СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E2 - 10668

V.B.Belyaev, J.Wrzecionko

APPROXIMATE FEW-BODY EQUATIONS FOR PION-NUCLEUS SCATTERING

1977

E2 - 10668

V.B.Belyaev, J.Wrzecionko*

APPROXIMATE FEW-BODY EQUATIONS FOR PION-NUCLEUS SCATTERING

* Permanent address: Institute for Nuclear Research, Warsaw.

Беляев В.Б., Вжециoнко Е. **E2 - 10668**

Приближенные 4-частичные уравнения для π-ядерного рассеяния

На основе метода многомерного сепарабельного разложения получены различные варианты интегральных уравнений для амплитуд упругого и неупругого рассеяния в системе четырех тел ($\pi+3N$). В качестве входных данных используются t-матрицы, описывающие рассеяние частиц на ядерных подсистемах с фиксированными центрами. Замена гамильтонианов ядерных подсистем (кластеров) операторами конечного ранга приводит к понижению размерности исходных четырехчастичных уравнений. Применяя Фаддеевскую технику, можно получить простую систему фредгольмовых уравнений, которые единым образом описывают упругое и неупругое рассеяние π-мезонов на легчайших ядрах. В случае упругого рассеяния получены простые одномерные уравнения, решения которых в области высоких энергий π-мезонов даются решением модели рассеяния на системе фиксированных центров.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1977

Belyaev V.B., Wrzecionko J. **E2 - 10668**

Approximate Few-Body Equations for Pion-Nucleus Scattering

Using many-dimensional separable expansions, several kinds of integral equations for the elastic ($\pi+{}^3He\to\pi+{}^3He$) and inelastic ($\pi+{}^3He\to\pi+N+d$) amplitudes have been obtained. As an unput, these equations include the matrices, which describe the scattering of pions on the nuclear subsystem with fixed scatterers. For inelastic processes the Faddeev-type equations have been obtained.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1977

I. Introduction

Two aspects of the problem necessarily arise, when one considers the pion-nucleus interaction. The first concerns the nucleon degree of freedom. Here we are dealing with the many-body system, description of which is difficult by itself. Nevertheless, at least for few-body systems, we have refined mathematical formalism, which allows one, in principle, to solve the corresponding, dynamical equations/1/. The second aspect is related to mesonic degree of freedom in nuclei. We mean the following processes: the excitation of nucleon isobars, reactions of one and double exchange of pions, pion capture by nuclei, mesic currents, π-condensation and others. It is obvious that studying of these effects is now very important. However, the extraction from experiment of even rough information about the mentioned effects, (as has been pointed out in literature) is complicated by uncontrolled model assumptions on the mechanism of the corresponding processes. Indeed, to describe the scattering of pions on nuclei for $A \geqslant 4$, the exact many-body dynamical equations are not still formulated even. Therefore it is difficult to say, in which sense the model solution (for example, given by optical/2/, or Glauber/3/ theory) could approximate the exact one.

Nevertheless, there is a scheme/4/, in which the dynamics of the many-particle system is taken into account in a definite sense. We mean the theory of scattering particles on n fixed centers. It has been successfully applied[5] to scattering of pions on lightest nuclei. An attractive feature of this approach is a consistent treatment of the multiparticle character of scattering pions on nucleons in nuclei. Moreover, this theory permits one to find a solution for the separable pion-nucleon interactions in explicit analytical form.

It is clear that the model of scattering on the system of fixed scatterers should be valid in the high energy region and small momentum transfer, since one could neglect the kinetic energy of the scatterers. For low and medium energy region the applicability of the model is questionable. Therefore, in this region it is desirable to develop a scheme, which contains the above advantages of the fixed center model, and also takes into account the motion of nucleons in necleus in a consistent way. Such a possibility appears if one uses the method of multidimensional separable expansion which has been applied[6] in the three-body bound state problem.

Below we shall derive one-dimensional integral equations for the pion-nucleus elastic scattering amplitudes. Inhomogeneous terms of these equations coincide with t-matrix for the fixed-center model.

Using the Faddeev technique, in the framework of the approach, developed in this paper, it is also possible to include inelastic processes of the type $\pi + He^3 \to \pi + N + d$.

In section II we shall present physical arguments in favour of the developed method. We shall give also the formal scheme for deriving equations for the amplitudes which describe elastic scattering pions by the system of 3 bounded nucleons. In section III this scheme is extended to describe in a unique way the elastic and inelastic processes of pion-nucleus interaction.

