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DIQUARK SPECTRUM
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DIQUARK SPECTRUM IN TWO-DIMENSIONAL QCD

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Спектр дикварков в двумерной квантовой хромодинамике

В двумерной квантовой хромодинамике изучаются одновременно кварк-кварк и кварк-антикварк каналы с помощью метода континуального интегрирования по билокальным переменным. Находится эффективное билокальное действие, условие стационарности которого дает уравнения, являющиеся релятивистскими аналогами уравнений для функций Грина теории сверхпроводимости. Вычисляется спектр дикварковых связанных состояний и показывается, что дикварки в рассматриваемой модели не могут существовать в свободном состоянии.

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Diquark Spectrum in Two-Dimensional QCD

The quark-antiquark and quark-quark channel of two-dimensional QCD are simultaneously studied by using bilocal functional techniques. The stationarity condition for the effective bilocal action yields a relativistic analogue of the Green's function equations of superconductivity. The spectrum of the diquark bound states is computed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. Introduction

There are many reasons to believe that Quantum Chromodynamics (QCD) is the basic theory of hadrons. 't Hooft has investigated, for example, the particle spectrum of a two-dimensional QCD/1/. In this case one is able to solve the dynamical equations of the theory. As has been found there, two-dimensional QCD exhibits quark confinement and yields a linearly increasing mass spectrum for the colour singlet quark-antiquark bound-state mesons. Recently, a lot of interesting questions such as unitarity, Regge behaviour, deep inelastic scattering, form factor behaviour, e^+e^--annihilation have been studied within this model/2-5/. Moreover, introducing bilocal fields it is possible to derive from two-dimensional QCD an infinite-component nonpolynomial field theory for the bound state mesons/6/. A complete effective Lagrangian of this model should, however, contain colour-singlet mesons as well as baryons. As a first step towards this aim we investigate in this paper besides the quark-antiquark channel the quark-quark channel and the question concerning diquark bound states. Baryons could then arise (for $SU(3_c)$) as bound states of diquarks and quarks.

For this purpose, we have introduced two types of bilocal fields for the $q\bar{q}$ - and qq -channels. The stationarity condition for the effective bilocal action yields then a system of equations for the quark Green's functions which is the relativistic analogue of the equations in the theory of superconductivity/7,8,9/. The second functional derivative of the effective action evaluated at the stationary point determines the kernel of the Bethe-Salpeter equation for the $q\bar{q}$ - and qq -channels.

In Sect. 2 we derive the effective bilocal action for the meson and diquark sector. Sect. 3 deals with the quark and diquark spectrum.

2. Bilocal Variables

Let us start with the generating functional of Green's functions of two-dimensional QCD in the light-cone gauge. In terms of dynamical variables we get/6/

$$Z(\eta,\eta^+) = C \int Dq\, Dq^+ \exp i \Big\{ -(qq^+, iG^{-1}) - i\frac{(2g)^2}{2}(qq^+, K qq^+) + \eta^+ q + q^+ \eta \Big\} . \tag{1}$$

Here we have used the short-hand notations

$$(qq^+, K qq^+) = q_B q_A^+ K_{AB,CD}\, q_D q_C^+$$

$$= \int d^2x\, d^2y\, d^2x'\, d^2y'\; q_{b\beta}(y)\, q^+_{a\alpha}(x)\, K_{a\alpha\, b\beta;\, g\gamma\, d\delta}(xy;y'x')\, q_{d\delta}(x')\, q^+_{g\gamma}(y') ,$$

$$K_{a\alpha\, b\beta;\, g\gamma\, d\delta}(xy;y'x') = \sum_{n=1}^{N_c^2-1} \lambda^{(n)}_{\alpha\delta}\, \lambda^{(n)}_{\gamma\beta}\, \delta_{ad}\, \delta_{gb}\, T^{-1} D(x-y)\, \delta(x-x')\, \delta(y'-y)$$

$$\equiv k_{\alpha\delta,\gamma\beta}\, \delta_{ad}\, \delta_{gb}\, T^{-1} D(x-y)\, \delta(x-x')\, \delta(y'-y) , \tag{2}$$

