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ON THE POSSIBLE EXISTENCE OF FLUCTUONS IN NUCLEI

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Буров В.В. и др. **E2 - 10680**

О возможном существовании флуктонов в ядрах

Проведен анализ выходов высокоэнергетических частиц в адрон-ядерных столкновениях на основе флуктуационного механизма. Сечение образования кумулятивных частиц получено в рамках партонной модели. Приводится расчет и сравнение с экспериментом $p+A \to \pi^- + X$ и $p+A \to p+X$ реакций.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Burov V.V. et al. **E2 - 10680**

On the Possible Existence of Fluctuons in Nuclei

The theoretical analysis of the inclusive nuclear reactions with the production of pions at large momenta is given. It is shown that the assumption on the cooperation of nucleons with joining their partons into "fluctuons" allows one to understand the main regularities under consideration. The data can analogously be interpreted on the production of protons and other particles in the proton-nucleus collisions.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

Recently the problem is widely discussed concerning the nature of the high energy pion production in the inclusive proton nucleus reactions[1], the pions of energy 2-5 times larger (the order of the cumulative effect) than the kinematically allowed limit in an elementary $pp\pi$-reaction. The same problem exists for other high momentum products from inclusive nuclear reactions at high energies[2]. Let us consider first the reactions of cumulative pion production.

2. FERMI MOTION AND RELATIVIZATION OF NUCLEAR WAVE FUNCTIONS

Since the nucleons in nuclei are not at rest and a number of them is moving towards an incident particle with "intranuclear" velocity, this naturally results in increasing kinematical limit in an elementary process. This is just the Fermi motion effect. It can be calculated using the impulse approximation diagram of Fig. 1, where the lower vertex $\omega_1(p_f)$ is the impulse distribution of a nucleon in the nucleus and the upper vertex $E_\pi \frac{d^3\sigma}{dp_\pi^3}(pp_f \to \pi + ...)$ is the invariant cross

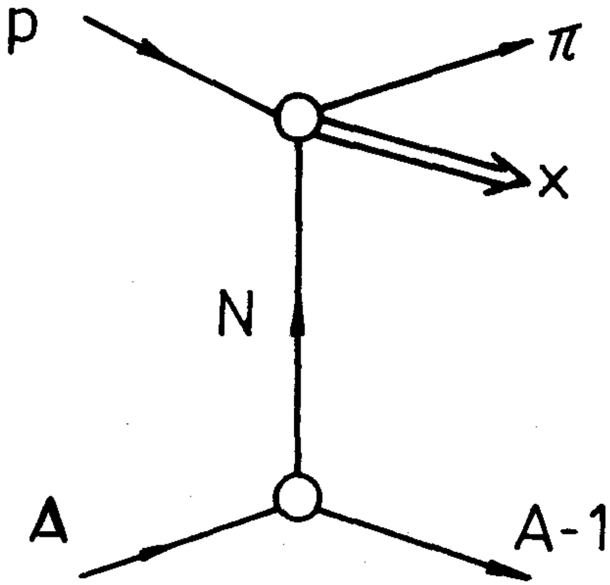

Fig. 1. The diagram of the impulse approximation for the interaction of the incident proton with nuclear nucleon in the pion production process.

section of the elementary reaction. Thus the invariant cross section of pion production on nucleus is [3]

$$E_\pi \frac{d^3\sigma}{dp_\pi^3}(pA \to \pi + \ldots) = \int \Re E_\pi \frac{d^3\sigma}{dp_\pi^3}(pp_f \to \pi + \ldots)\omega_1^-(p_f)\, d^3 p_f/(2\pi)^3, \quad (1)$$

where the factor $\Re$ contains off-shell effects, and redefinition of the flux of incident particles in terms of the kinematic variables of a virtual nucleon. The factor $\Re$ implies that the nucleon in the nucleus is ready to behave like an elementary particle in the pion production.

