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Горизонт событий вокруг частицы, окруженной статическим потенциалом запирания

Решается уравнение Эйнштейна для случая сферически-симметричного потенциала запирания, а затем обсуждается положение горизонтов событий в рамках теории гравитации, а также в рамках "сильной черной дыры", предложенной Реками и Касторина.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Event horizons Around a Particle Surrounded by a Static Confinement Potential

After solving one of the Einstein's equations in the case of a spherically symmetric confinement potential, the locations of event horizons are discussed in gravitational theory and in the "strong black hole" picture of Recami and Castorina.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. In elementary particle physics, in order to explain the confinement of quarks in mesons and hadrons it is widely accepted to take a two body "confinement potential" in the form

$$V(r) = \frac{e^{-mr}}{r}(a_0 + K_0 r^2). \qquad (1)$$

For instance, ref.[1] gives in the case of the charmonium for the $c-\bar{c}$ interaction

$$a = -0,2, \quad K_0 = -\frac{a_0}{a^2}, \quad a = 0.2\,\mathrm{fm}, \quad m = 0.$$

In the $cc(\bar{c}\bar{c})$ interaction the potential is supposed to be repulsive; $V_{cc} = V_{\bar{c}\bar{c}} = -V_{c\bar{c}}$, i.e., $a_0 = +0.2$ in eq. (1).

Our aim here is to investigate the space-time structure around an object (c or $\bar{c}$) which for the following arguments, tentatively, is supposed to exist alone, surrounded by a field representing (1).

The relativistic considerations necessary to carry out our program are quite straightforward, we quote ref.[2] only, where similar calculations for the Yukawa field have been performed.

2. The total potential energy of an object described by a distribution ρ

$$\int \rho(x)\,dx = 1$$

self-interacting through (1) is

$$W = \frac{1}{2}\int \rho(x)\,\rho(x')\,\frac{e^{-m|x-x'|}}{|x-x'|}(\alpha + K|x-x'|^2)\,d^3x\,d^3x'. \qquad (2)$$

Introducing

$$\Phi(x) = \int \rho(x')\,\frac{e^{-m|x-x'|}}{|x-x'|}(\alpha + K|x-x'|^2)\,d^3x',$$

W takes the form

$$W = \frac{1}{2}\int \rho(x)\,\Phi(x)\,d^3x.$$

A direct calculation gives

$$\Delta\Phi - m^2\Phi = -4\pi\alpha\rho + K\phi,$$

$$\phi = \int \rho(x')\,e^{-m|x-x'|}\left(\frac{2}{|x-x'|} - 4m\right)d^3x'.$$

Substituting ρ into (2), after performing the integration over x', W is

$$W = \frac{1}{8\pi\alpha}\int (|\nabla\Phi|^2 + m^2\Phi^2 + K\Phi\phi)\,d^3x, \qquad (3)$$

therefore the energy density in our case is the following:

$$u(x) = \frac{1}{3\pi\alpha}(|\nabla\Phi|^2 + m^2\Phi^2 + K\Phi\phi). \qquad (4)$$

One might notice the non-definiteness of (4), which from the point of view of field theory, seems to be related to an indefinite metric quantization [3,4].

For a point source

$$\Phi = \frac{e^{-mr}}{r}(a+Kr^2), \qquad \phi = \frac{e^{-mr}}{r}(2-4mr),$$

and

$$u = \frac{1}{8\pi a} e^{-2mr} \Big[\frac{a^2}{r^4} + \frac{2a^2 m}{r^3} + \frac{2a^2 m^2}{r^2} - \frac{4a\,Km}{r} +$$

$$+ 3K^2 + 4a\,Km^2 - 6K^2 mr + 2K^2 m^2 r^2 \Big]. \qquad (5)$$

Writing

$$ds^2 = e^{\nu(r)} c^2 dt^2 - e^{\lambda(r)} dr^2 - r^2 (d\theta^2 + \sin^2\theta\, d\phi^2),$$

one of the Einstein's equations gives [2] :

$$-\frac{8\pi G}{c^4} u = e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) - \frac{1}{r^2}, \qquad (6)$$

where G is the gravitational constant. With the notation

$$g(r) = e^{-\lambda(r)}.$$

eq. (6) reads

$$rg' + g = 1 - \frac{8\pi G}{c^4} r\, u.$$

for which the solution is

$$g = 1 - \frac{2m_0}{r} - \frac{8\pi G}{c^4 r} \int r^2 u(r)\, dr, \qquad (7)$$

where $m_0 = GMc^{-2}$ is an integration constant. Substituting (5) into eq. (7) one gets

$$g(r) = 1 - \frac{2m_0}{r} + \frac{G}{c^4} e^{-2mr} \left(\frac{\alpha}{r^2} + \frac{\alpha m}{r} + 2Kmr - \frac{K^2}{\alpha} r^2 + \frac{K^2 m^2}{\alpha} r^3 \right). \quad (8)$$

