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Эберт Д., Первушин В.Н. **E2 - 10730**

Эффективный билокальный лагранжиан для двумерной квантовой хромодинамики

Исследуется модель двумерной квантовой хромодинамики с помощью метода континуального интегрирования по билокальным полевым переменным. Находится эффективный билокальный лагранжиан. Первая и вторая вариации эффективного билокального действия ведут к уравнениям для кваркового и мезонного спектра, описывающим заключение кварков внутри адронов. Локальная теория поля с цветными кварками формулируется непосредственно в терминах полей связанных безцветных состояний. В результате получается бесконечно-компонентная неполиномиальная теория поля.

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Ebert D., Pervushin V.N. **E2 - 10730**

An Effective Bilocal Lagrangian for Two-Dimensional QCD

Bilocal path-integral technique is used to investigate two-dimensional QCD. The first and second variations of the effective bilocal action yield equations for the quark and meson spectrum. An infinite-component non-polynomial field theory of colour-singlet bound-state mesons is obtained.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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I. Introduction

There is much hope at present that quantum chromodynamics (QCD) - the Non-Abelian gauge theory of coloured quarks and gluons - is a good candidate for a realistic theory of hadrons /1/. It is further expected that the severe infrared singularities of QCD confine quarks within the observable hadrons. The hadrons should arise themselves in this picture in the form of colour-singlet quark bound-states. An important step for the understanding of these ideas was the investigation of a two-dimensional version of QCD by 't Hooft /2/. Recently, a number of interesting theoretical and phenomenological questions such as Regge behaviour of bound-state scattering amplitudes, form factor asymptotics, deep inelastic scattering and e^+e^- annihilation have been studied within this model /3-6/. It exists also an interesting relationship between two-dimensional QCD and certain dual string models /7/.

The aim of this paper is to derive an effective Lagrangian for two-dimensional QCD that contains directly the fields of the colour-singlet bound-state mesons. We use the bilocal functional techniques applied in ref. /8-9/ to an Abelian gauge theory of quarks and gluons. As can be shown the stationarity of the respective bilocal action favours planar ladder diagrams. Finally, an infinite-component non-polynomial field theory of bound-state mesons is obtained. In Sect.II two-dimensional QCD is formulated in terms of bilocal variables. Sect.III contains a discussion of the quark spectrum. The meson spectrum as well as a modified perturbation theory are discussed in Sect. IV. Finally, in Sect. V and VI the structure of the new effective action and a current matrix element for two-dimensional e^+e^--annihilation are investigated.

II. Model and method

A) Definition of the model

The Lagrangian of two-dimensional quantum chromodynamics (TDQCD) reads $\left(A_{\mu,\alpha\beta} = \sum_{n=1}^{N_c^2-1} A_\mu^{(n)} \lambda^{(n)}_{\alpha\beta} \right)$

$$L = \frac{1}{4} G_{\mu\nu,\alpha\beta} G^{\mu\nu}{}_{,\beta\alpha} + \bar{q}_{a\alpha} (i \gamma^\mu D_\mu - m_a) q_{a\alpha} \tag{1}$$

$$G_{\mu\nu,\alpha\beta} = \partial_\mu A_{\nu,\alpha\beta} - \partial_\nu A_{\mu,\alpha\beta} + g \left[A_\mu, A_\nu \right]_{\alpha\beta}$$

$$D_\mu q_{a\alpha} = \partial_\mu q_{a\alpha} + g A_{\mu,\alpha\beta} q_{a\beta}, \tag{2}$$

where q, A_μ are quark and gluon fields, respectively. The indices $\alpha = 1, 2, \dots, N_C$ denote colour labels, the index $a = 1, 2, \dots, N_f$ denotes flavour. The local gauge group $SU(N_c)$ is exactly conserved, whereas the global group $SU(N_f)$ is broken by the quark mass term $\bar{q}_{a\alpha} m_a q_{a\alpha}$ (unless $m_a = m$). $\lambda^{(n)}$ are the matrix representatives of the generators of $SU(N_c)$ normalized according to $\mathrm{tr}\, \lambda^{(n)} \lambda^{(m)} = T \delta^{nm}$, $q_{a\alpha} = \begin{pmatrix} q_{1a\alpha} \\ q_{2a\alpha} \end{pmatrix}$ is a 2-Dirac spinor.