For definiteness we restrict our consideration to the 4-body system $\pi + 3N$. Its Hamiltonian can be written in the form:

$$H = h_0 + h_c + V_N + V_\pi \quad ,$$
$$V_N = \sum_{i \neq j = 1}^{3} V_N^{ij} \quad , \quad V_\pi = \sum_{i=1}^{3} V_{\pi N}^{i} \quad , \tag{1.1}$$

where V_N^{ij} is the nucleon-nucleon potential, $V_{\pi N}^{i}$ is the pion-nucleon potential, h_0 is the kinetic energy of the relative motion of pion and center of mass of nucleus, h_c is the kinetic energy of nucleons.

In the center of mass 4-body system, the scattering wave function of pions on the 3-nucleon bound state, fulfils the boundary condition:

$$\langle \vec{r}_{12}, \vec{r}_3, \vec{\rho} | \Psi \rangle \equiv \Psi(\vec{r}_{12}, \vec{r}_3, \vec{\rho}) \simeq \Psi^0(\vec{r}_{12}, \vec{r}_3, \vec{\rho}) + $$
$$+ T \chi(\vec{r}_{12}, \vec{r}_3) e^{i\varkappa\rho}/\rho \qquad (|\vec{\rho}| \to \infty) \, , \tag{1.2}$$
$$\Psi(\vec{r}_{12}, \vec{r}_3, \vec{\rho}) \to 0 \qquad (|\vec{r}_{12}| \to \infty \, , |\vec{r}_3| \to \infty) \tag{1.3}$$

Here $\vec{r}_{12}, \vec{r}_3, \vec{\rho}$ are the Jacoby coordinates of particles, $\varkappa = \sqrt{2\mu_\pi E}$, μ_π - pion-nucleus reduced mass, E is the total energy of the system, T is the elastic scattering amplitude, $\langle \vec{r}_{12}, \vec{r}_3 | \chi \rangle \equiv \chi(\vec{r}_{12}, \vec{r}_3)$ is the ground state wave function of the 3N-system.

Due to (1.3) the following condition takes place:

$$\int |\Psi(\vec{r}_{12}, \vec{r}_3, \vec{\rho})|^2 d\vec{r}_{12}\, d\vec{r}_3 \leq M, \tag{1.4}$$

i.e., Ψ is square-integrable with respect to nucleon variables.

Under these circumenstances, it is possible to approximate the nuclear part (i.e., $h_c + V_N$) of the Hamiltonian (1.1) by the finite rank operator. For example:

$$(h_c + V_N)|\Psi\rangle \simeq |\varphi\rangle\langle\varphi|h_c + V_N|\varphi\rangle\langle\varphi|\Psi\rangle , \tag{1.5}$$

where $\langle \vec{r}_{12}, \vec{r}_3 | \varphi\rangle$ is the auxiliary function with finite norm. If $|\varphi\rangle$ is the normalized eigenstate of the nuclear Hamiltonian, then $\langle\varphi|h_c + V_N|\varphi\rangle = \mathcal{E}$ is the binding energy of the 3-nucleon system. The overlap integral $\langle\varphi|\Psi\rangle$ is unknown function which depends only on variable $\vec{\rho}$. Approximation (1.5) permits one to find simple integral equation for the elastic scattering amplitude.

In a more general case, if the considered system consists of a few subsystems (clusters) one can replace the parts of the total Hamiltonian, corresponding to these clusters, by finite rank operators. Such a procedure, permits one to reduce significantly the dimension of integral equations.

To use a procedure of the type (1.5), we shall write the equation for the 4-body ($\pi + 3N$) amplitude, separating the Hamiltonian of the bounded 3N subsystem.

Let us introduce the following set of Green functions:

$$G(E) = (H - E)^{-1};\quad G_c(E) = (h_0 + h_c + V_N - E)^{-1};$$

$$G_c(E) = (h_0 - E)^{-1};\ G_{cc}(E) = (h_0 + h_c - E)^{-1};\ G_\pi(E) = (h_0 + V_\pi - E)^{-1}. \tag{1.6}$$

The 4-body transition operator can be defined as usually:

$$T = V_\pi - V_\pi G(E) V_\pi . \tag{1.7}$$

We introduce an auxiliary operator

$$T^0 = V_\pi - V_\pi G_\pi(E) V_\pi = V_\pi - V_\pi G_0(E) T^0 . \tag{1.8}$$

For the total Green function, one can find the equation:

$$G(E) = G_\pi(E) - G_\pi(E) H_c G(E), \quad \text{where } H_c = h_c + V_N . \tag{1.9}$$

Substituting (1.9) into (1.7) and using (1.8), we derive the equation for the transition operator:

$$\begin{aligned} T &= T^\circ + T^\circ G_0(E) H_c G_c(E) T = \\ &= T^\circ + T^\circ [G_0(E) - G_c(E)] T . \end{aligned} \tag{1.10}$$