$\lambda^{(n)}_{\alpha\delta}$ are the matrix representatives of the generators of the local colour gauge group $SU(N_c)$ normalized according to $\mathrm{tr}\, \lambda^{(n)} \lambda^{(m)} = T \delta^{nm}$ ($\lambda^{0} = \sqrt{\frac{T}{N_c}}\, \mathbb{1}$). The gluon propagator $D(x)$ and the quark propagator $G(x)$ are given by

$$D(x) = \int \frac{d^2k}{(2\pi)^2} \Big[\frac{-i}{k_-^2} \Big] e^{-ikx}$$

$$G(x) = \int \frac{d^2k}{(2\pi)^2} \Big[\frac{i k_-}{2k_+k_- - m_\alpha^2 + i\varepsilon} \Big] e^{-ikx} \tag{3}$$

$$k_\pm = \frac{1}{\sqrt{2}}(k_0 \pm k_1)$$

The capital indices A in eq. (2) denote a triplet of indices (a, α, x), where $a = 1, \dots, N_f$ and $\alpha = 1, \dots, N_c$ are indices of the global flavour group $SU(N_f)$ or of the colour gauge group $SU(N_c)$, respectively, and x is an integration variable (summation or integration over repeated indices is understood). Finally, C is a normalization factor guaranteeing that $Z(0,0) = 1$. As can easily be seen, the interaction term (2) leads to attractive forces in the colour-singlet quark-antiquark channel. Indeed, we have

$$\{N_c\} \times \{N_c^*\} = \{1\} + \{N_c^2-1\}$$

$$k_{\alpha\beta,\gamma\delta} = \sum_{n=1}^{N_c^2-1} \lambda^{(n)}_{\alpha\delta}\, \lambda^{(n)}_{\gamma\beta} = T \Big(\frac{N_c^2-1}{N_c} P_1 - \frac{1}{N_c} P_{N_c^2-1} \Big)_{\alpha\beta,\gamma\delta} , \tag{4}$$

where P_1 , $P_{N_c^2-1}$ are the projection operators on the respective $q\bar{q}$ -channels. As has been shown in ref./1,6/ there arise then meson bound states in the colour-singlet $q\bar{q}$ -channel. These bound states are most easily introduced into the generating functional by linearizing the four-quark interaction (2) in (qq^+) using a bilocal field $\Sigma(x,y)$. Moreover, an effective Lagrangian of QCD should also contain the colour- singlet baryon bound states which are built from N_c quarks (for $SU(N_c)$) in the completely antisymmetric combination $\varepsilon_{i_1 i_2 \ldots i_{N_c}} q_{i_1} q_{i_2} \ldots q_{i_{N_c}}$ /10/. This could possibly be achieved by adding and subtracting suitable combinations of N_c quark fields to the exponent of eq. (1) and linearizing them by multi-local fields $B(x_1, x_2, \ldots, x_{N_c})$. As a first step towards this aim we investigate in this paper besides the qq^+ colour-singlet sector the qq (antisymmetric)-sector of two-dimensional QCD. Using

$$\{N_c\} \times \{N_c\} = \{A\} + \{S\}$$

$$-\tilde{k}_{\alpha\beta,\gamma\delta} \equiv \sum_{n=1}^{N_c^2-1} \lambda^{(n)}_{\alpha\delta}\left(-\lambda^{(n)T}_{\gamma\beta}\right) = T\left(\frac{N_c+1}{N_c} P_A - \frac{N_c-1}{N_c} P_S\right)_{\alpha\beta,\gamma\delta} \tag{5}$$

$$\left(\lambda^{(n)T}_{\gamma\beta} = \lambda^{(n)}_{\beta\gamma}\right)$$

it is obvious that the gluon exchange leads to attraction in the antisymmetric channel $\{A\}$ and to repulsion in the symmetric channel $\{S\}$. P_A and P_S are the respective projection operators. We expect then the formation of diquark bound states in an antisymmetric colour configuration. In particular, for $SU(3_c)$ diquarks could possibly be bound together with quarks to build colour-singlet baryons. Adding and subtracting suitable terms we now use a four-quark interaction which exhibits explicitly the attractive forces in both the $q\bar{q}$ - and qq - channels*)

$$F = \left\{ (qq^+, P_1 K P_1\, qq^+) + ((qq)^+, P_A \tilde{K} P_A\, qq) \right\} + W_r(q, q^+) \tag{6}$$

W_r denotes, in the spirit of a Hartree-Fock approximation, a "residual" interaction which will be treated perturbatively.

