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# RELATIVISTIC STRING WITH MASSES AND CHARGES AT THE ENDS IN THE CONSTANT HOMOGENEOUS ELECTROMAGNETIC FIELD

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# RELATIVISTIC STRING WITH MASSES AND CHARGES AT THE ENDS IN THE CONSTANT HOMOGENEOUS ELECTROMAGNETIC FIELD

Барбашов Б.М. и др. **E2 - 10626**

Релятивистская струна с массами и зарядами на концах в постоянном однородном электромагнитном поле

Получены классические решения для струны с массами и зарядами на концах в постоянном однородном электромагнитном поле. Рассматривается такой класс движений, когда параметр временной эволюции $\tau$ пропорционален собственному времени каждой из частиц на концах струны. Решениями уравнений движения и граничных условий оказываются почти периодические функции, полученные в виде рядов Фурье. Связи на фурье-амплитуды возникают как следствие ортогональной калибровки и выбранной параметризации. Найдены масса и полный канонический импульс струны.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Barbashov B.M. et al. **E2 - 10626**

Relativistic String with Masses and Charges at the Ends in the Constant Homogeneous Electromagnetic Field

The classical solution for the string with masses and charges at the ends in the constant homogeneous electromagnetic field is obtained. The dynamical equations are solved for the class of motions when the time evolution parameter $\tau$ is proportional to the proper times of the string ends.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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Relativistic string with masses and charges at the ends is one of the possible models for the quark confinement in hadrons [1]. The string models with massive ends or with charged ends only were considered in papers [2-4].

The masses at the string ends result in the non-linear boundary conditions which do not allow us to obtain the general solutions. However, this problem can be solved when we restrict ourselves to the motions of the string with massive ends for which the time evolution parameter $\tau$ is proportional to the proper times of the string ends [4].

In this paper there is considered just this class of the motions of the string which has, in addition to the masses, the charges at its ends and is placed in an external constant homogeneous electromagnetic field.

The solutions of the equations of motion obeying the linear boundary conditions are almost periodic functions which can be represented in terms of Fourier series. Restriction on the normal modes are the consequence of the orthonormal gauge and the used parametrization. The mass and the canonical momentum of the string are obtained. The transition to the quantum theory cannot be made as the eigenfunctions of the boundary problem are not orthogonal.

The action of the relativistic string with masses $m_i$ and charges $g_i$ at the ends placed in the electromagnetic field $F_{\mu\nu}(x)=\partial_\mu A_\nu(x)-\partial_\nu A_\mu(x)$ has the form

$$S=-\gamma\int_{\tau_1}^{\tau_2}d\tau\int_{\sigma_1}^{\sigma_2}d\sigma\sqrt{(\dot{x}x')^2-\dot{x}^2x'^2}-\sum_{i=1}^{2}\int_{\tau_1}^{\tau_2}d\tau\left\{m_i\sqrt{\dot{x}^2(\sigma_i,\tau)}+g_i\dot{x}_\nu(\sigma_i,\tau)A_\nu(x)\right\}, \quad (1)$$

where $\sigma_1=0,\ \sigma_2=\ell$ , $\gamma$ is a constant with dimension $L^{-2}$ , $\dot{x}_\nu=\partial x_\nu(\sigma,\tau)/\partial\tau,\ x'_\nu=\partial x_\nu(\sigma,\tau)/\partial\sigma$ .

The variation of the action (1) results in the equations of motion

$$\ddot{x}_\mu(\sigma,\tau)-x''_\mu(\sigma,\tau)=0 \quad (2)$$

and in the nonlinear boundary conditions

$$\begin{aligned} m_1\frac{\partial}{\partial\tau}\left(\frac{\dot{x}_\nu}{\sqrt{\dot{x}^2}}\right)&=\gamma x'_\nu+g_1F_{\nu\mu}\dot{x}_\mu,\ \sigma=\sigma_1, \\ m_2\frac{\partial}{\partial\tau}\left(\frac{\dot{x}_\nu}{\sqrt{\dot{x}^2}}\right)&=-\gamma x'_\nu+g_2F_{\nu\mu}\dot{x}_\mu,\ \sigma=\sigma_2. \end{aligned} \quad (3)$$

In addition two subsidiary conditions $(\dot{x}\pm x')^2=0$ have been imposed on the searched solutions $x_\mu(\sigma,\tau)$ as well as in a free string case.