It is easily seen, that eq. (1.8) determines the scattering amplitudes of pions on 3 fixed centers (nucleons). In the momentum representation, it has the form:

$$\begin{aligned} \langle \vec{k} | \tau^\circ(\vec{r}_{12}, \vec{r}_3) | \vec{k}' \rangle &= f(\vec{k}, \vec{k}', \vec{r}_{12}, \vec{r}_3) \langle \vec{k} | v_\pi | \vec{k}' \rangle - \\ &- \int \frac{d\vec{k}''}{(2\pi)^3} f(\vec{k}, \vec{k}'', \vec{r}_{12}, \vec{r}_3) \frac{\langle \vec{k} | v_\pi | \vec{k}'' \rangle}{\frac{k''^2}{2\mu_\pi} - E} \cdot \langle \vec{k}'' | \tau^\circ(\vec{r}_{12}, \vec{r}_3) | \vec{k}' \rangle , \end{aligned} \tag{1.11}$$

where

$$\langle \vec{k}, \vec{r}_{12}, \vec{r}_3 | T^\circ | \vec{k}', \vec{r}_{12}', \vec{r}_3' \rangle = \delta(\vec{r}_{12} - \vec{r}_{12}') \delta(\vec{r}_3 - \vec{r}_3') \langle \vec{k} | \tau^\circ(\vec{r}_{12}, \vec{r}_3) | \vec{k}' \rangle ,$$

$$\begin{aligned} f(\vec{k}, \vec{k}', \vec{r}_{12}, \vec{r}_3) &= \exp\left[i(\vec{k}' - \vec{k})\left(\frac{\vec{r}_{12}}{2} + \frac{\vec{r}_3}{3}\right)\right] + \exp\left[i(\vec{k}' - \vec{k})\left(-\frac{\vec{r}_{12}}{2} + \frac{\vec{r}_3}{3}\right)\right] + \\ &+ \exp\left[i(\vec{k} - \vec{k}')\tfrac{2}{3}\vec{r}_3\right] . \end{aligned} \tag{1.12}$$

It is known, that the above equation can be easily integrated for the separable π-nucleon interaction.

Bearing in mind for the elastic scattering of pions on the bound state of the 3 particles, for the nuclear part of the Hamiltonian we assume the following approximation:

$$H_c \approx H_c^N = \sum_{\mu,\nu=1}^{N} | \chi_\mu \rangle \sigma_{\mu\nu} \langle \chi_\nu | . \tag{1.13}$$

Expansion (1.13) of the nuclear Hamiltonian implies a different variants of replacement of H_c by finite rank operator H_c^N . Each variant corresponds to a definite choice of matrices $\sigma_{\mu\nu}$ and "state vectors" $|\chi_\mu\rangle$. We assume that the functions

$\langle \vec{r}_{12}, \vec{r}_3 | \chi_\mu \rangle \equiv \chi_\mu(\vec{r}_{12}, \vec{r}_3)$ form an ortonormal set of functions with finite norm, and $\chi_1(\vec{r}_{12}, \vec{r}_3)$ corresponds to the ground state of 3-nucleon system.

Hence,

$$\sigma_{\mu\nu} = \langle \chi_\mu | H_c | \chi_\nu \rangle , \quad \sigma_{\mu 1} = \mathcal{E}\, \delta_{\mu 1} ,$$

where $\mathcal{E}$ is the binding energy. For H_c given by (1.13) the Green function $G_c(E)$ has the form:

$$\langle \vec{K}, \vec{r}_{12}, \vec{r}_3 | G_c(E) | \vec{K}', \vec{r}\,'_{12}, \vec{r}\,'_3 \rangle = (2\pi)^3 \delta(\vec{K} - \vec{K}') \Big[\delta(\vec{r}_{12} - \vec{r}\,'_{12}) \delta(\vec{r}_3 - \vec{r}\,'_3) - \tag{1.14}$$

$$- \sum_{\mu\nu} \chi_\mu(\vec{r}_{12} \vec{r}_3) \Gamma_{\mu\nu}(K, E) \bar{\chi}_\nu(\vec{r}\,'_{12}, \vec{r}\,'_3) \Big] ,$$

where

$$\Gamma_{\mu\nu}(K, E) = \sum_\lambda \frac{\sigma_{\mu\lambda}}{\frac{K^2}{2\mu_\pi} - E} \left[\frac{K^2}{2\mu_\pi} - E + \hat{I} \hat{\sigma} \right]^{-1}_{\lambda\nu} , \quad I_{\mu\nu} = \langle \chi_\mu | \chi_\nu \rangle \tag{1.15}$$