Now the main problem is to choose a wave function of the nucleus $\psi(r)$ for the calculation

$$\omega_1(\vec{p}_f) = \sum_n | \int \psi_n(\vec{r}) \exp[i\vec{p}_f\vec{r}]\, d\vec{r}\,|^2, \qquad (2)$$

where n are the quantum numbers of states engaged in the reaction. The calculations /4/ for ^{12}C given in Fig. 2a show that the kinematical limit for the elementary process can be exceeded approximately by a factor of two. However, the discrepancy between theory and experiment is still of 4-5 orders of magnitude. Then, it is obvious that the nuclear wave functions should be made of the relativistic ones since we consider the large momenta in nuclei. Here we use the "relativization" procedure /5/ where the plane wave in the usual Fourier transform (2) changes to its relativistic generalization

$$\xi(\vec{p},\vec{r}) = \left(\frac{\sqrt{\vec{p}^{\,2} + m^2} - \vec{p}\vec{n}}{m} \right)^{-1-irm}; \quad (\vec{r} = r\vec{n};\ n^2 = 1). \qquad (3)$$

Expanding it into the series over spherical harmonics, we have, for instance, for the s-wave function

$$\omega_1^s = |4\pi \int \frac{\sin(\chi r)}{pr} \psi_s(r) r^2\, dr\,|^2, \qquad (4)$$

where

$$\mathrm{sh}\,\chi = p/m. \qquad (5)$$

There are analogous relations for other partial waves.

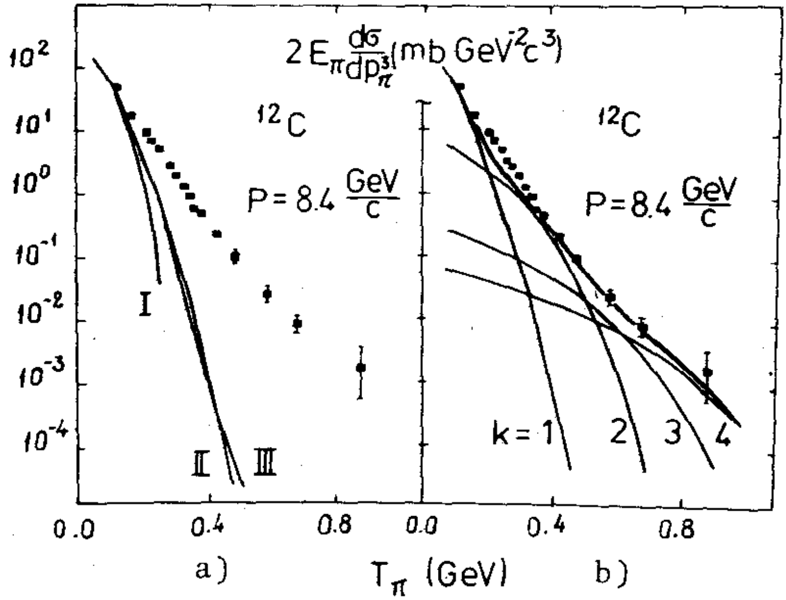

Fig. 2. a) The calculation of the pion production invariant cross section on ^{12}C: I - the production on nucleons at rest, II - taking into account the Fermi motion, III - taking into account the relativistic effects; b) the contributions to the cross sections from separate fluctuons with mass $M_k = km$, where k is the order of cumulativity.

The calculations of the same reaction give the results which are denoted by curve III in Fig.2a. It is seen that the relativisation procedure, at least at the given momenta, gives a small effect and do not change the above conclusions that Fermi motion cannot explain the cumulative effect.

Therefore, one may conclude that these processes are indeed of a cumulative character, i.e., an incident particle collides with a whole group of k nuclear nucleons [6]. Then the scale invariance allows one to introduce their mass $M_k = km$ into the cross section of the elementary process expressed in terms of the Feynman scale variables $x_k = p_{||}^{\pi}/p_{||\,max}^{\pi}(M_k)$ and of the total energy s_k in c.m.s. of interacting subsystems. It is important to know what reasons make these k nucleons cooperate in a uniquel object and behave with some probability β_k like an elementary particle of mass M_k.

Below we shall consider this phenomenon using the fluctuon mechanism, following the papers [7,8].

3. FLUCTUONS IN NUCLEI?

First the idea of fluctuons [9] in nuclei was used in order to interpret the nature of the "deuteron peak" in the pA-scattering cross-section at large momentum transfers [10] and also to understand the pd-scattering cross section behaviour [11].

