ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E2 - 10645

E.Sokatchev

COMMENTS ON A ONE-DIMENSIONAL SUPERGRAVITY MODEL

1977

E2 - 10645

E.Sokatchev

COMMENTS ON A ONE-DIMENSIONAL SUPERGRAVITY MODEL

Submitted to "Physics Letters"

Сокачев Э.С. E2 - 10645

Замечания к одномерной модели супергравитации

Одномерная модель супергравитации Бринка и др. выведена с помощью идеи о супертоке. Найдена минимальная группа инвариантности теории, содержащая гравитацию и суперсимметрию. Обсуждается незамыкание алгебры локальной суперсимметрии.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1977

Sokatchev E. E2 - 10645

Comments on a One-Dimensional Supergravity Model

The one-dimensional supergravity model of Brink et al. [1] is derived using the supercurrent concept. The simplest invariance group containing gravity and supersymmetry is found. The non-closing algebra of local supersymmetry is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1977

The superfield (SF) formulation of supergravity has widely been discussed and several approaches to it have been proposed [2,3,4]. In ref. [1] an interesting one-dimensional SF model of supergravity has been constructed using superspace geometric technique [2,4]. Although physically unrealistic, this model provides a simple and clear framework for testing and comparing the different approaches. In this letter we derive once more the results of ref. [1] using the supercurrent method of ref. [3]. We also make some comments on the problem of nonclosing algebra in local supersymmetry [5].

To start with, we consider a free massless SF in one-dimensional space (or two-dimensional superspace)

$$\Phi(x,\theta) = \phi(x) + \theta\psi(x). \tag{1}$$

It is not hard to see that the only action yielding reasonable equations for the component bosonic (fermionic) fields $\phi(\psi)$ is the following

$$S_0 = \int dx d\theta\, D\Phi . \partial\Phi . \tag{2}$$

Here $\partial \equiv \partial_x$ and

$$D = \partial_\theta + i\theta\partial_x \qquad (3)$$

is the ordinary flat space spinor derivative. It is interesting that the same term $D\Phi.\partial\Phi$ plays also the role of supercurrent [6]. Indeed, the free equation for Φ yields the following conservation law

$$\partial(D\Phi\partial\Phi) = 0 \qquad (4)$$

which contains the conservation laws of the energy-momentum tensor and the Noether supersymmetry current for the fields ϕ, ψ.

The main feature which distinguishes the gravitational SF from matter ones is its gauge nature. In our extremely poor case the "gravitational" SF $h(x,\theta)$ must be gauge degree of freedom as a whole. In other words, there is no free equation for h and the interaction of h with matter SF has to possess invariance under the following gauge transformation

$$\delta h = -\partial\lambda + \kappa\,\mathrm{m.t.} \qquad (5)$$

Here

$$\lambda = a(x) + \theta\epsilon(x) \qquad (6)$$

is an arbitrary SF (in Eq. (5) - $\partial\lambda$ is introduced instead of λ for convenience) and κ m.t. stands for multiplicative terms of the first and higher order in the coupling constant κ.

Now, following the concept of ref. [3] we can couple the gravitational SF h to the matter one Φ in the lowest order in κ through the supercurrent

$$S_{int} = \int dx d\theta (D\Phi.\partial\Phi + \kappa D\Phi.\partial\Phi.h + O(\kappa^2)) . \qquad (7)$$

In order to achieve invariance of this action under transformations (5), the SF Φ has to transform correspondingly. The only transformation which cancels the κ^1 variation of (7) is easily derived

$$\delta\Phi = \kappa [\lambda\partial\Phi - \frac{i}{2} D\lambda . D\Phi] . \qquad (8)$$

The most remarkable property of these transformations is that they form a group. Indeed,

$$[\delta_1, \delta_2]\Phi = \kappa[\lambda^{br}\partial\Phi - \frac{i}{2} D\lambda^{br} . D\Phi] \qquad (9)$$

with a Lie-bracket parameter

$$\lambda^{br} = \kappa(\lambda_1\partial\lambda_2 - \lambda_2\partial\lambda_1 - \frac{i}{2} D\lambda_1 D\lambda_2). \qquad (10)$$

Further, ordinary flat space supersymmetry is included in Eq. (8):

$$\delta\Phi = \epsilon i(\partial_\theta - i\theta\partial_x) \qquad (11)$$

when $a = 0$ and ϵ = const in Eq. (6). The commutator of two local $\epsilon(x)$-transformations gives pure general coordinate transformation with parameter $a^{br} = -2i\epsilon_1\epsilon_2$.

