,\theta)$ are finite polynomials in anticommuting spinors $\psi(x)$. Note that unlike the case of internal symmetries /5/ canonical transformation (4) includes necessarily field derivatives.

3. Inverting eq. (4), one expresses the superfield $\Phi_k(x,\theta)$ in terms of the nonlinear realization quantities:

$$\Phi_k(x,\theta) = \Phi^{\sigma}_k(\tilde{x},\ \theta - \psi(\tilde{x})), \tag{6}$$

where $\tilde{x}_\mu(x,\theta)$ is a solution of the nonlinear equation

$$\tilde{x}_\mu(x,\theta) + \frac{1}{2i}\bar{\psi}(\tilde{x})\gamma_\mu\theta = x_\mu. \tag{7}$$

Solving eq. (7) by iterations one may represent $\tilde{x}_\mu(x,\theta)$ as a finite polynomial in nilpotent quantities θ, $\psi(x)$ and derivatives of $\psi(x)$.

Relation (6) is the supersymmetric counterpart of the polar decomposition of linear multiplets of internal symmetry, the Goldstone field $\psi(x)$ being an analog of angle variables and the components of superfield $\Phi_k^{\sigma}(x,\theta)$ being analogous to radial variables.

The r.h. side of eq. (6) is the superposition of the following θ-polynomials:

$$\hat{\sigma}_k(x,\theta) \overset{\text{def}}{=\!=} \sigma_k(\tilde{x}(x,\theta)), \tag{8}$$

$$\hat{\psi}_a(x,\theta) \overset{\text{def}}{=\!=} \psi_a(\tilde{x}(x,\theta)) - \theta_a . \tag{9}$$

Each of them transforms under supertranslations (2), (3) by linear law (1), i.e., possesses itself the properties of superfield. To verify this one has to take into account that the group variation of functions $\psi(\tilde{x})$, $\sigma_k(\tilde{x})$ consists both of their change at the point $\tilde{x}_\mu$ according to rules (2), (3) and of the change due to the shift of $\tilde{x}_\mu$ induced by the transformation of function $\psi(\tilde{x})$ in eq. (7) (coordinates x_μ and θ_a remain unaffected).

So, given nonlinearly transforming fields $\psi_a(x), \sigma_k(x)$ by the substitution $x_\mu \to \tilde{x}_\mu(x,\theta)$ one can construct linearly transforming superfields with any external spin. From spinors $\psi(x)$ alone it is possible to form superfields either by multiplying basic superfields (9) or by applying algorithm (8) to the covariant derivative $\nabla_\rho \psi(x)$ /2/ which transforms like fields $\sigma(x)$, i.e., by law (3). First method is restricted to the construction of scalar, pseudoscalar, spinor and pseudovector superfields only.

4. Relation (4) makes it possible to represent any renormalizable model with spontaneously broken supersymmetry (linear σ-model) in terms of the nonlinear realization, in the complete analogy with the case of internal symmetries /6,7/. To get the nonlinear representation of the linear σ-model one has to do as follows. In the invariant action written through superfields $\Phi(x,\theta)$ and their covariant derivatives $D_\beta \Phi(x,\theta)$

$$S = \int d^4x\, d^4\theta\, L(\Phi(x,\theta),\, D_\beta \Phi(x,\theta)) \tag{10}$$

one performs the change of variables x_μ, θ_a as suggested by eq. (4), after that action (10) takes the form

$$S = \int d^4x\, d^4\theta\, J_B(x,\theta)\, L(\theta, \sigma_k(x), \nabla_\rho \sigma_k(x), \nabla_\lambda \psi(x)), \tag{11}$$

where $\sigma_k(x)$ are components of superfields $\Phi^{\sigma}(x,\theta)$, $\psi(x)$ is connected by the equivalence transformation with the Goldstone spinor of the linear realization, ∇_μ denotes the nonlinear covariant derivative. The quantity $J_B(x,\theta)$ is the Jacobian of transformation (4). It is calculated according to the general rules of changing variables in Grassman integrals /8/ and has the following structure:

$$J_B(x,\theta) = \det T(x) \det M(x,\theta), \tag{12}$$

where

$$T_\mu^\nu(x) = \delta_\mu^\nu - \frac{1}{2i}\, \partial^\nu \bar{\psi}(x)\, \gamma_\mu \psi(x), \tag{13}$$

$$M_\mu^\nu(x,\theta) = \delta_\mu^\nu - \frac{1}{2i}\, \nabla^\nu \bar{\psi}(x)\, \gamma_\mu \theta . \tag{14}$$

The integration measure $d^4x \det T$ arising in (11) coincides with the invariant volume element introduced in ref. /2/. It contains the kinetic term of the Goldstone spinor $\psi(x)$. On imposing the covariant condition $\sigma_k(x)=0$, action (11) reduces to the nonrenormalizable nonlinear action of ref.[2] supplemented with certain nonminimal interactions of field $\psi(x)$. In more detail the transition to the nonlinear parametrization will be discussed in forthcoming publications. It is worth noting that the parametrization of the action by the fields $\psi(x)$, $\sigma_k(x)$ is most convenient while analyzing low-energy consequences of the spontaneous supersymmetry breaking because the transformation of the Goldstone fermion $\psi(x)$ is separated from that of other fields.

Finally, canonical transition (4), (6) can be used to give the superfield form to the supersymmetric Higgs effect[9,4] treated up to now in the framework of nonlinear realization.

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