There were suggested the strongly compressional fluctuations of nuclear matter so that k nucleons find themselves in the small volume $V_\xi = \frac{4}{3}\pi r_\xi^3$ with r_ξ the fluctuon radius. Further, constructing the wave function of the k-fluctuon center of mass motion as an overlap integral in this volume of k single particle wave functions, one can show [7,8] that it is

$$\psi_k(R) \cong V_\xi^{\frac{k-1}{2}} \, n^{\frac{k}{2}}(R), \qquad (6)$$

where $n(R)$ is the nuclear density distribution. Then taking into account the normalization of the cross section and using the Poisson formula in the theory of fluctuations, the probability to find in a nucleus a fluctuon consisting of k nucleons and moving with the momenta $\vec{p}$ is as follows:

$$W_k(p) = \beta_k \omega_k(p) = \binom{A}{k} \alpha_k A^{1-k} \omega_k(p), \qquad (7)$$

where the momentum distribution probability

$$\omega_k(p) = (V_0 A)^{k-1} \left| \int n^{k/2}(R) e^{i\vec{p}\vec{R}} d\vec{R} \right|^2 \qquad (8)$$

obeys the normalization

$$\int \omega_k(p) d\vec{p} / (2\pi)^3 = 1. \qquad (9)$$

Here $\alpha_k = (V_\xi / V_0)^{k-1} = (\frac{r_\xi}{r_0})^{3(k-1)}$; V_0 and r_0- are the volume and radius of a nucleon, and β_k is the probability for the centers of k nucleons to occur in the volume V_ξ of the radius r_ξ. The pion production cross section in the impulse approximation is

$$E_\pi \frac{d^3\sigma}{dp_\pi^3}(pA \to \pi + \dots) = \sum_k \beta_k \int \Re E_\pi \frac{d^3\sigma}{dp_\pi^3}(pF(p_f) \to \pi + \dots) \times \qquad (10)$$

$$\times \omega_k(p_f) d\vec{p}_f / (2\pi)^3 .$$

As the underintegral $p + \text{fluctuon} \to \pi + \dots$ invariant cross section, we take that for the $p + p \to \pi + \dots$ process parametrized in the Feynman scaling variable $x_1 = p_{||}^\pi / p_{||\,max}^\pi$ in ref. /3/. Note, that for fluctuons with $k>1$ this form does not obey the quark counting rule for $x_k \lesssim 1$ /12/. However, a correct power behaviour of invariant cross sections should occur only in the region $x_k \simeq 1$ where, due to a strongly decreasing cross section $E_\pi \frac{d^3\sigma}{dp_\pi^3}(k)$ at $x_k \to 1$, important is the next $(k+1)$fluctuon. The latter is confirmed by the calculations given in Fig. 2b where separate yields of each k-fluctuon in ^{12}C and their sum are shown.

Figure 3 demonstrates the calculation and comparison with experiment of the yield of π-meson for nuclei ^{27}Al, ^{64}Cu and ^{208}Pb. For these nuclei the fluctuons with the number of nucleons $k=2,3,4$ were introduced. The size of fluctuons for all the nuclei occured to be $r_\xi = 0.5 \div 0.7$ fm, i.e., of an order of the NN-force repulsive core radius. It should be noted that the probability of production of fluctuons with $k=2$ in nuclei $\beta_2 = \frac{1}{2}\frac{V_\xi}{V_0} \simeq 6\%$ is consistent with an 8% six quark component in the deuteron obtained in ref. /13/ by the analysis of the data on $pd\pi$ inclusive reactions and elastic ed-scattering at large momentum transfer.

And finally, eq. (10) says that the yield of cumulative pions depends on the atomic weight of nucleus-target as $\binom{A}{k}A^{1-k} \approx A$ (if $A \gg k$). However, at comparatively low energies T_π, in (10) the absorption of pions by a nucleus results in the appearance of an additional factor proportional to $A^{-1/3}$ /9/. At high T_π this factor is equal to unity. Then in the A^n -dependence of cross sections the power of the exponent n changes from 2/3 at low energies to $n \simeq 1$ at high energies what is consistent with the experiment /1/.