*) The effective four-quark interaction in the curly bracket of eq. (6) leads to weight factors in the dynamical equations for the $q\bar{q}$ - and qq -sector that agree with perturbation theory.

For this purpose, we write

$$W_r(q, q^+) = W_r\left(\frac{1}{i}\frac{\delta}{\delta\eta^+}, \frac{1}{i}\frac{\delta}{\delta\eta}\right)$$

and take it outside the functional integral. We now linearize the effective four-quark interaction in the expressions (qq^+) and (qq) by performing real and complex bilocal Gauss integrations/6,11/

$$\exp\left\{\frac{(2gi)^2}{2}(qq^+, P_1 K P_1\, qq^+)\right\} = C_1 \int D\Sigma \exp\left\{\frac{1}{2}\frac{1}{(2gi)^2}(\Sigma, K^{-1}\Sigma) + i(qq^+ P_1)\Sigma\right\} \tag{7}$$

$$\exp\left\{\frac{(2gi)^2}{2}((qq)^+, P_A \tilde{K} P_A\, qq)\right\} = C_2 \int D\Lambda D\bar{\Lambda} \exp\left\{-\frac{2}{(2gi)^2}(\bar{\Lambda}, \tilde{K}^{-1}\Lambda) + (P_A qq)^+ \Lambda + \bar{\Lambda}(P_A qq)\right\} \tag{8}$$

$$K^{-1}_{AB,CD} K_{DC,EF} = \delta_{AB,EF} \equiv \delta_{AF}\delta_{EB}, \quad etc.$$

The bilocal "fields" Σ, Λ are treated as c -number (Bose) fields. To have bilocal sources that couple directly to the bilocal fields we add in eqs. (7,8) source terms $P_1 R$, $P_A Q$ and perform the change of variables

$$\begin{aligned} P_1 R + \Sigma &= X \\ P_A Q + \Lambda &= Y \\ \overline{Q P_A} + \bar{\Lambda} &= \bar{Y} \end{aligned} \tag{9}$$

Using eqs. (6-9) the functional integral over the quark fields in eq. (1) may now be taken

$$\int Dq\, Dq^+ \exp \frac{i}{2}\left\{(q^T, q)\, L \begin{pmatrix} q \\ q^T \end{pmatrix} + (q^T, q)\begin{pmatrix} \eta \\ -\eta^T \end{pmatrix} + (\eta^T, -\eta)\begin{pmatrix} q \\ q^T \end{pmatrix}\right\} \tag{10}$$

$$= i\left[-\det L\right]^{1/2} \exp -\frac{i}{2}(\eta^T, -\eta)\, L^{-1}\begin{pmatrix} \eta \\ -\eta^T \end{pmatrix}$$

where

$$L = \begin{pmatrix} i\, G_{X_1}^{-1} & ; & -2\, i Y_A \\ -2(\overline{i\, Y_A}) & ; & -i\, G_{X_1}^{-1\,T} \end{pmatrix} \qquad (11)$$

and

$$\begin{aligned} i\, G_{X_1}^{-1} &= i\, G^{-1} - X_1 \qquad ; \quad X_1 = P_1 X \\ (G_X^T)_{AB} &= (G_X)_{BA} \qquad ; \quad Y_A = P_A Y \end{aligned} \qquad (12)$$