Hence, in this approximation eq. (1.10) in mixed representation can be written:

$$\langle \vec{K}, \vec{r}_{12}, \vec{r}_3 | T | \vec{K}', \vec{r}\,'_{12}, \vec{r}\,'_3 \rangle = \delta(\vec{r}_{12} - \vec{r}\,'_{12}) \delta(\vec{r}_3 - \vec{r}\,'_3) \langle \vec{K} | \tau^0(\vec{r}_{12}, \vec{r}_3) | \vec{K}' \rangle +$$

$$\sum_{\mu\nu} \int \frac{d\vec{K}''}{(2\pi)^3} d\vec{r}\,''_{12} d\vec{r}\,''_3 \chi_\mu(\vec{r}_{12}, \vec{r}_3) \Gamma_{\mu\nu}(K'', E) \bar{\chi}_\nu(\vec{r}\,''_{12}, \vec{r}\,''_3) \langle \vec{K}, \vec{r}_{12}, \vec{r}_3 | T | \vec{K}', \vec{r}\,'_{12}, \vec{r}\,'_3 \rangle \times \tag{1.16}$$
$$\times \langle \vec{K} | \tau^0(\vec{r}_{12}, \vec{r}_3) | \vec{K}'' \rangle .$$

Introducing the amplitudes

$$\langle \vec{K} | \tau_{\mu\nu} | \vec{K}' \rangle \equiv \int d\vec{r}_{12} d\vec{r}_3 d\vec{r}\,'_{12} d\vec{r}\,'_3 \, \bar{\chi}_\mu(\vec{r}_{12}, \vec{r}_3) \langle \vec{K}, \vec{r}_{12} \vec{r}_3 | T | \vec{K}', \vec{r}\,'_{12}, \vec{r}\,'_3 \rangle \times$$
$$\times \chi_\nu(\vec{r}\,'_{12}, \vec{r}\,'_3) ,$$

$$\langle \vec{K} | \tau^0_{\mu\nu} | \vec{K}' \rangle \equiv \int d\vec{r}_{12} d\vec{r}_3 \, \bar{\chi}_\mu(\vec{r}_{12}, \vec{r}_3) \langle \vec{K} | \tau^0(\vec{r}_{12}, \vec{r}_3) | \vec{K}' \rangle \times \tag{1.17}$$
$$\times \chi_\nu(\vec{r}_{12}, \vec{r}_3) ,$$

we derive the system of the one-dimensional integral equations from eq. (1.16):

$$\langle \vec{K} | \tau_{\mu\nu} | \vec{K}' \rangle = \langle \vec{K} | \tau^0_{\mu\nu} | \vec{K}' \rangle + \int \frac{d\vec{K}''}{(2\pi)^3} \sum_{\lambda\sigma} \langle \vec{K} | \tau^0_{\mu\lambda} | \vec{K}'' \rangle \times \tag{1.18}$$
$$\times \Gamma_{\lambda\sigma}(K'', E) \langle \vec{K}'' | \tau_{\sigma\nu} | \vec{K}' \rangle .$$

If we keep in eq. (1.13) only the first term, the equation for the elastic scattering amplitud of pions on the bound state, is as follows:

$$\langle\vec{K}|\mathcal{E}_{11}|\vec{K}'\rangle \equiv \langle\vec{K}|\mathcal{E}|\vec{K}'\rangle = \langle\vec{K}|\mathcal{E}^{\circ}|\vec{K}'\rangle +$$

$$+\varepsilon\int\frac{d\vec{K}''}{(2\pi)^3}\langle\vec{K}|\mathcal{E}^{\circ}|\vec{K}''\rangle\frac{\langle\vec{K}''|\mathcal{E}|\vec{K}'\rangle}{\left(\frac{K''^2}{2\mu_\pi}-E\right)\left(\frac{K''^2}{2\mu_\pi}-E+\varepsilon\right)}, \tag{1.19}$$

where $\langle\vec{K}|\mathcal{E}^{\circ}|\vec{K}'\rangle$ is the scattering amplitude of pions on three fixed centers averaged over the wave function of the ground state of the 3-nucleon system.

It is known that in the traditional theory based on the optical potential, the kernel of the Lippman-Schwinger equation for elastic scattering of pioms on nuclei, is given by the two body t-matrices, averaged over the ground-state wave function of nuclei. The higher order terms in the Watson expansion for the optical potential are always, neglected. On the other hand,in eq. (1.19) inhomogeneous term and the kernel are given by the 4-body scattering matrix pions on fixed nucleons. It means that all rescattering effects are automatically included into the kernel.

This also means that equation (1.19) represents the real extention beyond the optical model approach.

