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## ON THE TWO- AND THREE-POINT FUNCTIONS FOR CONFORMAL SUPERFIELDS



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# ON THE TWO- AND THREE-POINT FUNCTIONS FOR CONFORMAL SUPERFIELDS

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О двух- и трехточечных функциях конформных суперполей

В работе рассмотрен вопрос о построении двух- и трех-точечных функций конформных суперлолей, преобразующихся по ранее найденным представлениям конформной супералгебры. В отличие от предыдущих работ эдесь не предполагается инкаких соотношений между параметрами d и z, характеризующими данное представление этой алгебры.

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On the Two- and Three-Point Functions for Conformal Superfields

The two- and three-point functions for conformal superfields transforming according to the previously found representations of the conformal superalgebra are constructed. Unlike the examples given in the previous papers no relations betweeen the parameters characterizing a given representation (d and z) are supposed.

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#### INTRODUCTION

In the present paper we construct the invariant two- and three-point functions for superfields transforming according to representations of the conformal superalgebra introduced in paper /1/. We recall that an arbitrary representation of the mentioned kind is determined by its Lorentz structure and by two complex numbers d and z. The corresponding generators of the representation are their differential operators in the space of functions of the variables  $x_{\mu}$ ,  $\theta_{a}^{+}$ ,  $\xi_{a}^{-}$ , where  $x_{\mu}$  are the coordinates of a point in the Minkovski space, and  $\theta_{a}^{+}$  and  $\xi_{a}^{-}$  are spinor mutually anticommuting variables satisfying the relations.

$$\frac{1}{2}[(1+iy_5)\theta^+]_a = \theta^+_a - \frac{1}{2}[(1-iy_5)\xi^-]_a = \xi^-_a.$$

In what follows we shall use the "nonphysical" form of the conformal superalgebra generators defined in ref.  $^{/3/}$ . We write down once more these generators for convenience :

 $\mathbf{P}_{\mu} = -\mathbf{i}\partial_{\mu} - \mathbf{i}\xi \,\overline{\gamma}^{\circ}\gamma_{\mu}\gamma^{\circ}\frac{\partial}{\partial\eta^{+}},$ 

$$\begin{split} S_{a}^{+} &= i(\gamma^{\circ}\frac{\partial}{\partial \eta^{+}})_{a} \quad , \qquad T_{a}^{-} &= i(\gamma^{\circ}\frac{\partial}{\partial \xi^{-}})_{a}, \\ T_{a}^{+} &= 8(x\gamma^{\nu}\eta^{+})_{a}\partial_{\nu} + (8d + 12z)\eta_{a}^{+} - 16\eta_{a}^{+}\xi^{-}\frac{\partial}{\partial \xi^{-}} - \\ &- 8(\gamma^{\mu}\xi^{-})_{a}[(2x_{\mu}x_{\nu} - x^{2}g_{\mu\nu})\partial^{\nu} + 2dx_{\mu}] - 8i\Sigma_{\mu\nu}(\sigma^{\mu\nu}\eta^{+})_{a} - \\ &- 8(\eta^{+}\gamma^{\circ}\eta^{+})(\gamma^{\circ}\frac{\partial}{\partial \eta^{+}})_{a} - 8i\Sigma_{\mu\nu}[(x^{\mu}\gamma^{\nu} - x^{\nu}\gamma^{\mu})\xi^{-}]_{a}, \\ \Pi &= -z + \eta^{+}\frac{\partial}{\partial \eta^{+}} + \xi^{-}\frac{\partial}{\partial \xi^{-}}, \\ S_{a}^{-} = 8(\gamma^{\nu}\eta^{+})_{a}\partial_{\nu} - 8(\gamma^{\nu}x\xi^{-})_{a}\partial_{\nu} - \\ &- (8d - 12z)\xi_{a}^{-} - 16\xi_{a}^{-}\eta^{+}\frac{\partial}{\partial \eta^{+}} - 8(\xi^{-}\gamma^{\circ}\xi^{-})(\gamma^{\circ}\frac{\partial}{\partial \xi^{-}})_{a} - \\ &- 8i\Sigma_{\mu\nu}(\sigma^{\mu\nu}\xi^{-})_{a}, \\ D &= -i\{d + x^{\mu}\partial_{\mu} + \frac{1}{2}\eta^{+}\frac{\partial}{\partial \eta^{+}} - \frac{1}{2}\xi^{-}\frac{\partial}{\partial \xi^{-}}\}, \\ M_{\mu\nu} &= \Sigma_{\mu\nu} + i\{x_{\mu}\partial_{\nu} - x_{\nu}\partial_{\mu} - \eta^{+}\gamma^{\circ}\sigma_{\mu\nu}\gamma^{\circ}\frac{\partial}{\partial \eta^{+}} - \\ &- \xi^{-}\gamma^{\circ}\sigma_{\mu\nu}\gamma^{\circ}\frac{\partial}{\partial \xi^{-}}\}, \\ K_{\mu} &= 2x^{\nu}\Sigma_{\mu\nu} + i\{(2x_{\mu}x_{\nu} - x^{2}g_{\mu\nu})\partial^{\nu} + 2x_{\mu}d\} - \\ &- i\eta^{+}\gamma^{\circ}\gamma_{\mu}\gamma^{\circ}\frac{\partial}{\partial \xi^{-}}, \end{split}$$