TDQCD becomes simplest in the light-cone gauge

$$A_- = \frac{1}{\sqrt{2}} (A_0 - A_1) = A^+ = 0 \tag{3}$$

$$\left(x^\pm = x_\mp = \frac{1}{\sqrt{2}} (x^0 \pm x^1) \; ; \; a \cdot b = a_+ b^+ + a_- b^- = a_+ b_- + a_- b_+ \right).$$

In this gauge the self-interaction of the gluon field vanishes and there are no Fadeev-Popov ghosts. The only dynamical variable is $\hat{q}_2 \equiv \begin{pmatrix} 0 \\ q_2 \end{pmatrix}$ /2,10,11/. After eliminating the dependent variables A_+, (q_1), the generating functional of Green's functions may be written as $(\hat{q}_2 \to q)$

$$\Xi(j,\eta,\eta^{+}) = C^{-1}\int Dq\, Dq^{+} \exp i \int d^2x\, d^2y \Big[q^{+}(x) [\text{illegible}] q(y) +$$

$$+ \left(\eta^{+}(x) q(y) + q^{+}(x)\eta(y)\right) [\text{illegible}] (x-y) +$$

$$+ i \frac{(2g_0)^2}{2} \sum_{n=1}^{N^2-1} \left(q^{+}(x)\lambda^{(n)} q(x)\right) [\text{illegible}] D(x-y) [\text{illegible}]$$

$$+ \frac{1}{2} \sum_{n=1}^{N^2-1} [\text{illegible}]\, j^{(n)}(x)\, D(x-y)\, j^{(n)}(y) \Big\} . \qquad (4)$$

The quark and gluon Green's functions (G, D) are given by the equations $(A_{ext}(x) = \int d^2y\, D(x-y)\, j(y))$

$$\int d^2y \left(G^{-1}(x,y) + 2g_0 A_{ext}(x)\delta^{(2)}(x-y)\right) G(y,z|j) = [\text{illegible}] \qquad (5)$$

$$G(x,y) = \frac{[\text{illegible}]\,\partial_-}{-2\partial_+\partial_- - m_0^2 + i\varepsilon}\,\delta^{(2)}(x-y) = \int \frac{d^2k}{(2\pi)^2} \left[[\text{illegible}] \right] e^{ik(x-y)} \qquad (6)$$

$$D(x-y) = \frac{i}{\partial_-^2}\,\delta^{(2)}(x-y) = \int \frac{d^2k}{(2\pi)^2} \left[-\frac{i}{k_-^2} \right] e^{ik(x-y)} . \qquad (7)$$

The generating functional (4) together with the propagators (5-7) just reproduce the Feynman rules of *TDQCD formulated* in ref. /2/.(We have chosen the normalization $\Xi(\ldots) = 1$).

B) Introduction of bilocal variables

Our next task is to perform the integration over the quark variable in eq. (4). For this purpose it is convenient first to rewrite the four-quark term of eq.(4)

$$F = \sum_{n=1}^{N^2-1} \int d^2x\, d^2y \left(q^{+}_{a\alpha}(x)\lambda^{(n)}_{ab} q_{b\beta}(x)\delta_{\alpha\beta}\right) [\text{illegible}] D(x-y) \left(q^{+}_{g\gamma}(y)\lambda^{(n)}_{gd} q_{d\delta}(y)\delta_{\gamma\delta}\right)$$

$$= -\int d^2x\, d^2y\, d^2x'\, d^2y'\, q_{b\beta}(y)\, q^{+}_{a\alpha}(x)\, K_{\alpha a b\beta,\, g\gamma d\delta}(x,y,y',x')\, q_{d\delta}(x')\, q^{+}_{g\gamma}(y')$$

$$\equiv -q_B\, q^{+}_A\, K_{AB,GD}\, q_D\, q^{+}_G , \qquad (8)$$

where

$$K_{a\alpha\, b\beta,\, g\gamma\, d\delta}(x\,y;\,y'x') = \sum_{n=1}^{N_c^2-1} \lambda^{(n)}_{\alpha\delta}\, \lambda^{(n)}_{\gamma\beta}\, \delta_{ad}\, \delta_{gb}\, T^{-1} D(x-y)\, \delta(x-x')\, \delta(y'-y).$$