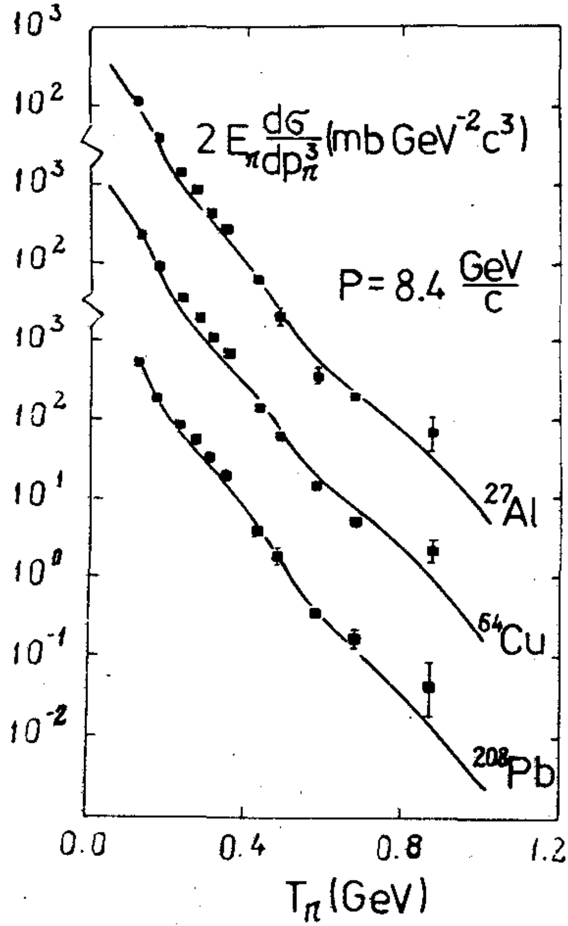

Fig. 3. The comparison of theoretical cross sections with experiment.

4. MICROSCOPIC APPROACH IN THE DESCRIPTION OF PION PRODUCTION ON NUCLEI

Up to now we worked out the phenomenological version of the nucleon-fluctuon interaction mechanism. However, to understand its nature one should pass over to the microscopic approach aimed at investigating the reactions within quark-parton models of elementary particles. The main idea of these models is that at asymptotically large energies $E \gg M$ the nucleon is a set of noninteracting partons, e.g.,/14/, quarks q, antiquarks $\bar{q}$, mesons μ, and others. The probability of finding, in this system, a parton with a given momentum is usually determined following the simplest statistical considerations, by integrating over the phase volume of the remaining partons and taking into account the conservation of the total nucleon momentum.

Now the pion production process in NN-collisions can be represented as a set of diagrams of Fig. 4a, where the main role is attributed to subprocesses

$$1)\ q\pi \to q\pi \qquad 2)\ \pi\pi \to \pi\pi \qquad 3)\ q\bar{q} \to \pi\bar{\pi}. \qquad (11)$$

One can obtain the probabilities to find these partons in nucleons with a part $x = p_i/p$ of the total nucleon momentum as follows:

$$G_{i/N}(x) \sim x^{\gamma_i^{(1)}} (1-x)^{\gamma_i^{(2)}}. \qquad (12)$$

The cross sections of the subprocesses (11) can be taken as

$$\frac{d\sigma_{ij}}{dt} = \frac{\sigma_{ij}}{s^4}. \qquad (13)$$

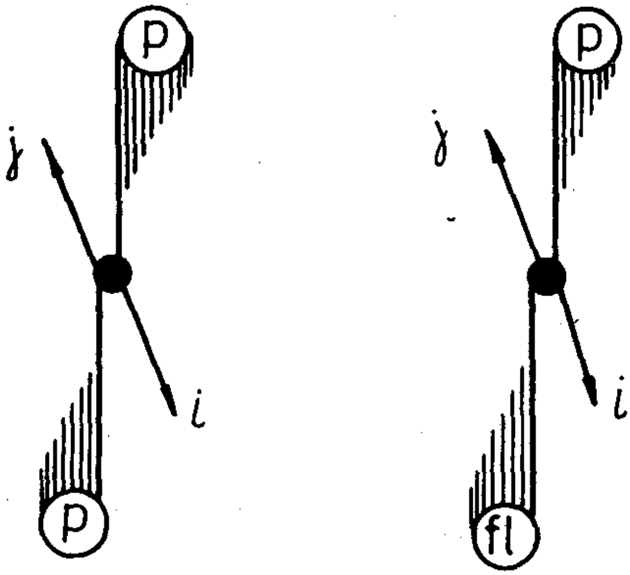

Fig. 4. a) The diagrams of elementary subprocesses resulting in the pion production in the nucleon-nucleon collisions. b) The same for the subprocesses with the nucleon-fluctuon interactions.

Here γ_i and σ_{ij} are the constants depending on the concrete assumptions of the parton model.