where

$$\eta_{\alpha}^{+} = \theta_{\alpha}^{+} + (\hat{\mathbf{x}}\xi^{-})_{\alpha} ,$$
  
$$\xi_{\alpha}^{-} = \xi_{\alpha}^{-} .$$

Constructing the two- and three-point functions we distinguish two cases:

a) In the first one an arbitrary two- or three-point function is constructed out of the superfields as well as of their conjugates. This case is discussed in the second section.

b) In the second case all two- and threepoint functions are constructed out either of the superfields only or of their conjugate fields only. We discuss this type of functions in the third section.

2. We introduce, first of all, the following notation. In accordance with paper  $^{/3/}$ , a superfield with arbitrary Lorentz structure is denoted by

$$\Phi_{\{\alpha_{p}\}} \| \beta_{q} \| (\mathbf{x}, \eta^{+}, \xi^{-}), \qquad (2.1)$$

where  $\{a_p\} = \{a_1, ..., a_p\}$   $\{\beta_q\} = \{\beta_1 ... \beta_q\}$ , and the brackets denote full symmetrization within the group of indices. The following identities are supposed to hold:

$$\frac{1}{2}(1+i\gamma_5)\beta_k\beta'_k \Phi_{\{\alpha_p\}}[\beta_q(-k)]\beta'_k\} =$$

$$= \frac{1}{2} (1 - i\gamma_5) a_k a'_k \Phi_{\{a_p(-k) | a'_k\} \{\beta_q\}} = 0,$$

where

$$\{\alpha_{p}(-k) | \alpha_{k}^{\prime}\} = \{\alpha_{1}, ..., \alpha_{k-1}, \alpha_{k}^{\prime}, \alpha_{k+1}, ..., \alpha_{p}\},$$

$$\{\beta_{q}(-k) | \beta_{k}^{\prime}\} = \{\beta_{1}, ..., \beta_{k-1}, \beta_{k}^{\prime}, \beta_{k+1}, ..., \beta_{q}\}.$$
(2.2)

Sometimes, when writing the full index structure is not necessary, we shall just write  $\Phi$ , meaning by "A" the hole group of indices ( $\{a_{p}\}, \beta_{q}\}$ ). The field transforming according to the conjugate representation is denoted by  $\Phi_{\{a_{p}\}, \beta_{q}\}}(x, \eta^{+}, \xi^{-})$  and  $\Phi_{A}(x, \eta^{+}, \xi^{-})$ , respectively\* Let

$$\Delta_{AB}(\mathbf{x}_{1}, \mathbf{x}_{2}, \eta_{1}^{+}, \eta_{2}^{-}, \xi_{1}^{-}, \xi_{2}^{+}) =$$

$$= \langle 0 | \Phi_{A}(\mathbf{x}_{1}, \eta_{1}^{+}, \xi_{1}^{-}) \widetilde{\Phi}_{B}(\mathbf{x}_{2}, \eta_{2}^{-}, \xi_{2}^{+}) | 0 \rangle$$