(9)

In the following we shall often abbreviate a triple of discrete or continuous indices and variables (a, α, x) by a capital A (summation over equal capital indices includes integrations over the continuous variable). Taking into account the decomposition $\{N_c\} \times \{N_c^*\} = \{1\} + \{N_c^2-1\}$ and denoting by P_1, $P_{N_c^2-1}$ the projection operators onto the colour singlet and (N_c^2-1) - -plet quark-antiquark channels, we may write also

$$k_{\alpha\beta,\gamma\delta} \equiv \sum_{n=1}^{N_c^2-1} \lambda^{(n)}_{\alpha\delta}\, \lambda^{(n)}_{\gamma\beta} = T\left\{ \frac{N_c^2-1}{N_c} P_1 - \frac{1}{N_c} P_{N_c^2-1} \right\}_{\alpha\beta,\gamma\delta}. \quad (10)$$

We see from eq.(10) that there exist attractive forces in the singlet $q\bar{q}$ -channel. Because we are mainly interested in the bound state problem, we concentrate from now on on the singlet channel writing

$$F = -\, q_B q_A^+ \,(P_1 K P_1)_{AB,GD}\, q_D q_G^+ + W(q, q^+). \quad (11)$$

W contains the projection of the interaction part into the non-resonant (N_c^2-1) -channel. We write then $W(q,q^+) = W\left(\frac{1}{i}\frac{\delta}{\delta q^+}, \frac{1}{i}\frac{\delta}{\delta q}\right)$ and take it outside the functional integral*). Let us now return to the task of performing the quark integrations. For this purpose we first linearize the four-quark term in the expression qq^+ by performing a bilocal Gauss integration /8/

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*) It is not difficult to include the term W into the functional integral. The factor exp $i\,W$ in Z will not be written down explititely in the following.

$$\exp\left\{\frac{(2g_1)^2}{2}(qq^+P_1)_{BA}K_{AB,CD}(P_1qq^+)_{DC}\right\}$$
$$= C_1\int D\Sigma\, \exp\left\{\frac{1}{2}\frac{1}{(2g_1)^2}(\Sigma, K^{-1}\Sigma) + i(qq^+P_1, \Sigma)\right\}$$

$$(\Sigma, K^{-1}\Sigma) \equiv \Sigma_{AB}K^{-1}_{BA,CD}\Sigma_{DC} \quad , \text{etc.},$$
$$K^{-1}_{AB,CD}K_{DC,EF} = \delta_{AB,EF} \equiv \delta_{AF}\delta_{EB}\,. \tag{12}$$

The bilocal fields Σ_{AB} appearing in eq. (12) are treated as commuting (Bose) variables. To have source terms coupling directly to the bilocal fields, we include also the term $(qq^+P_1)_{AB}R_{BA}$ into the action integral of eq. (4) and then introduce a new integration variable

$$X = P_1R + \Sigma\,. \tag{13}$$

Performing the manipulations (11)-(13) in eq. (4) the integration over q can be done by using the standard formula

$$\int Dq\, Dq^+\, e^{i(q^+Lq + \eta^+q + q^+\eta)} = [\det iL]\, e^{-i\eta^+L^{-1}\eta} \tag{14}$$

We obtain, finally, $(J=0)$

$$Z(\eta,\eta^+,R) \sim N\int DX\, \exp[iW_{eff}(X)]\,Z(\eta,\eta^+,R|X) \tag{15}$$

where $(X_1 = P_1X)$

$$W_{eff}(X) = -\frac{1}{2(2g_1)^2}(X, K^{-1}X) - i\,\mathrm{tr}\ln[iG^{-1} - X_1]$$

$$Z(\eta,\eta^+,R|X) = \exp\left\{-\eta^+G_{X_1}\eta + \frac{1}{2(2g_1)^2}(P_1R, K^{-1}P_1R) - \frac{1}{(2g_1)^2}(P_1R, K^{-1}X)\right\},$$

$$iG^{-1}_{X_1} = iG^{-1} - X_1 \tag{16}$$

G_X is the quark Green's function in an external bilocal field $X(x,y)$.