In the case of pion production from the proton-fluctuon collisions we use the same elementary subprocesses (11). However, now one of the partons here should belong to the fluctuons as a whole (see Fig. 4b). By analogy with the aforesaid, we can calculate the probability of finding a parton belonging to a set of k-nucleons. Then instead of (12) we will have [15]

$$G_{j/k}(x_k) \sim x_k^{\gamma_j^{(1)}} (1-x_k)^{\gamma_j^{(2)} + 6(k-1)} , \tag{14}$$

where

$$x_k = p_j / p_{fl} .$$

Now to write down the invariant cross section of the pion production we neglect Fermi-motion of nuclear nucleons. Then it is represented as integrals of sum of products of probabilities to find fluctuons from k-nucleons in a nucleus β_k to separate a parton by them (14), to separate a parton from an incident nucleon (12) and the cross sections of the parton-interactions (11). Thus we have

$$E_\pi \frac{d^3\sigma}{dp_\pi^3}(pA \to \pi + \dots) = \frac{1}{\pi} \sum_{ijk} \beta_k \{ \int dx\, dy\, s' \delta(s'+t'+u') \times \tag{15}$$

$$\times G_{i/N}(x) G_{j/k}(y) \frac{d\sigma_{ii}}{dt}(k, s', t', u') \dots \} .$$

The production cross section on a free nucleon is obtained by neglecting all the terms except for $k=1$ if $\beta_1 = 1$. The variables in (15) are

$$s' = kxys \qquad t' = yt \qquad u' = kxu , \tag{16}$$

where y is a fraction of the parton momentum in a fluctuon and x in a nucleon.

We stress here that expressions (12) and (14) are valid only in the asymptotic region and justify the laws of behaviour at $x \simeq 1$

which follow from the quark counting rule [12]. However, in practice the asymptotic region of s and t is not achieved in the processes studied, and as it was mentioned in §3, the main role in the formation of the spectrum of secondaries is attributed to the region $x < 1$. Therefore, it is natural to parametrize (12) and (14) so as to explain the available experimental data at least for the elementary $pp \to \pi + \dots$ cross section. As a first step let us introduce the parametrization of the following type

$$G \sim x^{\gamma^{(1)}} (1-x)^{\gamma^{(2)}\delta} \quad , \tag{17}$$

where δ is the parameter determined by the fit to experiment for the $pp \to \pi + \dots$ process [16], which gives us $\delta = 0.2$.

Now introducing a modified expression (17) into (19) we calculate the pion production cross-section on the nucleus ^{12}C. The results are given in Fig. 5. Note that the case $\delta = 1$ corresponds to the approach developed in ref. [17] where it was also used the combinatorial expressions for β_k and the asymptotic formula (14). Besides, the summation over k in (15) was replaced by the integral calculated by the saddle point method. It is seen from Fig. 5 that the asymptotic expressions ($\delta = 1$) for G cannot be used in the given energy region and a qualitative agreement with experiment at $\delta = 0.2$ taken from the analysis of the elementary $pp \to \pi + \dots$ cross section, confirms the validity of the basic assumption on the aforementioned mechanism. Analogous results can be obtained for ^{27}Al , ^{64}Cu and ^{208}Pb. It is interesting to note that the obtained values for β_k result in the same radius of

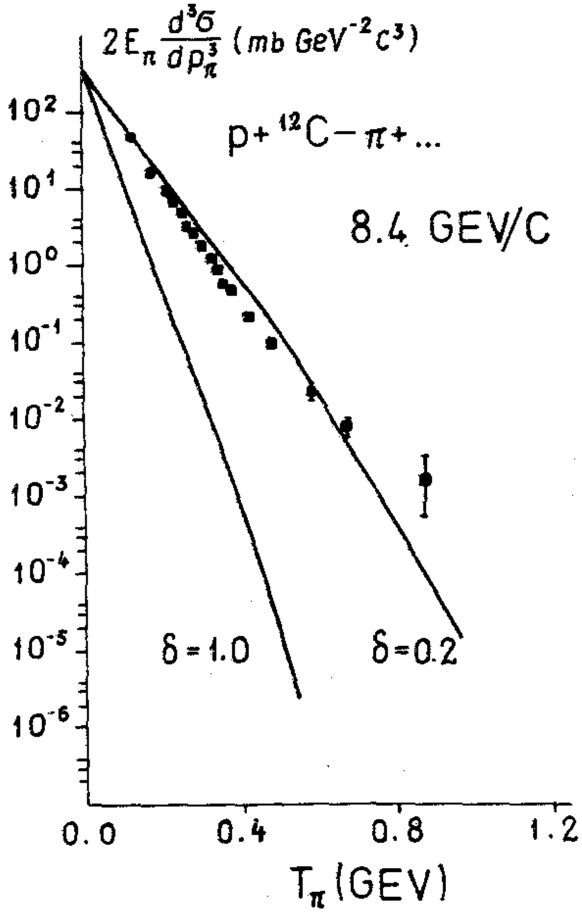

Fig. 5.The comparison with experiment of the pion production cross section on ^{12}C at $\delta = 0.2$ and 1.

fluctuons $r_f = 0.75$ fm (everywhere k was taken to be equal to 1,2,3,4).