$$(2.3)$$

and

$$\Gamma_{ABC}(\mathbf{x}_{1},\mathbf{x}_{2},\mathbf{x}_{3},\eta_{1}^{+},\eta_{2}^{+},\eta_{3}^{-},\xi_{1}^{-},\xi_{2}^{-},\xi_{3}^{+}) = (2.4)$$
$$= <0 |\Phi_{A}(\mathbf{x}_{1},\eta_{1}^{+},\xi_{1}^{-})\Phi_{B}(\mathbf{x}_{2},\eta_{2}^{+},\xi_{2}^{-})\tilde{\Phi}_{C}(\mathbf{x}_{3},\eta_{3}^{-},\xi_{3}^{+})|0>$$

are the two- and three-point functions, respectively \*.

<sup>\*</sup>Remark. We have written here the twoand three-point functions as functions of the nonphysical variables. In order to define the "physical" two- and three-point functions it is necessary to make the corresponding change of variables (see<sup>/3/</sup>) in formulae(2.3) and (2.4).

Producing an infinitesimal transformation of the superfields and taking into account the invariance of the vacuum state under the action of these transformations, a system of differential equations is obtained for the functions (2.3) and (2.4). As in papers  $^{/1,2,3/}$ , i

it is sufficient to examine the equations, corresponding to the generators  $S_{a}^{\pm}$ ,  $T_{a}^{\pm}$  only. We start with the two-point function. The corresponding equations have the following form:

$$\{i(\gamma^{\circ}\frac{\partial}{\partial\eta_{1}^{+}})_{\alpha}\delta_{BB}, + (S_{2\alpha}^{+})_{BB}\}\Delta_{AB}, = 0, \qquad (2.5)$$

$$\{i(\gamma^{\circ}\frac{\partial}{\partial \eta_{2}})_{\alpha} \delta_{AA}, + (S_{1\alpha})_{AA}, \} \Delta_{A'B} = 0, \qquad (2.6)$$

$$\{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{1}})_{\alpha} \delta_{BB'} + (T_{2\alpha})_{BB'}\} \Delta_{AB'} = 0, \qquad (2.7)$$

$$\{i(\gamma^{o} \frac{\partial}{\partial \xi_{2}^{+}})_{\alpha} \delta_{AA'} + (T_{1\alpha}^{+})_{AA} \lambda \Delta_{A'B} = 0, \qquad (2.8)$$

where  $S_{1\alpha}$ ,  $T_{1\alpha}^+$  are the generators of the representation under which the superfield 

ing solution:

$$\Delta_{AB}(\mathbf{x}_{1}, \mathbf{x}_{2}, \eta_{1}^{+}, \eta_{2}^{-}, \xi_{1}^{-}, \xi_{2}^{+}) = (2.9)$$
  
=  $\exp\{-i\eta_{2}^{-}\gamma^{\circ}S_{1}^{-} - i\xi_{2}^{+}\gamma^{\circ}T_{1}^{+}\}_{AA'}D_{A'B}(\mathbf{x}_{1}, \mathbf{x}_{2}, \eta_{1}^{+}, \xi_{1}^{-}),$ 

where  $D_{AB}$  is an unknown function. The exponent in the R.H.S. of equality (2.9) is determined as the global transformation produced over the function  $D_{AB}$  (see paper<sup>/3/</sup> and appendix I). We did not separate the finite dimensional part of this transformation since at this time it is not necessary.