Note that the same supercurrent method applied to the four-dimensional case [3] leads to analogous results for the chiral matter SF. The corresponding transformations are

$$\delta\Phi = \kappa[\lambda^\mu \partial_\mu \Phi + \frac{i}{8}(\bar{D}\tilde{\sigma}_\mu)^\alpha \lambda^\mu D_\alpha \Phi] . \qquad (12)$$

They form also a group and contain flat supersymmetry explicitly.

To proceed further, it is natural to suppose that the multiplicative terms in Eq.(5) are of the same type as in Eq.(8)

$$\delta h = -\partial\lambda + \kappa(\lambda\partial h - \frac{i}{2} D\lambda.\partial h). \qquad (13)$$

They prove to form the same group (10). The next step is to calculate the κ^2 variation of action (7) and thus we easily deduce the κ^2 interaction term

$$\frac{1}{2}\kappa^2 D\Phi.\partial\Phi.h^2. \qquad (14)$$

Then a simple recurrence formula leads to the complete action invariant under (8) and (13)

$$S_{int} = \int dx d\theta D\Phi.\partial\Phi.e^{\kappa h}. \qquad (15)$$

Putting

$$\Phi \equiv X, \ \lambda \equiv \xi^{\mu}, \ \epsilon \equiv -i\beta, \ e^{-\kappa h} \equiv \Lambda \qquad (16)$$

one can identify our final formulae (6), (8),(13),and (15) with the corresponding ones of ref.[1] though the latter are derived by quite different technique. There the starting point is a curved superspace (x, θ). Gravity is carried by four SF (vierbeins) and the initial invariance group (acting both in curved and tangent spaces) has six parameters. In order to limit this superfluous freedom, one has to fix the gauge properly and to restrict the transformations according to the fixed gauge. Note that the group structure (9),(10) has been overlooked in ref.[1] for the following reason. The aim of the authors has been to identify the transformations (8) and (13) with the common local supersymmetry ones [5]. So they have made a reparametrization

$$\beta \equiv i\epsilon \to a = e^{1/2}\beta, \qquad (17)$$

where e is the bosonic component of the SF Λ (16). It is clear now that this artificial field dependence of the parameters causes the well-known complications: the commutator of two local supersymmetry transformations yields general cooordinate transformation plus field dependent supersymmetry one. We think that this fact can throw new light on the algebraic structure of local supersymmetry.

In conclusion we wish to summarize what the simple model of ref.[1] demonstrates. The superspace geometric approach to supergravity supplies an inordinately wide framework. There exists a smaller subgroup and a minimal set of fields which provide the most economical description of supergravity with clear algebraic structure. The key to such a formulation may be the supercurrent concept.

I would like to acknowledge V.I.Ogievetsky for valuable remarks. I aslo thank E.A.Ivanov for helpful discussions.

REFERENCES

1. L.Brink,S.Deser,B.Zumino,P.DiVecchia, and P.Howe.Phys.Lett.,64B (1976)435.
2. D.V.Volkov and V.P.Akulov.JETP Lett., 18 (1973)312;
R.Arnowitt and P.Nath.Phys.Lett.,56B (1975)117;

R.Arnowitt,P.Nath, and B.Zumino.Phys.Lett., 56B(1975)81.

3. V.Ogievetsky and E.Sokatchev.Proceedings of the IV International Conference on Nonlocal and Nonlinear Field Theory, Alushta, April 1976 (p.183); JINR, E2-9985,Dubna (1976).

4. B.Zumino. CERN preprint TH 2120 (1975); V.Akulov, D.Volkov, and V.Soroka.JETP Lett., 22(1975)396; V.Akulov, D.Volkov, and V.Soroka. Kharkov preprint HFTI 77-10 (1977); J.Wess and B.Zumino. Phys. Lett., 66B(1977)361.

5. D.Z.Freedman,S.Ferrara, and P.van Nieuwenhuizen. Phys.Rev.,D13(1976)3214; S.Deser and B.Zumino.Phys.Lett., 62B(1976) 335.

6. S.Ferrara and B.Zumino.Nucl.Phys.,87B (1975)207.

Received by Publishing Department
on May 4, 1977