Further we shall consider only such string motions for which the parameter $\tau$ is proportional to the proper times of the string ends[4], i.e.,

$$\dot{x}^2(\sigma_i,\tau)=\frac{C_i^2}{m_i^2},\ i=1,2. \quad (4)$$

Here $C_i$ are arbitrary constants. In this case eqs. (3) become linear

$$\begin{aligned} \ddot{x}_\nu(0,\tau)&=q_1\left[x'_\nu(0,\tau)+\frac{g_1}{\gamma}F_{\nu\mu}\dot{x}_\mu(0,\tau)\right], \\ \ddot{x}_\nu(\ell,\tau)&=-q_2\left[x'_\nu(\ell,\tau)-\frac{g_2}{\gamma}F_{\nu\mu}\dot{x}_\mu(\ell,\tau)\right], \end{aligned} \quad (5)$$

where

$$q_i=\frac{C_i\gamma}{m_i^2},\ i=1,2.$$

To solve eqs. (2) and (5) we use for $x_\mu(\sigma,\tau)=\Psi_{1\mu}(\sigma+\tau)+\Psi_{2\mu}(\sigma-\tau)$ the Fourier integrals

$$\Psi_{1\mu}(\lambda)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}d\omega\, e^{-i\omega\lambda}\widetilde{\Psi}_{1\mu}(\omega),\quad \Psi_{2\mu}(\lambda)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}d\omega\, e^{i\omega\lambda}\widetilde{\Psi}_{2\mu}(\omega).$$

Further for simplicity we put $m_1=m_2=m$, $g_1=-g_2=g$ and denote $f_{\mu\nu}=\frac{g}{\gamma}F_{\mu\nu}$. Substituting $\Psi_{1\mu}$ and $\Psi_{2\mu}$ into (5) gives the following equations for functions $\widetilde{\Psi}_1$ and $\widetilde{\Psi}_2$:

$$\begin{gathered}[i\omega+q(1+f)]_{\mu\nu}\widetilde{\Psi}_{1\nu}(\omega)=[-i\omega+q(1-f)]_{\mu\nu}\widetilde{\Psi}_{2\nu}(\omega),\\ [-i\omega+q(1+f)]_{\mu\nu}\widetilde{\Psi}_{1\nu}(\omega)=[i\omega+q(1-f)]_{\mu\nu}\widetilde{\Psi}_{2\nu}(\omega)\,e^{2i\omega\ell}.\end{gathered}\tag{6}$$

This set of homogeneous equations has non-trivial solutions $\widetilde{\Psi}_{1\mu}(\omega)$ and $\widetilde{\Psi}_{2\mu}(\omega)$ if the frequencies $\omega$ are roots of the transcendental equation

$$\mathrm{tg}\,\omega\ell=\frac{2\omega q}{\omega^2-q^2(1-d)},\tag{7}$$

where
$$d=\frac{1}{2}\frac{g^2}{\gamma^2}\left[E^2-H^2\pm\sqrt{(H^2-E^2)^2+4(\vec{E}\vec{H})^2}\right].$$

$\omega$ are symmetrical with respect to zero therefore these may be numbered so that $\omega_0=0$, $\omega_{-n}=-\omega_n$, $n=1,2,\ldots$. As a result, the Fourier integral representations of $\Psi_{1\mu}(\lambda)$ and $\Psi_{2\mu}(\lambda)$ reduce to the Fourier series. And now $x_\mu(\sigma,\tau)$ can be expanded as follows

$$x_\mu(\sigma,\tau)=(1-f^2)^{-1}_{\mu\nu}Q_\nu+\frac{qP_\mu\tau}{\gamma(2+q\ell)}-\frac{qf_{\mu\nu}P_\nu\sigma}{\gamma(2+q\ell)}+\frac{i}{\sqrt{2\gamma}}\sum_{\substack{n=-\infty\\ n\neq0}}^{+\infty}\frac{1}{\omega_n}\mathcal{V}_{n\mu\nu}(\sigma)\alpha_{n\nu}e^{-i\omega_n\tau},\tag{8}$$

where $\mathcal{V}_{n\mu\nu}(\sigma)=\delta_{\mu\nu}\cos\omega_n\sigma-\left(\frac{\omega_n}{q}-if\right)_{\mu\nu}\sin\omega_n\sigma$, $\alpha_{-n}=\alpha_n^*$.

The Fourier amplitudes $\alpha_n$ in expansion (8) must obey the equations:

$$(d-f^2)_{\mu\nu}\alpha_{n\nu}=0, \ \mu,\nu=0,1,2,3. \qquad (9)$$

For $q\to\infty$, $\omega_n\to\frac{n\pi}{\ell}$ and solution (8) turns into the solution to the massless string in a constant homogeneous electromagnetic field[2]. On the other hand, for $f_{\mu\nu}=0$ we obtain solution for the relativistic string with point masses attached to the ends[4].

The number of linearly independent solutions of eqs. (9) is equal to $4-r$, where $r$ is the rank of the matrix $(d-f^2)_{\mu\nu}$. We can show that $r=2$ except for the case when electric and magnetic fields are equal in magnetude and perpendicular to each other ( $E^2-H^2=0$ , $\vec{E}\vec{H}=0$ ). In the last case $r=1$.