III. Quark Spectrum

One of the advantages of the path-integral formulation of field theory is that one can go beyond usual perturbation theory by applying the method of stationary phase to the action integral. To get equations for the quark spectrum we consider the stationary points of the integrand of eq. (15) in the case of vanishing external sources /8/

$$\frac{\delta W_{eff}}{\delta \chi}\Big|_{\chi_1 = M} = 0 . \tag{17}$$

From eq. (17), we obtain

$$M = -i(ig)^2 K D_1 \frac{G_M}{i} . \tag{18}$$

Eq. (18) is just the Schwinger-Dyson equation for the mass operator M and the dressed quark propagator G_M in an approximation where vertex and gluon propagator corrections are absent. In momentum space eq. (18) reads ($M_{\alpha\beta}(p) = M(p)\delta_{\alpha\beta}$)

$$M(p_-) = i(ig)^2 \frac{N_c^2-1}{N_c} \int \frac{d^2k}{(2\pi)^2} \Theta(|k_-|-\lambda) \frac{1}{k_-^2} G_M(p+k) \tag{19}$$

$\Theta(|k_-|-\lambda)$ $(\lambda \to 0)$ is an infrared cut-off. Eq. (19) takes into account only planar diagrams. It was studied by 't Hooft /2/ who showed that the approximation by planar diagrams is a good one in the limit $N_c \to \infty$, $g^2 N_c$ fixed. Eq. (19) admits the solution

$$M(p_-) = \frac{g^2}{\pi} \frac{N_c^2-1}{N_c} \left(\frac{\operatorname{sign} p_-}{\lambda} - \frac{1}{p_-} \right), \tag{20}$$

$$G_M(p) = \frac{i p_-}{2p_+p_- - m^2 + i\varepsilon - p_- M(p_-)} . \tag{21}$$

As we see the poles of the quark propagator tend to infinity if $\lambda \to 0$ ($M(p_-) \to \infty$) leaving us with the expression

$$G_M(p) \sim \lambda \left[\frac{-i}{\frac{g^2}{\pi} \frac{N_c^2-1}{N_c} \operatorname{sign} p_-} \right] \tag{22}$$

This vanishing of the quark pole in $G_M(p)$ is an example of an infrared confinement of quarks*).

IV. Modified perturbation theory and meson spectrum

Let us now formulate a modified perturbation theory that uses the dressed propagator (21) as a lowest order term. For this purpose we shift the integration variable X in eq.(15)

$$X(x,y) = M(x-y) + \phi(x,y) \tag{23}$$

and expand the integrand around M. This yields

$$Z(\eta,\eta^{\dagger},R) \sim N' \int D\phi \, \exp i \left\{ W^{\phi}_{free} + W^{\phi}_{int} \right\} \times Z(\eta,\eta^{\dagger},R \mid M+\phi) , \tag{24}$$

where

$$W^{\phi}_{free} = \frac{1}{2i}(\phi, D_{\phi}^{-1}\phi) \equiv \frac{1}{2i}\left(\phi, \left[(2g\lambda)^2 K\right]^{-1}\left[1-(2g\lambda)^2\left(\frac{G_M}{i}(KP_1)\frac{G_M}{i}\right)P_1\right]\phi\right) \tag{25}$$

$$W^{\phi}_{int} = i \sum_{n=3}^{\infty} \frac{1}{n} \operatorname{tr}\left(\frac{G_M}{i}\phi\right)^n . \tag{26}$$

Here D_{ϕ} denotes the propagator of the bilocal field

$$D_{\phi}^{-1} = \frac{\delta^2 W(X)}{\delta X \, \delta X}\Big|_{X=M} .$$

As follows from eq. (25) it satisfies the inhomogeneous Bethe-Salpeter equation for quark-antiquark scattering in ladder approximation

$$D_{\phi} = (2g\lambda)^2 K + (2g\lambda)^2 \left(\frac{G_M}{i}(KP_1)\frac{G_M}{i}\right) P_1 D_{\phi} . \tag{27}$$

*) In ref. /3,4/ one can find also other realizations of the confinement mechanism using a gluon propagator with a "regular" cut-off.