III

We consider now the inelastic processes of the type:

$$\pi + He^3(T) \to N + d + \pi.$$

Bearing in minde the three cluster character of the final state the total Hamiltonian (1.1) of the system can be divided in the following way: $H = H_0 + H_d + \mathcal{V}_1 + \mathcal{V}_2 + \mathcal{V}_3$,

$$H_d = h_d + V_{N_1N_2},\ \mathcal{V}_1 = V_{\pi N_3},\ \mathcal{V}_2 = V_{\pi N_1} + V_{\pi N_2},\ \mathcal{V}_3 = V_{N_1N_2} + V_{N_2N_3}, \tag{2.1}$$

where H_c is the kinetic energy of the relative motion in the system $N+d+\pi$, h_d is the kinetic energy of the relative motion of nucleons in the deuteron, $V_{\pi N_i}$ is the interaction potential of the pion with i-th nucleon, $V_{N_i N_j}$ is the nucleon-nucleon potential.

Let us introduce the Green functions:

$$g(E) = (H-E)^{-1}, \qquad g_c(E) = (H_c - E)^{-1},$$
$$g_d(E) = (H_c + H_d - E)^{-1}, \qquad \bar{g}(E) = (H - H_d - E)^{-1}. \tag{2.2}$$

Following Faddeev[1], one can introduce the operators $M_{\alpha\beta}$ and $M^c_{\alpha\beta}$ (α, β = 1,2,3):

$$M_{\alpha\beta} = v_\alpha \delta_{\alpha\beta} - v_\alpha g(E) v_\beta,$$
$$M^c_{\alpha\beta} = v_\alpha \delta_{\alpha\beta} - v_\alpha \bar{g}(E) v_\beta. \tag{2.3}$$

It can be easily shown that they satisfy the following equations:

$$M_{\alpha\beta} = t_\alpha \delta_{\alpha\beta} - t_\alpha g_d(E) \sum_{\lambda\neq\alpha} M_{\lambda\beta},$$
$$M^c_{\alpha\beta} = t^c_\alpha \delta_{\alpha\beta} - t^c_\alpha g_c(E) \sum_{\lambda\neq\alpha} M^c_{\lambda\beta}, \tag{2.4}$$

where the "channel" t-matrices t_α and t^c_α are given by the solution of the Lippman-Schwinger equations:

$$t_\alpha = v_\alpha - v_\alpha g_d(E) t_\alpha;$$
$$t^c_\alpha = v_\alpha - v_\alpha g_c(E) t^c_\alpha; \tag{2.5}$$

The second eq. (2.5) for α =2,3 gives t-matrices for scattering on two fixed centers.

The operators $M_{\alpha\beta}$ and $M^c_{\alpha\beta}$ define the four-body transition operators T and $\tilde{T}$

$$T = \sum_{\alpha\beta} M_{\alpha\beta} \quad ; \quad \tilde{T} = \sum_{\alpha\beta} M^{o}_{\alpha\beta} \tag{2.6}$$

It is easy to see that the following equation holds

$$T = \tilde{T} + \tilde{T}\, g_o(E) H_\alpha g_\alpha(E) T \tag{2.7}$$

The last equation is a generalization of eq. (1.10) in which breake up channel is included explicitly since the operator $\tilde{T}$ is the three-cluster amplitude, while T^{o} in eq. (1.10) is the two-body amplitude. The eq. (2.7) has non-fredholmian kernel, becouse it contains disconnected parts produced by the inhomogeneous term $\tilde{T}$. Applying Faddeev procedure, one can obtain the system of Fredholm equations. To this end we introduce a new operator $W^{o}_{\alpha\beta}$ (by formula)

$$M^{o}_{\alpha\beta} = t^{o}_{\alpha}\, \delta_{\alpha\beta} + W^{o}_{\alpha\beta}. \tag{2.8}$$

Then

$$\tilde{T} = \sum_{\alpha\beta} M^{o}_{\alpha\beta} = \sum_{\alpha} t^{o}_{\alpha} + \sum_{\alpha\beta} W^{o}_{\alpha\beta} = T^{oo} + T_F. \tag{2.9}$$

Inserting (2.9) into (2.7) and introducing

$$\begin{aligned} T_{1\alpha} &= t^{o}_{\alpha} + t^{o}_{\alpha}\, g_o(E) H_\alpha g_\alpha(E)(T_1 + T_2) \; ; \; T_1 + T_2 = T \\ T_1 &= T^{oo} + T^{oo} g_o(E) H_\alpha g_\alpha(E)(T_1 + T_2), \\ T_2 &= T_F + T_F\, g_o(E) H_\alpha g_\alpha(E)(T_1 + T_2), \end{aligned} \tag{2.10}$$

we derive finally the desirable system of equations:

$$\begin{aligned} T_{1\alpha} &= t_\alpha + t_\alpha\, g_o(E) H_\alpha g_\alpha(E) T_2 + t_\alpha\, g_o(E) H_\alpha g_\alpha(E) \sum_{\gamma \neq \alpha} T_{1\gamma} \\ T_2 &= T_F + T_F\, g_o(E) H_\alpha g_\alpha(E)\Big(T_2 + \sum_{\alpha} T_{1\alpha}\Big), ^{x)} \end{aligned} \tag{2.11}$$

---

x) It is easy to see that in the lower order of iteration for $W^{o}_{\alpha\beta}$ $\left(W^{oo}_{\alpha\beta} = \begin{cases} 0 \;;\; \alpha = \beta \\ t^{o}_{\alpha}\, g_o(E)\, t^{o}_{\beta} ; \alpha \neq \beta \end{cases}\right)$ the amplitude T_F does not contain the disconnected parts. That means, kernel of the second eq. (2.11) is fredholmian.