The generating functional now reads

$$\begin{aligned} Z(\eta, \eta^+) = {} & C' \exp i \frac{(2gi)^2}{2} W_r\left(\frac{1}{i}\frac{\delta}{\delta\eta^+}, \frac{1}{i}\frac{\delta}{\delta\eta}\right) \times \\ & \times \int DX \int DY \int D\overline{Y} \exp\left[i W_{eff}(X, Y, \overline{Y})\right] Z(\eta, \eta^+, R, Q, \overline{Q} \,|\, X, Y, \overline{Y}), \end{aligned} \qquad (13)$$

where the effective action W_{eff} and the source-dependent term $Z(\,.\,.\,|X, Y, \overline{Y})$ are given by

$$\begin{aligned} W_{eff}(X, Y, \overline{Y}) = {} & \frac{1}{2}\frac{1}{i(2gi)^2}(X, K^{-1}X) - i \operatorname{tr} \ln\left[i G^{-1} - X_1\right] \\ & + \frac{2i}{(2gi)^2}(\overline{iY}, \widetilde{K}^{-1} iY) - \frac{i}{2}\operatorname{tr}\ln\Big[1 + \frac{G_{X_1}^T}{i} 2(\overline{iY_A}) \times \\ & \times \frac{G_{X_1}}{i} 2(iY_A)\Big] \end{aligned} \qquad (14)$$

and

$$\begin{aligned} Z(\,.\,.\,|X, Y, \overline{Y}) = \exp\Big\{ & -i\eta^+ G_n \eta + \frac{i}{2}\eta^+ G_a \eta^+ - \frac{i}{2}\eta \overline{G}_a \eta \\ & + \frac{1}{2}\frac{1}{(2gi)^2}(P_1 R, K^{-1} P_1 R) - \frac{1}{(2gi)^2}(P_1 R, K^{-1} X) - \frac{2}{(2gi)^2}(\overline{P_A Q}, \widetilde{K}^{-1} P_A Q) \\ & - i \frac{2}{(2gi)^2}(\overline{P_A Q}, \widetilde{K}^{-1} iY) + i \frac{2}{(2gi)^2}(\overline{iY}, \widetilde{K}^{-1} P_A Q) \Big\} \end{aligned} \qquad (15)$$

Here

$$\begin{aligned} G_n &= \left[1 + \frac{G_{X_1}}{i} 2(iY_A) \frac{G_{X_1}^T}{i} 2(\overline{iY_A})\right]^{-1} \frac{G_{X_1}}{i} \\ G_a &= -\left[1 + \frac{G_{X_1}}{i} 2(iY_A) \frac{G_{X_1}^T}{i} 2(\overline{iY_A})\right]^{-1} \frac{G_{X_1}}{i} 2(iY_A) \frac{G_{X_1}^T}{i} \\ \overline{G}_a &= \frac{G_{X_1}^T}{i} 2(\overline{iY_A}) \frac{G_{X_1}}{i}\left[1 + 2(iY_A)\frac{G_{X_1}^T}{i} 2(\overline{iY_A}) \frac{G_{X_1}}{i}\right]^{-1} \end{aligned} \qquad (16)$$

are the normal and anomalous Green's functions of quarks moving in external bilocal fields X , Y .

3. Particle Spectrum

a) Quark sector

Considering the stationary points of the effective bilocal action (14) immediately leads to dynamical equations for the quark spectrum of two-dimensional QCD. Let us take

$$\delta W_{eff}(X, Y, \overline{Y}) = 0$$

or

$$\frac{\delta W_{eff}}{\delta X} = 0 \quad , \quad \frac{\delta W_{eff}}{\delta Y} = 0 \quad ; \quad \frac{\delta W_{eff}}{\delta \overline{Y}} = 0 \,. \qquad (17)$$

Denoting the projections of the solutions $(X, Y, \overline{Y})$ of eq. (17) by

$$P_1 X = M \quad , \quad P_A 2(iY) = \Delta \quad , \quad P_A 2(\overline{iY}) = \overline{\Delta} \qquad (18)$$

and taking into account the definitions (16), the stationarity conditions (17) yield the following coupled system of equations

$$\begin{aligned} M &= (2gi)^2 K P_1 \, G_n(M, \Delta, \overline{\Delta}) \\ \Delta &= -(2gi)^2 \widetilde{K} P_A \, G_a(M, \Delta, \overline{\Delta}) \\ \overline{\Delta} &= (2gi)^2 \, \overline{G}_a(M, \Delta, \overline{\Delta}) P_A \widetilde{K} \,. \end{aligned} \qquad (19)$$