We use also the notation

$$[\text{illegible}] = \langle \Phi_{\alpha\beta}(x,y)\,\Phi_{\gamma\delta}(x',y')\rangle_{\Phi} \equiv \overline{\Phi_{\alpha\beta}(x,y)\,\Phi_{\gamma\delta}(x',y')} \qquad (28)$$

where [illegible] $\rangle_{\Phi}$ abbreviates the path-integral average with the weight factor exp [illegible] W^{Φ}_{free} [illegible]. The bilocal integral in eq.(24) may now be evaluated perturbatively by expanding exp [illegible] W^{Φ}_{int} [illegible] and $Z(n,q,R|M,\Phi)$ in powers of Φ and applying a Wick theorem; for example

$$\langle \Phi_1\Phi_2\Phi_3\Phi_4\rangle_{\Phi} = \overline{\Phi_1\Phi_2}\,\overline{\Phi_3\Phi_4} + \overline{\Phi_1\Phi_3}\,\overline{\Phi_2\Phi_4} + \overline{\Phi_1\Phi_4}\,\overline{\Phi_2\Phi_3}$$

$$[\text{illegible}] \qquad (29)$$

The "free field" equation for the bilocal field follows from the variational principle

$$[\text{illegible}] \Rightarrow D_{\Phi}^{-1}\,\Gamma = 0 \qquad (30)$$

or, explicitely, taking the Fourier transform

$$[\text{illegible}] \int \frac{d^2k}{(2\pi)^2}\,\frac{1}{k_-^2}\,G_M(p+k)\,\Gamma(p+k,r)\,G_M(p+k-r) \qquad (31)$$

Eq.(31) is recognized as the homogeneous BS-equation for the vertex function of quark-antiquark bound states in the colour-singlet state in ladder approximation. The arguments p, r denote a quark momentum and the total momentum of the quark-antiquark pair as shown in fig. 1. The eqs. (19) and (31) were the starting point of 't Hoofts investigations of TDQCD. To be self-contained, we recapitulate shortly the properties of the solutions of eq. (31). Introducing the integrated wave function

$$h(p_-,r) = \int dp_+\,G_M(p)\,\Gamma(p,r)\,G_M(p-r) \qquad (32)$$

into eq. (31) yields the new equation $(x = p_-/r_-)$

$$\mu_k^2\,h_k(x) = H\,h_k(x) \equiv \left(\frac{\alpha_1}{x} + \frac{\alpha_2}{1-x}\right)h_k(x) - P\int_0^1 \frac{h_k(y)}{(y-x)^2}\,dy\,, \qquad (33)$$

where

$$\alpha_{1,2} = m_{1,2}^2 \Big/ \frac{g^2}{\pi}\frac{N_c^2-1}{N_c} - 1$$

$$M_K^2 = (2r_+r_-)_K = \frac{g^2}{\pi}\frac{N_c^2-1}{N_c}\,\mu_K^2 \qquad (34)$$

and $\mathcal{P}$ is the principal value symbol. One obtains the following results /2/

(i) H is positive-definite and self-adjoint on the space of functions which vanish at $x=0$ [$x=1$] like [illegible] where $\pi\beta_a \cot \pi\beta_a = -\alpha_a$. H has only a discrete spectrum. The eigenfunctions are complete and orthogonal

$$\sum_k h_k(x)\,h_k(x') = \delta(x-x')$$

$$\int_0^1 h_m(x)\,h_n(x)\,dx = \delta_{mn} \qquad (35)$$

(ii) For large k the eigenfunctions and eigenvalues may be approximated by

$$h_k(x) \simeq \sqrt{2}\,\sin\pi k x \qquad (k \gg 1 \text{ [illegible]}$$

$$\mu_k^2 \simeq \pi^2 k \qquad (36)$$

For completeness, we quote also the normalization and orthogonality relations of the vertex functions Γ_k. They may be derived in a standard way from the BS-equations (27), (31). We get*)

$$i\,\mathrm{tr}\int\frac{d^2q}{(2\pi)^2}\,\bar\Gamma_k(q,r)\left[\frac{1}{2r_-}\frac{\partial}{\partial r_+}\frac{G_M}{i}\left(q+\frac{r}{2}\right)\gamma_+\frac{G_M}{i}\left(q-\frac{r}{2}\right)\gamma_+\right]_{r^2=M_k^2}\Gamma_k(q,r) = 1 \qquad (37)$$

$$i\,\mathrm{tr}\int\frac{d^2q}{(2\pi)^2}\,\bar\Gamma_\ell(q,r)\Big[\frac{G_M}{i}\left(q+\frac{r_\ell}{2}\right)\gamma_+\frac{G_M}{i}\left(q-\frac{r_\ell}{2}\right)\gamma_+$$

$$-\frac{G_M}{i}\left(q+\frac{r_k}{2}\right)\gamma_+\frac{G_M}{i}\left(q-\frac{r_k}{2}\right)\gamma_+\Big]\Gamma_k(q,r) = 0 \qquad (38)$$