Formulae (2.11) are the exact four-body equations presented in the form suitable for applying the approximations of the (1.3) type.

From the definition of $W^c_{\alpha\beta}$ we see that it is small when all three clusters do not approach to each other closely. In this case, we put:

$$M^c_{\alpha\beta} \approx t^c_\alpha \delta_{\alpha\beta} \tag{2.12}$$

and the equations are simplified considerably. Instead of (2.11), we have an analog of Faddeev equations but for four-particle amplitudes $T_{1\alpha}$:

$$\begin{gathered} T_{1\alpha} = t_\alpha + t_\alpha g_c(E) H_d g_d(E) \sum_{\beta\neq\alpha} T_{1\beta} \\ T = \sum_{\alpha=1}^{3} T_{1\alpha} \; , \; T_2 \equiv 0 \, . \end{gathered} \tag{2.13}$$

Now we would like to show that eq. (2.13) within the approximation:

$$H_d = \sum_{\mu\nu} |\chi_\mu\rangle \varkappa_{\mu\nu} \langle \chi_\nu| , \tag{2.14}$$

(compare with (1.13)) is reduced to the system of two-dimensional integral equations.

Let us introduce the mixed Jakobian variables

$$\begin{aligned} \vec{k}_1 &= (m\vec{p}_\pi - m_\pi \vec{p}_N)/(m+m_\pi) \; ; \; \vec{q}_1 = \vec{p}_d \, , \\ \vec{k}_2 &= (m_\pi \vec{p}_d - 2m\vec{p}_\pi)/(2m+m_\pi) \; ; \; \vec{q}_2 = \vec{p}_N \\ \vec{k}_3 &= (2m\vec{p}_N - m\vec{p}_d)/3m \qquad ; \; \vec{q}_3 = \vec{p}_\pi \\ \vec{r} &= \vec{r}_{12} \qquad ; \quad \vec{p}_d + \vec{p}_\pi + \vec{p}_N = 0 \, . \end{aligned} \tag{2.15}$$

In this representation, for the Green function $g_d(E)$ in approximation (2.14) we have

$$\begin{aligned} \langle \vec{k}_\alpha, \vec{q}_\alpha, \vec{r} | g_\alpha(E) | \vec{k}'_\alpha, \vec{q}'_\alpha, \vec{r}' \rangle &= (2\pi)^6 \delta(\vec{k}_\alpha - \vec{k}'_\alpha)\, \delta(\vec{q}_\alpha - \vec{q}'_\alpha) \Big[\delta(\vec{r} - \vec{r}') \times \\ &\times \Big(\frac{k_\alpha^2}{2\mu_\alpha} + \frac{q_\alpha^2}{2\bar{\mu}_\alpha} - E \Big)^{-1} - \sum_{\mu\nu} \chi_\mu(\vec{r})\, \Gamma^\alpha_{\mu\nu}(k_\alpha, q_\alpha)\, \bar{\chi}_\nu(\vec{r}') \Big] , \end{aligned} \tag{2.16}$$

where

$$\Gamma^{\alpha}_{\mu\nu}(k_\alpha,q_\alpha) = \left(\frac{k_\alpha^2}{2\mu_\alpha}+\frac{q_\alpha^2}{2\bar{\mu}_\alpha}-E\right)^{-1}\sum_\lambda \varkappa_{\mu\lambda}\left[\frac{k_\alpha^2}{2\mu_\alpha}+\frac{q_\alpha^2}{2\bar{\mu}_\alpha}-E+I\varkappa\right]^{-1}_{\lambda\nu}$$

$$I_{\mu\nu} = \langle\chi_\mu|\chi_\nu\rangle \quad ; \tag{2.17}$$

$$\frac{1}{\mu_1} = \frac{1}{m}+\frac{1}{m_\pi} \quad ; \quad \frac{1}{\bar{\mu}_1} = \frac{1}{2m}+\frac{1}{m_\pi+m}$$

$$\frac{1}{\mu_2} = \frac{1}{2m}+\frac{1}{m_\pi} \quad ; \quad \frac{1}{\bar{\mu}_2} = \frac{1}{m}+\frac{1}{m_\pi+2m}$$

$$\frac{1}{\mu_3} = \frac{3}{2}\cdot\frac{1}{m} \quad ; \quad \frac{1}{\bar{\mu}_3} = \frac{1}{m_\pi}+\frac{1}{3m}$$