Later on we shall have to determine the function  $D_{A'B}$  from equations (2.5) and (2.7). A system of equations for  $D_{A'B}$  (which we do not write here) after some algebraic manipulations (we must commute the differential operators of the equations with the exponent) is obtained. The latter shows that:

a)  $D_{AB}$  does not depend on  $\eta_1^+$  and  $\xi_1^-$ ; b) A nontrivial solution can exist only if

 $z_1 = z_2$   $d_1 = d_2$ ; (2.10)

c) The function  $D_{AB}$  satisfied the equations of ordinary conformal invariance. Then, a nontrivial solution exists if and only if

$$\mathbf{p} = \mathbf{s} \quad \mathbf{q} = \mathbf{r} \quad (2.11)$$

If all these conditions hold the function  $D_{AB}$  is determined up to an arbitrary constant and has the following form:

$$\times (\hat{\mathbf{x}}_{12} \gamma^{\circ} (1 - i\gamma_5)) \{\beta_q\} \{\gamma_q\},$$

$$\mathbf{x}_{12} = \mathbf{x}_1 - \mathbf{x}_2,$$
(2.12)

where

$$\hat{\mathbf{x}}_{12} \{\alpha_{p}\} \{\delta_{p}\} = \sum \prod_{i=1}^{p} \hat{\mathbf{x}}_{12\alpha_{i}\delta_{i}}$$
$$\hat{\mathbf{x}}_{12} \{\beta_{q}\} \{\gamma_{q}\} = \sum \prod_{i=1}^{q} \hat{\mathbf{x}}_{12\beta_{i}\gamma_{i}}$$

and summation runs through all permutations of the indices  $a_i$  and  $\beta_i$ , respectively. Thus, the solution is determined. To obtain its explicit form it is necessary to produce the global transformation (2.9) over the function  $D_{AB}$  (formula (2.12)). It is more convenient to do the latter, if we return to the physical variables:

$$\theta_{1a}^{+} = \eta_{1a}^{+} - (\hat{x}_{1}\xi_{1}^{-})_{a} \qquad \theta_{2a}^{-} = \eta_{2a}^{-} - (\hat{x}_{2}\xi_{2}^{+})_{a}$$

$$\xi_{1a}^{\prime-} = \xi_{1a}^{-} \qquad \xi_{2a}^{\prime+} = \xi_{2a}^{+} .$$
(2.13)

Since the general case is rather cumbersome, we confine ourselves to two examples.

a) Two-point function of scalar superfields  $(d = d_1 = d_2; z = z_1 = z_2)$ 

$$\Delta(\mathbf{x}_{1},\mathbf{x}_{2},\theta_{1}^{+},\theta_{2}^{-},\xi_{1}^{-},\xi_{2}^{+}) = C|Y_{12}^{2}|^{-d} \times \\ \times (1-16i\xi_{2}^{+}\gamma^{\circ}\theta_{1}^{+})^{-\frac{1}{2}(d-\frac{3}{2}z)} \quad (1-16i\theta_{2}^{-}\gamma^{\circ}\xi_{1}^{-})^{-\frac{1}{2}(d-\frac{3}{2}z)} \times$$

$$h(z_{12},\xi_1^-,\xi_2^+,\theta_2^-)^{-\frac{1}{2}(d-\frac{3}{2}z)},$$
 (2.14)

where

$$Y_{12\mu} = x_{12\mu} - 8i\theta_{2}^{-}\gamma^{\circ}\gamma_{\mu}\theta_{1}^{+},$$

$$Z_{12\mu} = (1 - 16i\xi_{2}^{+}\gamma^{\circ}\theta_{1}^{+})^{1/2}Y_{12\mu}\Lambda_{\mu}^{\nu}(\xi_{2}^{+},\theta_{1}^{+}), \qquad (2.15)$$

$$\Lambda_{\mu}^{\nu}(\lambda_{2}^{+},\theta_{1}^{+}) = \delta_{\mu}^{\nu} - 16i\xi_{2}^{+}\gamma^{\circ}\sigma_{\mu}^{\nu}\theta_{1}^{+} - 48\delta_{\mu}^{\nu}\theta_{1}^{+}\gamma^{\circ}\theta_{1}^{+}\xi_{2}^{+}\gamma^{\circ}\xi_{2}^{+},$$

$$h(z_{12}^{-},\xi_{1}^{-},\xi_{2}^{+},\theta_{2}^{-}) = 1 - 16i\xi_{2}^{+}\gamma^{\circ}\hat{z}_{12}\xi_{1}^{-} - 128\xi_{2}^{+}\gamma^{\circ}\hat{z}_{12}\theta_{2}^{+}\xi_{1}^{+}\gamma^{\circ}\xi_{1}^{-}. \qquad (2.16)$$

b) Two-point function of the spinor superfields\*

\*Remark: Note that the functions (2.14), (2.17) reduce to the corresponding functions of superfields, belonging to the invariant subspaces, if a relation between d and z holds under which an invariant subspace exists, although we do not suppose the existence of an invariant subspace in the present paper.