*) In writing eqs. (37,38) we used the convention that the argument q of Γ is now the relative momentum of the quark-antiquark pair. For real $h_K(x)$ we have also $\overline{\Gamma_k(q,r)/i} = \Gamma_k(q,r)/i$.

Using the explicit expression of the normalized BS-vertex functions

$$\frac{1}{r}\,\Gamma_k(x) = -2i\left(\frac{N_c^2-1}{N_c^2}\right)^{1/2}\left(g^2\,\frac{N_c^2-1}{N_c\,\pi}\right)^{1/2}\frac{2g}{\lambda}\,\times$$

$$\times\left[\theta\big(x(1-x)\big) + \frac{\lambda}{2|r|}\left(\frac{\alpha_1}{x}+\frac{\alpha_2}{1-x}-\mu_k^2\right)\right]\varphi_k(x) \tag{39}$$

one may rewrite the bilocal propagator $\mathcal{D}_\varphi$ in the final form (cf.fig.2)

$$\mathcal{D}_\varphi(x,x';r) = (2g)^2\,\varepsilon_{\alpha\beta,\gamma\delta}\,\frac{\lambda}{r_-^2(x'-x)^2} - \sum_k \frac{1}{r^2 M_k^2}\left[\frac{1}{r_-}\Gamma_k(x)\right]\left[\frac{1}{r_-}\overline{\Gamma}_k(x')\right] \tag{40}$$

As is shown by eq.(40) the bilocal propagator describes the propagation of the quark-antiquark bound states of the colour--singlet sector of *TDQCD. It coincides with the* $q\bar{q}$- scattering amplitude of ref. /3/. In bound-state scattering amplitudes the term $\frac{1}{\lambda}\,\theta(x(1-x))$ in the vertex (39) just compensates the λ factor of the quark propagator (22) yielding finite results in the limit $\lambda \to 0$.

V. Infinite-component field theory for bound state mesons

It has been proposed in ref. /9/ that a field theory in bilocal variables may be interpreted as an infinite-component field theory for the bound states of the theory. Let us apply this idea to TDQCD. For this purpose, we expand the bilocal fields in terms of the complete set of vertex functions (39). The latter may be considered as the "plane-wave solutions" of the "free field" equations $\mathcal{D}_\varphi^{-1}\,\Gamma = 0$. We write $(\phi_1 = P_1\varphi)$

$$\phi_1(x,y) = \sum_k \int\frac{d^2r}{(2\pi)^2}\int\frac{d^2q}{(2\pi)^2}\,e^{-i\left[r\left(\frac{x+y}{2}\right)+q(x-y)\right]}\,\Gamma_k(q,r)\,\varphi_k(r) \tag{41}$$

where the $q\bar{q}$ -mesons are described by hermitian fields $\varphi_k(X)$ $\left(X = \frac{x+y}{2}\right)$. Inserting eq. (41) in eq. (25) and using eqs. (37,38), one gets the diagonalized contribution

$$W^{\Phi}_{free} = \frac{1}{2}\sum_k \int \frac{d^2r}{(2\pi)^2}\, \varphi_k(-r)\left[r^2 - M_k^2\right]\varphi_k(r). \tag{42}$$