5. PRODUCTION OF CUMULATIVE PROTONS ON NUCLEI

It was already mentioned that the production of cumulative pions [1] and protons [2] on nuclei has similar features, in particular, this concerns the exponential decay of invariant cross sections with increasing transferred momentum. Therefore, one can use the same mechanism as in §4 for the analysis of the reaction $pA \to p + \ldots$.

The main diagrams in this case will be those (Fig. 6) where k is a fluctuon separated in the nucleus and i is a parton corresponding to the elementary subprocess, which results in the nucleon production. Now we write down a cross section in the form analogous to (15)

$$E_p \frac{d^3\sigma}{dp_p^3}(pA \to p + \ldots) = \sum_{ik} \beta_k \int dx \{ G_{i/k}(x) \times \times E_p \frac{d^3\sigma}{dp_p^3}(p + i \to p + \ldots) \}. \qquad (18)$$

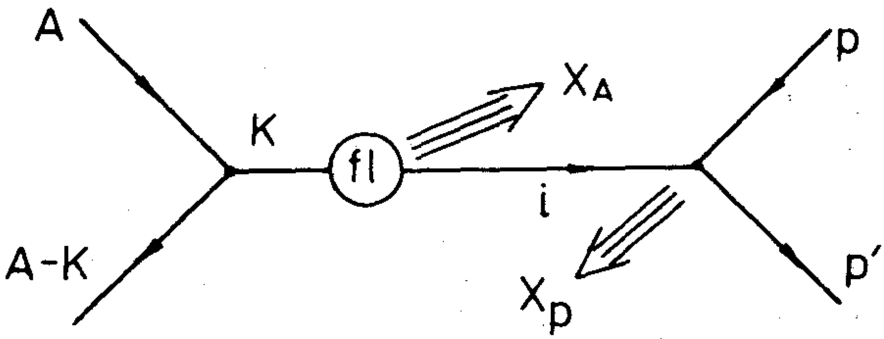

Fig. 6. The main diagrams of processes resulting in the proton production in the proton-nucleus collisions.

For qualitative estimates some simplifications can be made due to the fact that the elementary cross section in (18) restricts strongly the lower limit of the integration $x_{min} \sim \frac{t}{ks} \cong x_k = \frac{x_1}{k}$ which gives the main contribution to the integral. This results in the following simplified expression

$$E_p \frac{d^3\sigma}{dp_p^3}(pA \to p + \ldots) \cong \sum_k \tilde{\beta}_k \left(1 - \frac{x_1}{k}\right)^{6(k-1)\delta} E_p \frac{d^3\sigma}{dp_p^3}(pp \to p + \ldots, x_k), \quad (19)$$

where

$$\tilde{\beta}_k = \frac{\kappa}{6(k-1) + \kappa} \beta_k \quad (20)$$

with the effective parameter κ of an order of unity. On the tails of distributions for $k \gg 1$ we can use the approximations

$$(1 - x_1/k)^{6(k-1)\delta} \simeq e^{-6x_1\delta} \quad (21a)$$

$$k! \simeq e^{k \ln k - k} \quad (21b)$$

$$E_p \frac{d^3\sigma}{dp_p^3}(pp \to p + \ldots, x_k) \simeq f(p_\perp)\,\theta(1 - x_1/k) \quad (21c)$$

$$\sum_k \sim \int dk. \quad (21d)$$

Then (19) is simplified and is of the following analytical form:

$$E_p \frac{d^3\sigma}{dp_p^3}(pA \to p + \ldots) = \text{const}\, \frac{\kappa A \sqrt{x_1}}{6(x_1 - 1) + \kappa} \exp\{-x_1(6\delta + \ln|x_1| + \ln|\beta_0| - 1)\} \simeq C\exp(-T_p/T_0), \quad (22)$$

where

$$x_1 = \frac{p_p^L (E_1 + m)\cos\theta - p_1 E_p^L}{m p_1} \qquad (23)$$

$$T_0 = \frac{m}{(1-\cos\theta)(6\delta + \ln|\frac{t}{s}| + \ln|\beta_0| - 1)}; \ \beta_0 = \frac{V_\xi}{V_0}. \qquad (24)$$

