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TWO-LOOP DIVERGENCES OF FIELD THEORIES WITH NONLINEAR SYMMETRY

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Двухпетлевые расходимости в теориях с нелинейными симметриями

С использованием инвариантной формулировки метода фонового поля перенормировки получена общая формула для двухпетлевых контрчленов в теориях с нелинейными симметриями.

Основным моментом является построение контрчленов из инвариантов группы симметрии в терминах форм Картана.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1977

Kazakov D., Pushkin S. **E2 - 10655**

Two-Loop Divergences of Field Theories with Nonlinear Symmetry

A general formula is obtained for two-loop counterterms of the field theories with the nonlinear realization of symmetry group.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1977

In the recent paper[1] we have proposed an invariant formulation of background-field method of renormalization for the Lagrangians with nonlinear realization of a symmetry group. The formalism developed allows us to obtain counterterms, which eliminate ultraviolet divergences in a manifestly invariant form without considering the equations of motion. The counterterms are constructed of a small number of beforehand known invariants of a group, furthermore the procedure of the determination of the coefficients for the invariants enables us to calculate the minimal possible number of diagrams.

In the present note we adduce the results of the application of the proposed method to the calculations of the two-loop counterterms at the theories with nonlinear realization of a semisimple symmetry group $G = H \otimes H$ with spontaneously broken symmetry of vacuum. It is of interest in connection with the investigation of the possibility of the dynamical restoration of symmetry in the case of nonlinear realizations.

By the phenomenological Lagrangians method[3] the Lagrangian invariant under the group G can be written in the form:

$$\mathcal{L} = \frac{1}{2C_2} \mathrm{Sp}\, \omega_\mu(A)\omega_\mu(A) = \frac{1}{2} \omega^i_\mu(A)\omega^i_\mu(A), \qquad (1)$$

where $\omega_\mu(A)$ are differential Cartan forms of the group G. They can be defined via the finite transformations of the group by the equation

$$G^{-1}(A)\partial_\mu G(A) = i[\omega_\mu(A) + \Theta_\mu(A)] = i[\omega^i_\mu X_i + \Theta^\alpha_\mu X_\alpha]. \qquad (2)$$

Here X_i are the generators of the subgroup $H = G/H$ taken in the adjoint representation (H is the vacuum stability subgroup), C_2 is the quadratic Casimir operator. The parameters of the group(A)are identified with the fields of particles.

In paper[1] the following expression for one-loop counterterms

$$\Delta \mathcal{L}_1 = \frac{1}{3\cdot(16\pi^2\epsilon)}\left[Sp\,\omega_\mu\omega_\mu\omega_\nu\omega_\nu + \frac{1}{2}Sp\,\omega_\mu\omega_\nu\omega_\mu\omega_\nu\right] \quad (3)$$

was obtained, using the continuous dimensional regularization, where $\epsilon = d-4/2$, and $d \to 4$ is space-time dimension.

To obtain n-loop counterterms it is necessary:[1]

(i) to change variables $A \to A(+)\varphi$ in the initial Lagrangian, where the symbol (+) means addition with taking account of the geometry of curved isospace (guotient space G/H), i.e., $G(A) \to G(A)G(\varphi)$ or $\omega_\mu(A) \to \bar{\omega}_\mu(A,\varphi)$ and $\bar{\omega}_\mu(A,\varphi)$ is given by the formula[4]

$$\bar{\omega}^i_\mu(A,\varphi) = \sum_{n=0}^{\infty}(-1)^n(\mathcal{M}^n_\varphi)^i_\ell\left[\frac{\omega^\ell_\mu(A)}{(2n)!} + \frac{(\mathcal{D}_\mu\varphi)^\ell}{(2n+1)!}\right], \quad (4)$$

where $(\mathcal{D}_\mu\varphi)^\ell = \partial_\mu\varphi^\ell + i(\Theta_\mu(A)\varphi)^\ell$ is the covariant derivative, $(\mathcal{M}^0)^i_\ell = \delta_{i\ell}$, $\mathcal{M}^i_\ell = -f^i_{km}f^m_{j\ell}\varphi^k\varphi^j$; f^i_{jk} are the structure constants of the group.

Field φ is the quantum internal field, and field **A** is the background one;

(ii) to expand $\mathcal{L}(A(+)\varphi)$ into the powers of φ up to φ^{2n} and then to carry out the expansion of the counterterms of lower order up to φ^{2n-2} . The expansion of the counterterms reproduces the subtraction in the subgraphs.