In writing eq. (42), higher order (off mass-shell) terms $O\left((r^2-M_k^2)^2\right)$ arising by expanding the square-bracket in eq. (25) as well as non-resonant contributions have been omitted. Eq. (42) may indeed be interpreted as the free effective action of an infinite-component field theory with meson fields. The mass spectrum appearing in the Klein-Gordon operator $(-\Box_X - M_k^2)$ is here given by eq. (34). The self-interactions of the meson fields are described by the non-polynomial interaction part (cf.fig.3)

$$\begin{aligned} W^{\Phi}_{int} &= i \sum_{n=3}^{\infty} \frac{1}{n}\, tr\left(\frac{G_M}{i}\Phi_1\right)^n \\ &= \sum_{n=3}^{\infty} \sum_{k_1 \ldots k_n} \frac{(i)^{1-n}}{n} \int \prod_{i=1}^{n} \frac{d^2 r_i}{(2\pi)^2}\,(2\pi)^2\,\delta^{(2)}\Big(\sum_1^n r_i\Big)\, V_{k_n \ldots k_1}(r_n \ldots r_1) \\ &\qquad \times \prod_{i=1}^{n} \varphi_{k_i}(r_i)\,, \end{aligned} \tag{43}$$

$$\begin{aligned} V_{k_n \ldots k_1}(r_n \ldots r_1) &= \int \frac{d^2 q}{(2\pi)^2}\, tr\, \Gamma_{k_n}(\xi_n, r_n)\, G_M\Big(\xi_{n-1} + \frac{r_{n-1}}{2}\Big)\, \Gamma_{k_{n-1}}(\xi_{n-1}, r_{n-1}) \ldots \\ &\quad \times \; G_M\Big(\xi_1 + \frac{r_1}{2}\Big)\, \Gamma_{k_1}(\xi_1, r_1)\, G_M(q)\,, \end{aligned}$$

$$\xi_k = q + \sum_{i=1}^{k-1} r_i + \frac{r_k}{2}\,. \tag{44}$$

Starting from a local field theory of quarks and gluons we arrived thus at an infinite-component non-polynomial field theory for the colour-singlet bound states of TDQCD.

VI. Currents

As we have seen a characteristic feature of TDQCD is the existence of a large series of resonances dominating the scattering amplitude. The scaling behaviour at small distances should be explained in contrast to this by the parton-like behaviour of quarks. As the quarks do not contribute in this model to the spectrum, the question arises how the composite mesons can simulate in the scaling region the effects of the asymptotic free quark interaction.

Let us investigate as an example the two-point function of the (colour-singlet) electromagnetic current

$$j_\mu(x) = \sum_{\alpha=1}^{N_c} \bar{q}_{a\alpha}(x)\,\gamma_\mu\, Q^{ab}\, q_{b\alpha}(x) \quad , \tag{45}$$

where Q is the charge matrix. We have

$$\begin{aligned} T_{\mu\nu}(r) &= \int d^2x\, e^{irx} \langle j_\mu(x)\, j_\nu(0)\rangle \\ &= \int d^2x\, e^{irx}\, \mathrm{tr}\, \gamma_\mu Q\, G(x,x;0,0)\,\gamma_\nu Q \end{aligned} \tag{46}$$

The matrix element T_{--} may easily be computed from the two-particle quark-antiquark Green's function of the $\hat{q}_1$ field (the + components of $j_\mu(x)$ couple to $\hat{q}_1 = \binom{q_1}{c}$ and have to be treated slightly different). We obtain

$$\begin{aligned} G(xy,x'y') &= \frac{1}{2}\, \frac{\delta^2 Z}{\delta R(x,y)\,\delta R(x'y')}\Big|_{R=\eta=\eta'=0} \\ G(xy,x'y') &\approx \underset{1}{G_H}(x'y)\,\underset{1}{G_H}(xy) - \underset{1}{G_H}(x'y)\,P_1\,\underset{1}{G_H}(xy')\,P_1 \\ &\quad - \int d^2\bar{x}\, d^2\bar{y}\, d^2u\, d^2v\, \underset{1}{G_H}(\bar{x}y)\,P_1 \left[\underset{1}{G_H}(x\bar{y})\right] P_1\, D_\phi(\bar{y}\bar{x},uv)\,\underset{1}{G_H}(vy')\,P_1 \\ &\quad \times \underset{1}{G_H}(u,x')\,P_1 \,, \end{aligned} \tag{47}$$

where W^{ψ}_{int} has been neglected in a first order approximation. If we now remove the infrared cut-off, i.e. $\lambda \to 0$, the first two terms of eq. (47) do not contribute, whereas the third term yields

$$T_{--}(r) \sim i \frac{N_c}{\pi} \sum_k \frac{1}{r^2 - M_k^2} \langle Q_k \rangle^2 + O(\lambda) . \tag{48}$$