In momenum space we get ($M_{\alpha\beta}(p) = M(p)\,\delta_{\alpha\beta}$; flavour indices are suppressed)

$$\begin{aligned} M(p_-) = {} & i \frac{g^2}{\pi^2} \frac{N_c^2 - 1}{N_c} \frac{1}{N_c} \int dk_- \frac{\theta(|k_- - p_-| - \lambda)}{(k_- - p_-)^2} \times \\ & \times \int dk_+ \left\{\left[\mathbb{1} + \frac{G_M(k)}{i} \Delta(k_-) \frac{G_M^T(k)}{i} \overline{\Delta}(k_-)\right]^{-1} \cdot \frac{G_M(k)}{i}\right\}_{\delta\delta} \end{aligned} \qquad (20)$$

$$\Delta_{\alpha\beta}(p_-) = -i\,\frac{g^2}{\pi^2}\,\frac{N_c+1}{N_c}\,(P_A)_{\alpha\beta,\gamma\delta}\int dk_-\,\frac{\theta(|k_--p_-|-\lambda)}{(k_--p_-)^2}\;\times$$
$$\times\int dk_+\left\{\left[\mathbb{1}+\frac{G_M(k)}{i}\Delta(k_-)\frac{G_M^T(k)}{i}\bar{\Delta}(k_-)\right]^{-1}\frac{G_M(k)}{i}\Delta(k_-)\frac{G_M^T(k)}{i}\right\}_{\delta\gamma}, \tag{21}$$

where

$$\frac{G_M(k)}{i} = -\frac{G_M^T(k)}{i} = \frac{k_-}{2k_+k_- - m_a^2 + i\varepsilon - k_-M(k_-)}, \tag{22}$$

$\theta(|k_-|-\lambda)$ is an infrared cut-off. The coupled system of the nonlinear equations (16),(19) for the normal and anomalous quark Green's functions and their respective normal and anomalous self-energies M and $\Delta, \bar{\Delta}$ is just the relativistic analogue of the equations in the theory of superconductivity /7-9/. For illustration, we give in Fig. 1 the graphical expressions for G_n, G_a, G_M, $\Delta, \bar{\Delta}$ and M. The content of the coupled equations (16,19) is shown in Fig. 2. Notice that in the case of trivial anomalous solutions $\Delta = \bar{\Delta} = G_a = \bar{G}_a = 0$ the above equations reduce to the Schwinger-Dyson equations for G_M, M in an approximation where vertex and gluon propagator corrections are absent. The respective equations have been solved by 't Hooft /1/ in the limit $N_c \to \infty$, g^2N_c fixed. The solutions are

$$M(k_-) = \frac{g^2}{\pi}\,\frac{N_c^2-1}{N_c}\left(\frac{\operatorname{sgn} k_-}{\lambda} - \frac{1}{k_-}\right) \tag{23}$$

with $G_M(k)$ given by eq. (22). In particular, it follows from eq. (23) that if $\lambda \to 0$ then $M(k_-) \to \infty$ and

$$G_M(k) \sim \lambda\left[\frac{-i}{\frac{g^2}{\pi}\frac{N_c^2-1}{N_c}\operatorname{sgn} k_-}\right] \tag{24}$$

This vanishing of the quark poles for $\lambda \to 0$ has been interpreted as an example of an infrared confinement of quarks*).

*) The limit $\lambda \to 0$ has to be performed at the end of all calculations. Other realizations of quark confinement with propagator poles have been discussed in ref. /2,3/.

We shall restrict ourselves in the following to the solution $M \neq 0$, $\Delta = \bar{\Delta} = 0$. The question of possible nontrivial solutions $\Delta \neq 0$ will be studied elsewhere (a solution $\Delta \neq 0$ would imply a new BCS-ground state containing Cooper pairs of quarks).*)

b) Diquark spectrum

Let us now expand the integrand of eq. (13) around the stationary point $P_1X = M$, $\Delta = \bar{\Delta} = 0$. Shifting the integration variable according to

$$X(x,y) = M(x-y) + \phi(x,y) \tag{25}$$

the generating functional (13) may be rewritten as

$$Z(\eta,\eta^+,R,Q,\bar{Q}) = \bar{C}\,\exp\left[i\frac{(2gi)^2}{2}W_r\left(\frac{1}{i}\frac{\delta}{\delta\eta^+},\frac{1}{i}\frac{\delta}{\delta\eta}\right)\right.$$
$$\times\int D\phi\int Dy\int D\bar{y}\,\exp i\left\{W_{free}^{\phi} + W_{free}^{y} + W_{int}\right\}$$
$$\times\, Z(\eta,\eta^+,R,Q,\bar{Q}\,|\,M+\phi,y,\bar{y}) \tag{26}$$