It is seen that the exponential decay of the cross section with increasing T_p is independent of the entrance channel energy and of masses of colliding particles ($T_0 \neq T_0(E, A_1, A_2)$). At the same time T_0 depends on the angle of detection of secondary protons. This is confirmed by the regularities observed in experiments and explains to some extent the name of this phenomenon, nuclear scaling /2/. However, it should be noted that the above formulae (21)-(24) and the expressions for the cross section in the form (22) are valid only in the asymptotic region, in particular at large orders of cumulativity of k and at the energies of outgoing protons $E_p \gg M_k$. In the region $E_p \lesssim m_p$ with available experimental data, more adequate is the approximation (19) where one can use the experimental cross section of the elementary $pp \to p + \ldots$ process, parametrized as a function of the Feynman variable x. Taking it again in the form (21c), one can perform the calculation by (19). The results thus obtained (the cross section in relative units) and their comparison with experiment /2/ are shown in Fig. 7. It appears that the parameter $\delta = 0.5$. The fact that its value is different from $\delta = 0.2$ used for the interpretation of the reactions $pp \to \pi + \ldots$ and $pA \to \pi + \ldots$ indicates the necessity to study in detail the elementary $pp \to p + \ldots$ process the cross section of which is taken here in a highly simplified form (21c)

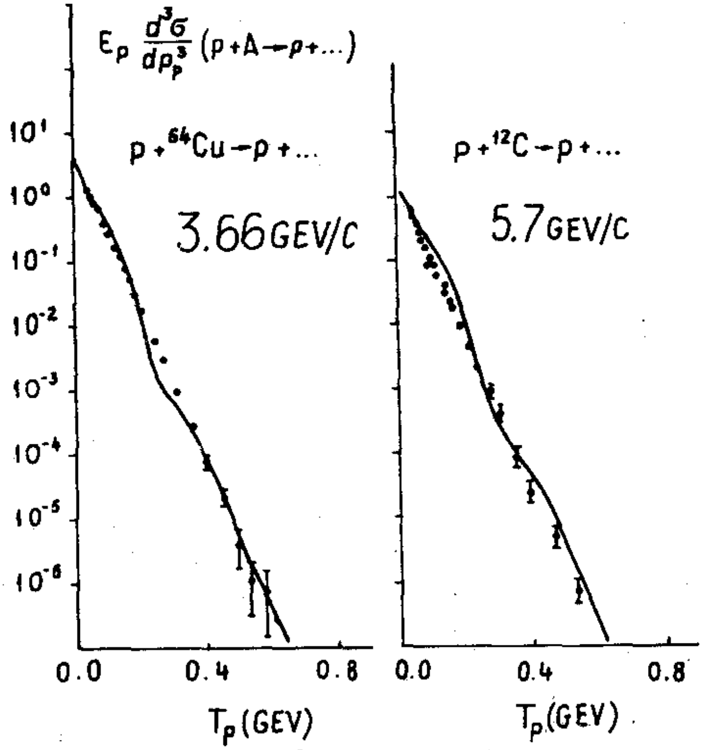

Fig. 7. The comparison of theoretical cross sections in the reactions of cumulative production of protons on nuclei with experiment.

6. CONCLUSION

1. The cumulative effect in the fragmentation of complex nuclei cannot be explained by Fermi-motion of nucleons and taking into account the effects of relativization of nuclear nucleons.

2. The idea of nuclear fluctuations gives a possibility to quantitatively understand

the main regularities of cumulative production of particles in proton-nucleus collisions.

3. The analysis of the aforesaid phenomena raises a new problem, namely the calculation of probability for nucleons to be in a small volume with dimensions of an order of the radius of the nucleon-nucleon core.

4. It is of interest for particle physics to investigate the distribution of partons in nucleons and fluctuons what is a new problem in the region of actually studied energies and transferred momenta.

5. It is also important to study experimentally other processes on nuclei at momentum transfers considerably larger than those investigated at present. To such processes refer, first, the reactions of elastic and quasielastic scattering of particles with momentum transfer of an order of $7 \div 10\ fm^{-1}$, i.e., those which have been achieved in the above considered reactions.

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