In the two-loop approximation the generating Lagrangian has the form

$$\mathcal{L}^{(2)}_{int} = \frac{1}{2}\left\{(\mathcal{D}_\mu\varphi)^i(\mathcal{D}_\mu\varphi)^i - \varphi\,\omega_\mu(A)\omega_\mu(A)\varphi\right\} +$$

$$+\frac{1}{2}\left\{-\frac{4}{3}\varphi(\mathcal{D}_\mu\varphi)\omega_\mu\varphi - \frac{1}{3}\varphi(\mathcal{D}_\mu\varphi)(\mathcal{D}_\mu\varphi)\varphi + \frac{1}{3}\varphi X^i\omega_\mu\varphi\varphi X^i\omega_\mu\varphi\right\}. \quad (5)$$

All coefficient functions in $\mathcal{L}_{int}$ are the products of forms $\omega_\mu(A)$ and $\Theta_\mu(A)$. This fact allows us to obtain manifestly invariant counterterms, written in terms of Cartan forms, without expansion over fields.

The construction procedure for counterterms is based on the initial group symmetry. Counterterms are constructed in the form

$$\Delta\mathcal{L} = a_1 I_1 + \cdots + a_N I_N\,, \quad (6)$$

where $I_1, \dots, I_N$ is the complete set of linearly independent invariants of the group, and $a_1, \dots, a_N$ are the functions of the regularization parameter defined by the contribution into invariants from different divergent diagrams.

The invariants are represented as traces of the products of Cartan forms and their covariant differentials[6], and if the continuous dimensional regularization is used, they become uniform structures of power $[D\omega]^{2K}[\omega]^{2(n+1-2K)}$, $K=0,1,\dots,n-1$; n is the number of loops. Linearly independent invariants are chosen allowing for the structure equations of group;

$$\begin{aligned} &\partial_\mu \theta_\nu - \partial_\nu \theta_\mu + i[\theta_\mu, \theta_\nu] = -i[\omega_\mu, \omega_\nu] \equiv C_{\mu\nu}, \\ &\partial_\mu \omega_\nu - \partial_\nu \omega_\mu + i[\theta_\mu, \omega_\nu] = -i[\omega_\mu, \theta_\nu] \equiv \tilde{C}_{\mu\nu} \end{aligned} \tag{7}$$

and in the two-loop approximation have the form

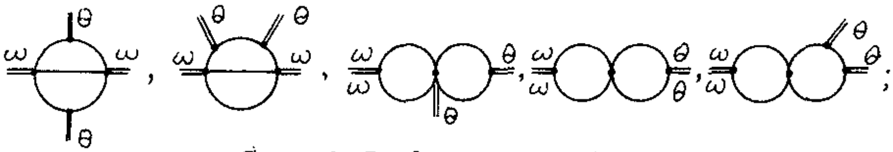
$$\begin{aligned} &I_1 = Sp\, \omega_\mu \omega_\mu \omega_\nu \omega_\nu \omega_\rho \omega_\rho, \quad I_2 = Sp\, \omega_\mu \omega_\nu \omega_\mu \omega_\nu \omega_\rho \omega_\rho, \\ &I_3 = Sp\, \omega_\mu \omega_\nu \omega_\nu \omega_\mu \omega_\rho \omega_\rho, \quad I_4 = Sp\, \omega_\mu \omega_\nu \omega_\rho \omega_\mu \omega_\nu \omega_\rho, \\ &I_5 = Sp\, \omega_\mu \omega_\nu \omega_\rho \omega_\mu \omega_\rho \omega_\nu, \\ &I_6 = Sp\, D_\mu \omega_\mu D_\nu \omega_\nu \omega_\rho \omega_\rho, \quad I_7 = Sp\, D_\mu \omega_\nu D_\mu \omega_\nu \omega_\rho \omega_\rho, \\ &I_8 = Sp\, D_\mu \omega_\mu D_\nu \omega_\rho \omega_\nu \omega_\rho, \quad I_9 = Sp\, D_\mu \omega_\nu D_\mu \omega_\rho \omega_\rho \omega_\nu, \\ &I_{10} = Sp\, D_\mu \omega_\mu \omega_\rho D_\nu \omega_\nu \omega_\rho, \quad I_{11} = Sp\, D_\mu \omega_\mu \omega_\nu D_\nu \omega_\rho \omega_\rho. \end{aligned} \tag{8}$$