Here the free actions W_{free}^{ϕ}, W_{free}^{y} for the field variables ϕ and y are determined by the quadratic parts of the effective bilocal action

$$W_{free}^{\phi} = \frac{1}{2i}(\phi, D_\phi^{-1}\phi) \equiv \frac{1}{2i}\left(\phi,[(2gi)^2K]^{-1}\left[1-(2gi)^2\left(\frac{G_M}{i}(KP_1)\frac{G_M}{i}\right)P_1\right]\phi\right)$$
$$W_{free}^{y} = \frac{1}{2i}(\bar{y}, D_y^{-1}y) \equiv \frac{1}{2i}\left(\bar{y},[-(2gi)^2\tilde{K}]^{-1}\left[1+(2gi)^2\left(-\frac{G_M^T}{i}(\tilde{K}P_A)\frac{G_M}{i}\right)P_A\right]y\right) \tag{27}$$

The interaction part W_{int} is given by $(\phi_1 \equiv P_1\phi)$

*) There are implications from the non-relativistic theory that superconductivity cannot arise in two-dimensional models, i.e., $\Delta \equiv 0$ /12/.

$$W_{int} = i\sum_{n=3}^{\infty}\frac{1}{n}\,\mathrm{tr}\left(\frac{G_M}{i}\phi_1\right)^n + \frac{i}{2}\sum_{n=2}^{\infty}\frac{1}{n}\,\mathrm{tr}\left[-\frac{G_M^T}{i}\,\overline{iY_A}\,\frac{G_M}{i}\,iY_A\right]^n + \frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\,\mathrm{tr}\left\{-\sum_{p+q>0}\left[\left(\frac{G_M}{i}\phi_1\right)^p\frac{G_M}{i}\right]^T\overline{(iY_A)}\left[\left(\frac{G_M}{i}\phi_1\right)^q\frac{G_M}{i}\right]\cdot iY_A\right\}^n \tag{28}$$

The first two contributions describe here the self-interactions of the ϕ and Y fields, the third term describes the ϕ, Y interaction. $\mathcal{D}_\phi$, $\mathcal{D}_Y$ are the propagators of the bilocal fields ϕ, Y. They satisfy inhomogeneous Bethe-Salpeter equations for $q\bar{q}$- or qq-scattering in the colour-singlet or antisymmetric channels, respectively,

$$\begin{aligned}\mathcal{D}_\phi &= (2gi)^2K + (2gi)^2\left(\frac{G_M}{i}(KP_1)\frac{G_M}{i}\right)P_1\mathcal{D}_\phi\\ \mathcal{D}_Y &= -(2gi)^2\tilde{K} - (2gi)^2\left(-\frac{G_M^T}{i}(\tilde{K}P_A)\frac{G_M}{i}\right)P_A\mathcal{D}_Y\end{aligned} \tag{29}$$

Varying the free actions (27), we easily get the Euler-Lagrange equations for the "free" bilocal fields

$$\mathcal{D}_\phi^{-1}\Gamma_1 = 0 \quad ; \quad \mathcal{D}_Y^{-1}\Gamma^A = 0 . \tag{30}$$

These are homogeneous BS-equations for the respective bound state vertex functions in ladder approximation. The first equation determines the $q\bar{q}$-meson spectrum [6] and has been solved by 't Hooft [1]; the second equation determines the diquark spectrum. Its explicit form reads

$$\Gamma^A(p,r) = -i(2g)^2\frac{N_c+1}{N_c}\int\frac{d^2k}{(2\pi)^2}\frac{1}{k_-^2}G_M(p+k)\Gamma^A(p+k,r)\times G_M(p+k-r), \tag{31}$$

where r is the total momentum of the qq pair (cf. Fig. 3). Eq. (31) is analogous in structure to the equation of the $q\bar{q}$-sector [1]. In particular, we have the right sign in front of the integral in order to get attraction. There is, however, now the important difference that the infrared singularities of the quark propagators and of the gluon kernel do not longer cancel (except for the special case $SU(2_c)$). The solutions of eq.(31) have to be totally antisymmetric if one interchanges the colour, flavour and space-time variables of the two quark constituents.