So, the vector $\alpha_{n\mu}$ can be represented as follows:

$$\alpha_{n\mu}=\sum_{i=1}^{4-r}C_n^i\mathcal{E}^i_\mu,$$

where $\mathcal{E}^i_\mu$ are the linearly independent solutions of eqs. (9).

It follows from the expansion (8) that the internal string excitations in the case under consideration has the property which is similar to the polarization property of the electromagnetic field. The number of different "polarization" states of the string excitation is always less than four, that means the internal degrees of freedom of the string in a constant homogeneous electromagnetic field decrease frmm 4 to $4-r$.

The subsidiary conditions result in the constraints on amplitudes $\alpha_n$

$$\alpha_{n\mu}\alpha_{m\mu}=0, \ \alpha_{n\mu}f_{\mu\nu}\alpha_{m\nu}=0, \ n\neq-m, \ n,m\neq0, \qquad (10)$$

$$\alpha_{n\mu}P_\mu=0,\ \alpha_{n\mu}f_{\mu\nu}P_\nu=0,\ n\neq 0, \qquad (11)$$

$$P_\mu(1-f^2)_{\mu\nu}P_\nu=-\frac{\gamma}{2}\frac{(2+q\ell)}{q^2}\sum_{n\neq 0}\left[(1-d+\frac{\omega_n^2}{q^2})|\alpha_n|^2+\frac{2i\omega_n}{q}\alpha_{n\mu}f_{\mu\nu}\alpha_{-n\nu}\right]. \qquad (12)$$

In addition it is necessary that at the string ends the conditions (4) hold which lead to the equalities:

$$P^2=-\frac{\gamma}{2}\left(\frac{2}{q}+\ell\right)^2\sum_{n\neq 0}|\alpha_n|^2+4m^2\left(1+\frac{q\ell}{2}\right)^2, \qquad (13)$$

$$\sum_{n\neq 0}\frac{\omega_n}{R_n}\left\{2\omega_n q d|\alpha_n|^2-i[\omega_n^2+q^2(1-d)]\alpha_{n\mu}f_{\mu\nu}\alpha_{-n\nu}\right\}=0, \qquad (14)$$

where $R_n=\omega_n^4+q^4(1-d)^2+2\omega_n^2q^2(1+d)$.

The conserved total canonical momentum of the string with massive ends in the constant homogeneous electromagnetic field is of the form

$$\Pi_\mu=\int_0^\ell \pi_\mu(\sigma,\tau)d\sigma=\frac{\gamma}{q}[\dot{x}_\mu(0,\tau)+\dot{x}_\mu(\ell,\tau)]+\gamma\int_0^\ell(\dot{x}_\mu+f_{\mu\nu}x'_\nu)d\sigma, \qquad (15)$$

where $\pi_\mu(\sigma,\tau)=-\dfrac{\partial\mathcal{L}}{\partial\dot{x}_\mu}$ - is the canonical momentum density.

Inserting the expansion (8) into (15) gives

$$\Pi_\mu=\left(1-\frac{q\ell}{2+q\ell}f^2\right)_{\mu\nu}P_\nu. \qquad (16)$$

It follows from formula (16) that vector $P_\mu$ is conserved as well as $\Pi_\mu$. According to the expansion (8) we can consider the vector $P_\mu$ as the total mechanical momentum of the string. Using eq. (13), we obtain for the squared mass of the string in the external electromagnetic field the following expression

$$M^2=P^2=-\frac{\gamma}{2}\left(\frac{2}{q}+\ell\right)^2\sum_{n\neq 0}|\alpha_n|^2+4m^2\left(1+\frac{q\ell}{2}\right)^2. \qquad (17)$$

If the string does not vibrate then its squared mass differs, nevertheless, from $4m^2$ :

$$M_o^2 = 4m^2(1+\frac{ql}{2})^2.$$

In terms of the harmonic oscillator amplitudes $a_n = \alpha_n/\sqrt{\omega_n}$ , $a_{-n} = a_n^*,\ n>0$ , we can represent eq. (17) as follows:

$$M^2 = -\gamma(\frac{2}{q}+l)^2 2\sum_{n=1}^{\infty}\omega_n a_n^* a_n + 4m^2(1+\frac{ql}{2})^2.$$

The roots of eq. (7) are not divisible by $\overset{\circ}{\omega} = \pi/l$ and, as a result, a greater part of degeneracy of the mass spectrum is removed.

The obtained classical solutions for the string with masses and charges at the ends in the external electromagnetic field do not allow us to quantize this system because the functions $\vartheta_{n\mu\nu}(\sigma)$ are not orthonormal.

The above solutions show that the introduction of the charges and masses into the relativistic string model changes essentially the mass spectrum, the degeneracy of this spectrum being removed. In addition, there is a possibility to specify the internal string excitations similar to the description of the polarization states of the classical electromagnetic field.

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