Here

$$\langle Q_k \rangle = r_- \, tr\, Q \int_0^1 dx \, h_k(x) \tag{49}$$

is the averaged electric charge of the quark-antiquark pair contained in the bound state meson. In obtaining eq. (48) we have used eqs. (39,40). The first order expression (48) coincides with the leading contribution in an $1/N_c$ expansion investigated in ref. /3/. Following the argumentation of these authors, the high energy limit of $T_{--}(r)$ is given by

$$T_{--}(r) \underset{r^2 \to -\infty}{\sim} \frac{N_c}{\pi} \frac{r_-^2}{r^2} \sum_k \left[tr\, Q \int_0^1 h_k(x) dx \right]^2$$

$$\sim \frac{N_c}{\pi} \frac{r_-^2}{r^2} tr\, Q^2 = \frac{N_c}{\pi} \frac{r_-^2}{r^2} \sum_{i=1}^{N_f} Q_{ii}^2 . \tag{50}$$

To obtain the second row of eq. (50) the completeness relation of the functions $h_k(x)$ has been used. Taking into account that $T_{--}^{free} = \frac{N_c}{\pi} \frac{r_-^2}{r^2} tr\, Q^2$ is just the contribution of the free massless quark loop, eq. (50) expresses the asymptotic freedom result

$$T_{--} \underset{r^2 \to -\infty}{\sim} T_{--}(g = 0, m = 0) \tag{51}$$

In this way, TDQCD gives some hint on the meaning of the concepts of "current" and "constituent" quarks /4/. In an analogous way one may get path-integral expressions for the quark and meson form factors as well as for current elements of deep inelastic lepton hadron scattering. These quantities were discussed for leading order in $1/N_c$ in ref. /3-6/.

VII. Concluding remarks

We have shown for the case of TDQCD that bilocal path-integral techniques are a useful tool for deriving a new effective Lagrangian for the bound-state mesons starting from a Lagrangian with elementary fields (quarks, gluons). In this approach the equations determining the quark and boson spectrum follow immediately by the variation of the effective bilocal action. Bound state scattering amplitudes (Green's functions) may then be calculated by varying the effective action with respect to bilocal source terms. All the calculations have been performed in the light-cone gauge $A_- = 0$. There are also considerations of more general gauges /12/ where one found some inconsistencies which are not yet solved.
It would be very desirable, of course, to carry out a similar program for four-dimensional QCD in order to obtain an effective Lagrangian including only (bilocal or trilocal (for $SU(3)$)) fields of "observable" colour-singlet mesons and baryons.

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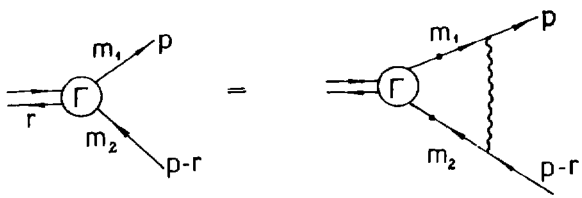

Fig.1: The quark-antiquark BS-equation for the bound state mesons.

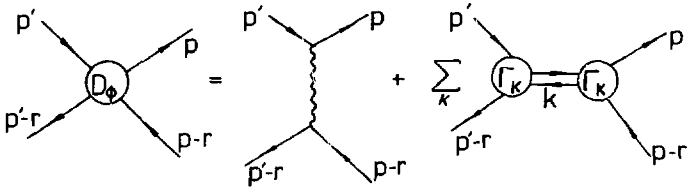

Fig.2: Decomposition of the bilocal propagator $\mathcal{D}_\phi$.

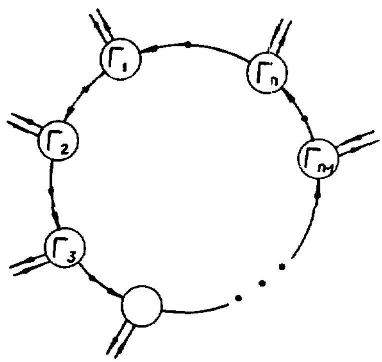

Fig.3: Graphical representation of the vertex part $V_{k_n \dots k_1}$ (comp.eq.44).

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