It is convenient to consider the integrated wave function

$$h^A(p_-,r) = \int dp_+\,G_M(p)\,\Gamma^A(p,r)\,G_M(p-r). \tag{32}$$

Inserting it into eq. (31) we obtain again an equation of the 't Hooft type

$$\nu^2 h^A(x) = \left(\frac{\beta_1}{x} + \frac{\beta_2}{1-x}\right)h^A(x) - P\int_0^1\frac{h^A(y)}{(y-x)^2}\,dy, \tag{33}$$

where $x = p_-/r_-$ and

$$\begin{aligned}2r_+r_- - \frac{2g^2}{\pi\lambda}|r_-|\left(\frac{N_c^2-1}{N_c} - \frac{N_c+1}{N_c}\right) &= \frac{g^2}{\pi}\,\frac{N_c+1}{N_c}\,\nu^2\\ \beta_{1,2} = m_{1,2}^2\Big/\frac{g^2}{\pi}\frac{N_c+1}{N_c} &- (N_c-1).\end{aligned} \tag{34}$$

The solutions of eq. (33) have to satisfy the boundary conditions $h^A(x)\sim x^{\gamma_a}\,[(1-x)^{\gamma_b}]$ at $x=0$ $[x=1]$, where $\pi\gamma_a\cot\pi\gamma_a = -\beta_a$. The system of the eigenfunctions of eq. (33) is complete and orthogonal. For large k one obtains

$$\begin{aligned}h_k^A(x) &\simeq \sqrt{2}\,\sin\pi k x \qquad (k\gg 1)\\ \nu_k^2 &\simeq \pi^2 k .\end{aligned} \tag{35}$$

This yields together with eq. (34) the following mass spectrum for the diquarks

$$M_k^2 = (2r_+r_-)_k = \frac{g^2}{\pi}\,\frac{N_c+1}{N_c}\,\nu_k^2 + \frac{2g^2}{\pi\lambda}|r_-|\,\frac{N_c(N_c-1)-2}{N_c} . \tag{36}$$

This spectrum is very similar to the single quark spectrum defined by the pole of the dressed quark propagator (22), (23)

$$M^2 = 2k_+k_- = \left(m_a^2 - \frac{g^2}{\pi}\frac{N_c^2-1}{N_c}\right) + \frac{g^2}{\pi\lambda}|k_-|\frac{N_c^2-1}{N_c} . \qquad (37)$$

Thus, for $N_c > 2$ the quark and diquark masses tend to infinity as $\lambda \to 0$. This is the expected result that after removing the infrared cut-off coloured quark states should not contribute. The diquarks are thus confined like the quarks itself. Notice also the exceptional role of $N_c = 2$ where the diquark spectrum coincides with the meson spectrum.

In ref./6/ we have represented the bilocal propagator of the meson field in terms of the BS-vertex functions. For completeness, we quote here also the analogous result, for D_y (flavour indices are suppressed; $x = p_-/r_-$, $x' = p'_-/r_-$)

$$D_y(x,x';r)_{\alpha\beta,\gamma\delta} = -(2g)^2\,\tilde{k}_{\alpha\beta,\gamma\delta}\,T^{-1}\frac{i}{r_-^2(x'-x)^2} - \sum_k \frac{i}{r^2-M_k^2}\left[\frac{1}{r_-}\Gamma^A_{k,\alpha\beta}(x)\right]\left[\frac{1}{r_-}\bar{\Gamma}^A_{k,\gamma\delta}(x')\right] \qquad (38)$$

$$\frac{1}{r_-}\Gamma^A_{k,\alpha\beta}(x) \equiv \Gamma^A_{k,\alpha\beta}(p,r) = -2i\left(\frac{N_c+1}{N_c}\right)^{1/2}\frac{2g}{\lambda}\left(\frac{g^2}{\pi}\frac{N_c+1}{N_c}\right)^{1/2} \times \left\{\Theta(x(1-x)) + \frac{\lambda}{2|r_-|}\left(\frac{\beta_1}{x}+\frac{\beta_2}{1-x}-\nu_k^2\right)\right\} h^A_{k,\alpha\beta}(x) . \qquad (39)$$