With the help of the structure equations (7) the invariants can be transformed into the form, where they are directly reproduced by the combinations of the coefficient functions from the Lagrangian (5), i.e., (see[1])

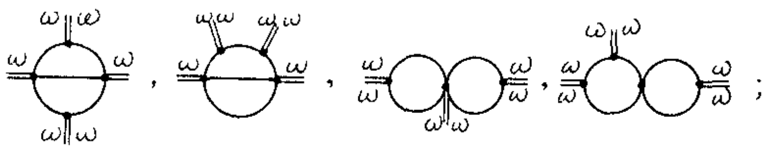
$$\begin{aligned} &J_1 = I_2 - I_3 = i\, Sp\, \omega_\mu \omega_\nu C_{\mu\nu} \omega_\rho \omega_\rho = -\tfrac{1}{2} Sp\, C_{\mu\nu} C_{\mu\nu} \omega_\rho \omega_\rho, \\ &J_2 = I_2 + I_3 = Sp[\omega_\mu \omega_\nu \omega_\mu \omega_\nu \omega_\rho \omega_\rho + \omega_\mu \omega_\nu \omega_\nu \omega_\mu \omega_\rho \omega_\rho], \\ &J_3 = -I_1 + 3I_2 + I_4 - 3I_5 = -i\, Sp\, C_{\mu\nu} C_{\rho\mu} C_{\nu\rho}, \end{aligned}$$

$$
\begin{aligned}
J_4 &= I_1 - I_2 + I_4 - I_5 = -\mathrm{Sp}\left[C_{\mu\nu} C_{\rho\mu}\omega_\nu\omega_\rho + C_{\mu\nu} C_{\rho\mu}\omega_\rho\omega_\nu\right], \\
J_5 &= -I_1 - I_2 + I_4 - I_5 = i\,\mathrm{Sp}\left[\omega_\mu\omega_\nu C_{\rho\mu}\omega_\nu\omega_\rho + \right. \\
&\quad \left. + \omega_\nu\omega_\mu C_{\rho\mu}\omega_\nu\omega_\rho + \omega_\mu\omega_\nu C_{\rho\mu}\omega_\rho\omega_\nu + \omega_\nu\omega_\mu C_{\rho\mu}\omega_\rho\omega_\nu\right], \qquad (9) \\
J_6 &= I_6,\quad J_7 = I_7,\quad J_8 = I_8,\quad J_9 = I_9,\quad J_{10} = I_{10},\quad J_{11} = I_{11}.
\end{aligned}
$$

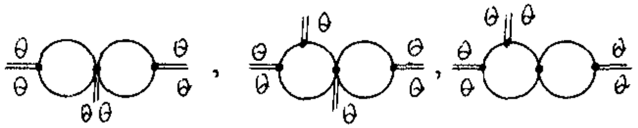
Coefficients a_i are defined from the following diagrams



$$\mathrm{Sp}\,\partial_\mu\Theta_\nu\partial_\mu\Theta_\nu\omega_\rho\omega_\rho \Rightarrow J_1$$

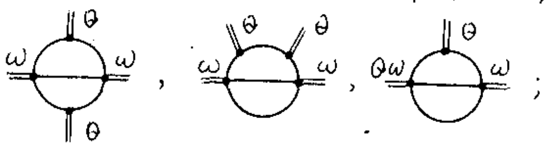


$$\mathrm{Sp}\,\omega_\mu\omega_\nu\omega_\mu\omega_\nu\omega_\rho\omega_\rho \Rightarrow J_2$$

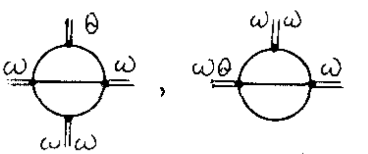


+ variations ;

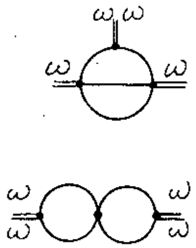
$$\mathrm{Sp}\,\Theta_\mu\Theta_\nu\Theta_\rho\Theta_\mu\Theta_\nu\Theta_\rho \Rightarrow J_3$$



$$\mathrm{Sp}\,\partial_\mu\Theta_\nu\partial_\rho\Theta_\mu\omega_\nu\omega_\rho \Rightarrow J_4$$



$$\mathrm{Sp}\,\omega_\mu\omega_\nu\partial_\rho\Theta_\mu\omega_\nu\omega_\rho \Rightarrow J_5$$