$g(r)=0$ gives the locations of the event horizons if there $\exp \nu(r)$ is finite. Note that the metric tends to the Minkowsi's one only if $m \neq 0$.

Let us suppose, however, that $mr \ll 1$ in the whole relevant interval. Then

$$g(r) \sim 1 - \frac{2m_0}{r} + \frac{G}{c^4} \left(\frac{\alpha}{r^2} - \frac{K^2}{\alpha} r^2 \right) \equiv f(r). \quad (9)$$

and $f=0$ leads to a fourth order equation. For $r \gg a$ one immediately observes the possibility of an even horizon (form (8) actually two) around $(\alpha > 0!)$

$$r^2 \sim \frac{\alpha c^4}{GK^2} .$$

with the above data for α and K $(\alpha = \alpha_0 hc)$

$$r \sim 5.5 \times 10^5 \text{ cm}$$

an enormously large value. The assumption $mr \ll 1$ leads to the ratio

$$\frac{m}{m_\pi} \ll 2.5 \times 10^{-19} ,$$

where m_π is the pion mass.

It is quite obvious that the starting formula (1) for the potential cannot be extrapolated for such distances. Indeed, as c and $\bar{c}$ separate the energy grows until a pair of light quarks is created, which together with c and $\bar{c}$ form a pair of mesons, therefore the whole former simple picture breaks down.

Lonely lived quarks, therefore, in reality do not seem to produce some type of "exotic" gravitational black holes.

3. Net we wish to discuss the ideas of Recami and Castorina [5,6] concerning the concept of "strong black holes" in our case.

According to the classical mechanics, two particles with the same mass due to their gravitational interaction move around each other in a flat space-time as if they were led by the force

$$F = -G\frac{M^2}{r^2}. \qquad (10)$$

General relativity tells us that the motion is actually a geodetic one in a curved space-time, the structure of which is described by Einstein's equations. In an attractive strong interaction, e.g., with

$$F = -\frac{a}{r^2}e^{-mr}, \qquad a>0, \qquad (11)$$

for those distances where $mr \ll 1$, , at the classical level one may say that the interaction is of a "gravitational" nature, but G is replaced by

$$G \to \frac{a}{M^2}. \qquad (12)$$

In analogy with the gravitational theory, let us suppose now that the expression (11) is just as crude a description of the reality as (10) is in the gravitational case. Instead, the motion of a strongly interacting particle (for attraction) is a geodetical one in a (strongly) curved space-time, where the (strong) space-time structure (measured by pion or gluon signals?) is determined by Einstein's equations, but according to (12) G is replaced by αM^{-2}.

Adopting this idea, introducing

$$K = -\frac{\alpha}{a^2}, \quad x = \frac{r}{a}, \quad A = \frac{Mc^2 a}{\alpha}, \quad G \to \frac{\alpha}{M^2},$$

the equation $f(r) = 0$, where f is given by (9) takes the form

$$x^4 - A^2 x^2 + 2Ax - 1 = 0. \tag{13}$$

The value of A with the data of ref./1/ ($M = M_c = 1.6$ GeV) is $A = 8.5$. With this value of A from eq. (13) one immediately observes that there are three positive real roots, two of them lie close to A^{-1}, the third one in the immediate neighbourhood of A . Keeping in mind the former picture this means, that a lonely lived quark produces a "strong black hole" for its antiparticle (attraction) of the radius:

$$R \sim aA = 1.7\,\text{fm}.$$

then the antiparticle falls down to $R \sim aA^{-1} = 0.02$ fm. The condition $mr \ll 1$ gives for the gluon mass

$$\frac{m}{m_\pi} \ll 1.$$

4. Apart from the above considerations, one might notice that one of the Einstein's equations (for the coefficient of dr^2) can exactly be solved for those situations where a particle is surrounded by a given field representing a spherically symmetric static potential V in the form

V = Yukawa times an arbitrary polynomial in r .

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