The operators t_α in the space of the four-body states have the form:

$$\langle\vec{k}_\alpha,\vec{q}_\alpha,\vec{r}\,|\,t_\alpha\,|\vec{k}'_\alpha,\vec{q}'_\alpha,\vec{r}\,'\rangle = (2\pi)^3\delta(\vec{q}_\alpha-\vec{q}'_\alpha)\langle\vec{k}_\alpha\vec{r}\,|\,\mathcal{T}_\alpha(q_\alpha)\,|\vec{k}'_\alpha\vec{r}\,'\rangle. \tag{2.18}$$

It is easy to show that for matrix elements of the operators $\mathcal{T}_\alpha$ there takes place the equation

$$\begin{aligned}\langle\vec{k}_\alpha,\vec{r}\,|\mathcal{T}_\alpha(q_\alpha)|\vec{k}'_\alpha\vec{r}\,'\rangle &= \delta(\vec{r}-\vec{r}\,')\langle\vec{k}_\alpha|\mathcal{T}^{\circ}_\alpha(\vec{r})|\vec{k}'_\alpha\rangle + \\ &+\frac{1}{(2\pi)^3}\sum_{\mu\nu}\int d\vec{k}''_\alpha\, d\vec{r}\,''\langle\vec{k}_\alpha|\mathcal{T}^{\circ}_\alpha(\vec{r})|\vec{k}''_\alpha\rangle\chi_\mu(\vec{r})\Gamma^{\alpha}_{\mu\nu}(k''_\alpha,q_\alpha)\times \\ &\times\bar{\chi}_\nu(\vec{r}\,'')\langle\vec{k}''_\alpha\vec{r}\,''|\mathcal{T}_\alpha(q_\alpha)|\vec{k}'_\alpha,\vec{r}\,'\rangle\end{aligned} \tag{2.19}$$

similar to (1.16). Here $\langle\vec{k}_\alpha|\mathcal{T}^{\circ}_\alpha(\vec{r})|\vec{k}'_\alpha\rangle$ is the amplitude of scattering on two-fixed centers[4].

Let us denote

$$\langle\vec{k}_\alpha|\mathcal{T}^{\circ}_{\alpha;\mu\nu}|\vec{k}'_\alpha\rangle \equiv \int d\vec{r}\,\bar{\chi}_\mu(\vec{r})\langle\vec{k}_\alpha|\mathcal{T}^{\circ}_\alpha(\vec{r})|\vec{k}'_\alpha\rangle\chi_\nu(\vec{r})$$

$$\langle\vec{k}_\alpha|\mathcal{T}_{\alpha;\mu\nu}|\vec{k}'_\alpha\rangle \equiv \int d\vec{r}\,d\vec{r}\,'\bar{\chi}_\mu(\vec{r})\langle\vec{k}_\alpha\vec{r}\,|\mathcal{T}_\alpha(q_\alpha)|\vec{k}'_\alpha\vec{r}\,'\rangle\chi_\nu(\vec{r}\,'). \tag{2.20}$$

Then from (2.19) there follow the one-dimensional integral equations:

$$\begin{aligned}\langle\vec{k}_\alpha|\mathcal{T}_{\alpha;\sigma\lambda}(q_\alpha)|\vec{k}'_\alpha\rangle &= \langle\vec{k}_\alpha|\mathcal{T}^{\circ}_{\alpha;\sigma\lambda}|\vec{k}'_\alpha\rangle + \\ &+\frac{1}{(2\pi)^3}\sum_{\mu\nu}\int d\vec{k}''_\alpha\langle\vec{k}_\alpha|\mathcal{T}^{\circ}_{\alpha;\sigma\mu}|\vec{k}''_\alpha\rangle\Gamma^{\alpha}_{\mu\nu}(k''_\alpha,q_\alpha)\langle\vec{k}''_\alpha|\mathcal{T}_{\alpha;\nu\lambda}(q_\alpha)|\vec{k}'_\alpha\rangle\end{aligned} \tag{2.21}$$