The generating functional (26) may now be calculated perturbatively by expanding the integrand in powers of ϕ and $y, \bar{y}$. The functional integral over these powers can then be taken by means of a Wick theorem using the bilocal propagators D_ϕ , D_y /6,13,14/. Finally, expanding the bilocal fields in terms of the "free" field solutions obtained from the BS-equation one can rewrite the effective actions (27,28) in the form of

an infinite-component nonpolynomial field theory for the bound states in the qq - and $q\bar{q}$ -channels/6/.

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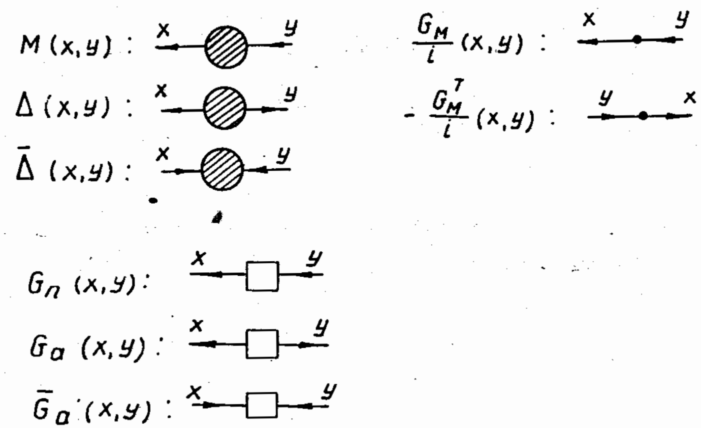

Fig. 1. Graphical representation of the normal and anomalous self-energies and quark Green's functions.

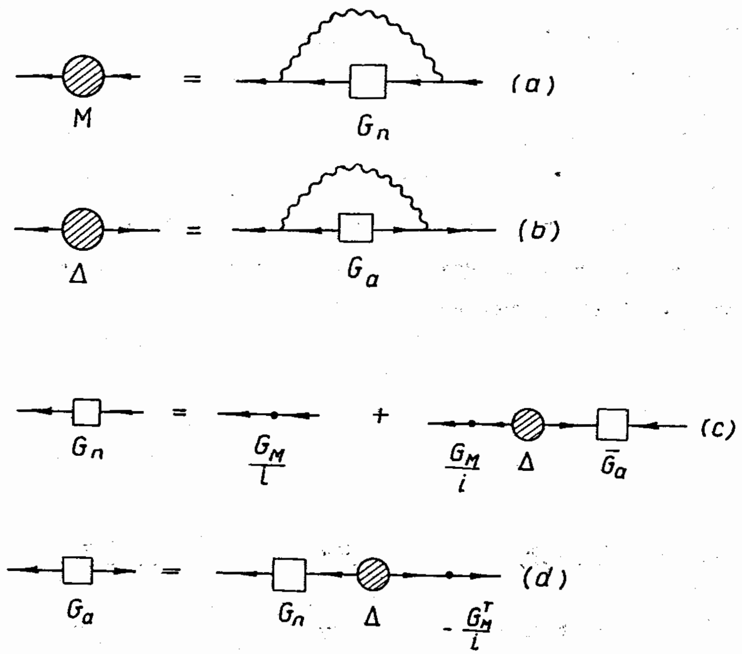

Fig. 2. Graphical representation of the coupled system of equations (16), (19).

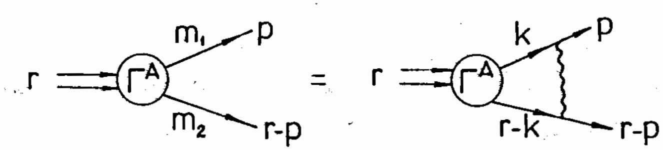

Fig. 3. The homogeneous quark-quark BS-equation.

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