$$
\begin{aligned}
&Sp\, \partial_\mu \omega_\mu \partial_\nu \omega_\nu \omega_\rho \omega_\rho \Rightarrow J_6 , \\
&Sp\, \partial_\mu \omega_\nu \partial_\mu \omega_\nu \omega_\rho \omega_\rho \Rightarrow J_7 , \\
&Sp\, \partial_\mu \omega_\mu \partial_\nu \omega_\rho \omega_\nu \omega_\rho \Rightarrow J_8 , \\
&Sp\, \partial_\mu \omega_\nu \partial_\mu \omega_\rho \omega_\rho \omega_\nu \Rightarrow J_9 , \\
&Sp\, \partial_\mu \omega_\mu \omega_\rho \partial_\nu \omega_\nu \omega_\rho \Rightarrow J_{10} , \\
&Sp\, \partial_\mu \omega_\mu \omega_\nu \partial_\nu \omega_\rho \omega_\rho \Rightarrow J_{11} .
\end{aligned}
$$

The final result of our calculations is

$$\Delta \mathcal{L}_2 = \sum_{i=1}^{11} \frac{2C_2}{9(16\pi^2\epsilon)^2}\, b_i I_i ,$$

$$b_1 = -\frac{1}{24}\left(1 + \frac{45\cdot 65}{2(N+2)}\right) + \frac{\epsilon}{96\cdot 3}\left(71 - \frac{2290}{N+2}\right),$$

$$b_2 = \frac{1}{48}\left(23 + \frac{15\cdot 697}{N+2}\right) - \frac{\epsilon}{96\cdot 6}\left(211 + \frac{80\cdot 161}{N+2}\right),$$

$$b_3 = -\frac{7}{16}\left(1 - \frac{30}{N+2}\right) + \frac{\epsilon}{96\cdot 2}\left(23 - \frac{2660}{N+2}\right),$$

$$b_4 = -\frac{1}{24}\left(1 - \frac{45\cdot 63}{2(N+2)}\right) + \frac{\epsilon}{96\cdot 3}\left(71 + \frac{1790}{N+2}\right), \quad b_8 = \frac{55\epsilon}{12(N+2)},$$

$$b_5 = \frac{1}{24}\left(1 - \frac{15\cdot 589}{2(N+2)}\right) - \frac{\epsilon}{96\cdot 3}\left(71 + \frac{1790}{N+2}\right), \quad b_9 = -\frac{1}{16} + \frac{13\epsilon}{16\cdot 9}\left(1 - \frac{10}{N+2}\right),$$

$$b_6 = -\frac{1}{12}\left(1 + \frac{5}{N+2}\right) + \frac{5\epsilon}{18}\left(1 - \frac{7}{4(N+2)}\right), \quad b_{10} = \frac{5}{12(N+2)} + \frac{35\epsilon}{72(N+2)},$$

$$b_7 = \frac{3}{16} - \frac{\epsilon}{32}\left(1 + \frac{40}{N+2}\right), \quad b_{11} = -\frac{1}{16} + \frac{5\epsilon}{16\cdot 54}\left(35 - \frac{176}{N+2}\right).$$

Here C_2 is the Casimir operator, N is the number of group parameters. We point once more, that invariants (8) do not include the products of traces. This reflects the so-called property of the algebraic duality /6/. To our mind, such products, even if they appear should not contribute to the counterterms. Otherwise they should be included in the initial set of the invariants (8).

Thus, we have obtained the general formula for the two-loop counterterms of the field theories with nonlinear realization of a semisimple symmetry group of the $G = H \otimes H$ type.

Note, that in the proposed approach the power of the invariants in the Cartan forms increases with the number of loops. This fact indicates that the pure-symmetry arguments do not lead to the closed form of the Lagrangian and to its renormalizability in the ordinary sense. The situation is of interest, when the coefficients for the higher invariants are not arbitrary, but are determined by the coefficients for lower ones. Physically it means dynamical restoration of a symmetry group which is more wide than the initial one. The role of the carrier of this symmetry would play the bound state of initial fields. In the simplest case this possibility was already analysed in paper /7/, where the dynamical appearance of the isoscalar σ -field was supposed, but ended in failure. However, in the consideration of more complicated variant of the σ -model, including, for example, isotensor σ -fields, the one-loop approximation can be interpreted in this manner. The question on its validity in the two-loop and higher approximations is now under consideration.

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