With the help of (2.20) the equations (2.13) for $\tilde{T}_{1\alpha}$ can be written in momentum representation explicitly. Using abbreviation

$$\langle \vec{k}_\alpha, \vec{q}_\alpha | \tilde{T}^{\alpha}_{\mu\nu} | \vec{k}'_\alpha \vec{q}'_\alpha \rangle = \int d\vec{r}\, d\vec{r}\,' \chi_\mu(\vec{r}) \langle \vec{k}_\alpha \vec{q}_\alpha \vec{r} | \tilde{T}_{1\alpha} | \vec{k}'_\alpha, \vec{q}'_\alpha, \vec{r}\,' \rangle \chi_\nu(\vec{r}\,') \tag{2.22}$$

from (2.13), it is easy to write the system of Faddeev-type equation for the amplitudes $\tilde{T}^{\alpha}_{\mu\nu}$:

$$\langle \vec{k}_\alpha, \vec{q}_\alpha | \tilde{T}^{\alpha}_{\mu\nu} | \vec{k}'_\alpha \vec{q}'_\alpha \rangle = (2\pi)^3 \delta(\vec{q}_\alpha - \vec{q}'_\alpha) \langle \vec{k}_\alpha | \tau_{\alpha;\mu\nu} | \vec{k}'_\alpha \rangle -$$
$$- \frac{1}{(2\pi)^3} \sum_{\sigma\lambda} \int d\vec{k}''_\alpha \langle \vec{k}_\alpha | \tau_{\alpha;\mu\sigma} | \vec{k}''_\alpha \rangle \Gamma^{\alpha}_{\sigma\lambda}(k''_\alpha, q_\alpha) \times$$
$$\times \Big[\sum_{\beta\neq\alpha} \langle a^{\alpha}_{\beta} \vec{k}''_\alpha + b^{\alpha}_{\beta} \vec{q}_\alpha ; c^{\alpha}_{\beta} \vec{k}''_\alpha + d^{\alpha}_{\beta} \vec{q}_\alpha | \tilde{T}^{\beta}_{\lambda\nu} | a^{\alpha}_{\beta} \vec{k}'_\alpha + b^{\alpha}_{\beta} \vec{q}'_\alpha ; c^{\alpha}_{\beta} \vec{k}'_\alpha + d^{\alpha}_{\beta} \vec{q}_\alpha \rangle \Big]$$

$$a^{\alpha}_{\beta} = d^{\beta}_{\alpha} \; ; \quad c^{\alpha}_{\beta} = - c^{\beta}_{\alpha} \; ; \quad b^{\alpha}_{\beta} = - b^{\beta}_{\alpha} \tag{2.23}$$

$$a^{1}_{2} = - \frac{2m}{m_\pi + 2m} \; ; \quad a^{1}_{3} = - \frac{2}{3} \, ,$$

$$a^{2}_{1} = - \frac{m}{m_\pi + m} \; ; \quad a^{2}_{3} = - \frac{1}{3} \, ,$$

$$a^{3}_{1} = - \frac{m_\pi}{m_\pi + m} \; ; \quad a^{3}_{2} = - \frac{m_\pi}{m_\pi + 2m} \; ;$$

$$b^{1}_{2} = \frac{m_\pi}{(m_\pi + m)} \frac{(3m + m_\pi)}{(2m + m_\pi)} \; ; \quad b^{1}_{3} = - \frac{3m + m_\pi}{3(m + m_\pi)} \; ;$$

$$b^{2}_{3} = \frac{2}{3} \frac{(m_\pi + 3m)}{(m_\pi + 2m)} \; ;$$

$$c^{1}_{2} = c^{2}_{3} = c^{3}_{1} = -1 ; \qquad \alpha, \beta = 1, 2, 3$$

The physical amplitudes for the breake up and elastic scattering can be found by integrating the amplitudes (2.22) with appropriate asymptotic initial and final wave functions.

In conclusion we should like to note that the "relativistic effects" in pion motion for equations (1.19) and (1.20) can easily be taken into account passing from the Lippman-Schwinger to Logunov-Tavkhelidze [7] or Kadyshevsky [8] equations. The developed formalism can be applied to the processes involving hyperons (for instance, $K^- + He^3 \to d + \Lambda(\Sigma) + \pi$) and the process of deuteron knocking out from nuclei by high energy pions.

REFERENCES

1. L.D.Faddeev. Sov.Phys.JETP, 12 (1961) 1014.
2. J.Hufner. Pions interact. with nuclei. M.P.I. - Heidelberg - 1975.
3. R.Mach. Czech. J. Phys., v. B29 (1976) 1248.
4. L.L.Foldy and J.D.Walecka. Ann. Phys., 54, 447 (1969).
5. B.F.Gibson. "Pion interaction with few-nucleon systems", Topical Meeting on π- Nuclei Int. Pittsburg (1976).
6. V.B.Belyaev, J.Wrzecionko, M.I.Sakvarelidze, A.T.Eliava. Journ. of Comp. Phys., 20, 495 (1976).
7. A.A.Logunov, A.N.Tavkhelidze. N.C., 29, 380 (1963).
8. V.G.Kadyshevsky. Nucl. Phys.,B6, 125 (1968).

Received by Publishing Department
on May 17, 1977