$$\Phi_{a}^{+}(x_{1},\theta_{1}^{+},\xi_{1}^{-}) \text{ and } \tilde{\Phi}_{\beta}^{+}(x_{2}^{-},\theta_{2}^{-},\xi_{2}^{+});(1+iy_{5})\Phi^{-}=(1-iy_{5})\Phi^{+}=0$$

$$\Delta_{a\beta}^{-+}(x_{1},x_{2}^{-},\theta_{1}^{+},\theta_{2}^{-},\xi_{1}^{-},\xi_{2}^{+}) = C|Y_{12}|^{-d-\frac{1}{2}} \times (1-16i\xi_{2}^{+}\gamma^{\circ}\theta_{2}^{+})^{-\frac{1}{2}\cdot(d-\frac{3}{2}\cdot z)} = C|Y_{12}|^{-d-\frac{1}{2}} \times (1-16i\xi_{2}^{+}\gamma^{\circ}\theta_{2}^{+})^{-\frac{1}{2}\cdot(d-\frac{3}{2}\cdot z)} (1-16i\theta_{2}^{-}\gamma^{\circ}\xi_{1}^{-})^{-\frac{1}{2}\cdot(d-\frac{3}{2}\cdot z)}(2\cdot17) \times h(z_{12}^{-},\xi_{1}^{-},\theta_{2}^{-},\xi_{2}^{+})^{-\frac{1}{2}\cdot(d-1-\frac{3}{2}\cdot z)} (\hat{Y}_{12}^{-}\gamma^{\circ}(1-iy_{5}^{-}))_{\alpha\beta},$$

where  $Y_{12}$ ,  $Z_{12}$  and h are determined by formulae (2.14), (2.15) and (2.16), respectively.

Now we pass to the three-point function  $\Gamma_{ABC}$  . The corresponding equations have the form:

$$\begin{split} & \{i[(\gamma^{\circ}\frac{\partial}{\partial\eta_{1}^{+}})_{a} + (\gamma^{\circ}\frac{\partial}{\partial\eta_{2}^{+}})_{a}]\delta_{CC} + (S_{3a}^{+})_{CC}, \{\Gamma_{ABC}, = 0, (2.18)\} \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\eta_{3}^{-}})_{a}\delta_{AA}, \delta_{BB}' + (S_{12a}^{-})_{(AA')(BB')}\}\Gamma_{A'B'C} = 0, (2.19) \\ & \{i[(\gamma^{\circ}\frac{\partial}{\partial\xi_{1}^{-}})_{a} + (\gamma^{\circ}\frac{\partial}{\partial\xi_{2}}), ]\delta_{CC}' + (T_{3a}^{-})_{CC}, \{\Gamma_{ABC}' = 0, (2.20)\} \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{AA'}, \delta_{BB'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{AA'}, \delta_{BB'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{AA'}, \delta_{BB'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{AA'}, \delta_{BB'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{AA'}, \delta_{BB'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{AA'}, \delta_{BB'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{AA'}, \delta_{BB'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{AA'}, \delta_{BB'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{AA'}, \delta_{BB'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{AA'}, \delta_{BB'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{AA'}, \delta_{BB'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{AA'}, \delta_{BB'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{AA'}, \delta_{B'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{A'}, \delta_{A'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{A'}, \delta_{A'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{A'}, \delta_{A'} + (T_{12a}^{+})_{(AA')(BB')}, ]\Gamma_{A'B'C} = 0, (2.21) \\ & \{i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{+}})_{a}\delta_{A'}, \delta_{A'} + (T_{12a}^{+})_{a}\delta_{A'},$$

equations (2.19) and (2.21) have the following solution:

 $\Gamma_{ABC}(\mathbf{x}_{1}, \mathbf{x}_{2}, \mathbf{x}_{3}, \eta_{1}^{+}, \eta_{2}^{+}, \eta_{3}^{-}, \xi_{1}^{-}, \xi_{2}^{-}, \xi_{3}^{+}) =$ 

$$= \exp\{-i\eta_{3}^{-}\gamma^{\circ}S_{12}^{-} - i\xi_{3}^{+}\gamma^{\circ}T_{12}^{-}\}_{(AB)(A'B')} \mathcal{I}_{A'B'C}^{\circ}(x_{1}, x_{2}, x_{3}, \eta_{1}^{+}, \eta_{2}^{+}, \xi_{1}^{-}, \xi_{2}^{-}),$$

(2.22)

where  $\mathcal{I}_{ABC}$  is an unknown function. Acting on (2.22) with equations (2.18) and (2.20), after some algebraic manipulations, we obtain for the function  $\mathcal{I}_{ABC}^{\circ}(\mathbf{x}_{1},\mathbf{x}_{2},\mathbf{x}_{3},\eta_{1}^{+},\eta_{2}^{+},\xi_{1}^{-},\xi_{2}^{-})$ a system of equations. The latter shows that:  $\mathbf{x}_{a}^{-} = \frac{1}{2}(\eta_{1a}^{+} - \eta_{2a}^{+}),$  (2.23)  $\Psi_{a}^{-} = \frac{1}{2}(\xi_{1a}^{-} - \xi_{2a}^{-}).$  (2.24)

b) A nontrivial solution can exist only if the quantity  $z = z_1 + z_2 - z_3$  obtains one of the following values:

z = 0, 1, 2, 3, 4. (2.25)

c) Under these conditions the function  $\mathcal{J}_{ABC}^{\circ}$  must satisfy the equations corresponding to the conformal subalgebra. The latter show that in the cases z = 1,2,3 a nontrivial three-point function does not exist. In the other two cases, i.e., z = 0,4 a nontrivial function exists provided that the following equality

 $p_1 + p_2 + p_3 = q_1 + q_2 + q_3$  (2.26) holds. If all these conditions are satisfied the function  $\mathcal{T}^\circ_{ABC}$  in the case z=0 is determined as

 $\mathcal{I}_{ABC}^{\circ}(\mathbf{x}_{1},\mathbf{x}_{2},\mathbf{x}_{3},\chi^{+},\Psi^{-}) = \mathcal{I}_{ABC}^{\circ}(\mathbf{x}_{1}-\mathbf{x}_{3},\mathbf{x}_{2}-\mathbf{x}_{3}), (2.27)$ 

where  $\Im_{ABC}^{\circ}$  is the ordinary conformal invariant three point function for fields with dimensions  $d_1$ ,  $d_2$  and  $d_3$ , respectively, while in the case z = 4, we have

 $\mathcal{I}_{ABC}(x_1, x_2, x_3, \chi^+, \Psi^-) = \mathcal{I}_{ABC}^{\circ}(x_1 - x_3, x_2 - x_3) \chi^+ \gamma^{\circ} \chi^+ \Psi^- \gamma^{\circ} \Psi^-,$ ere  $\mathcal{I}_{DC}^{\circ}$  is determined as in the pre-

where  $\mathcal{T}^{\circ}_{ABC}$  is determined as in the previous case.

Thus, the solution of the equations for the superconformal invariant three point functions is completely determined\* . In order to obtain the explicit form of the solution it is necessary to produce the global transformation (2.21). As in the case of the two-point function it is convenient to do this in terms of the "physical" variables. After this procedure the operator exponent preserves its form while the generators  $S_{12}^{-}$ and  $T_{12}^{+}$  act on the differences  $x_{13}$  and  $x_{23}^{-}$ only.

We give two examples:

a) Three-point function of scalar superfields. In the case z = 0, we have

\*Conformal invariant three point functions for frelds with an arbitrary spin were presented in several works, see f.i. .

$$\begin{split} &\Gamma(\mathbf{x}_{1},\mathbf{x}_{2},\mathbf{x}_{3},\theta_{1}^{+},\theta_{2}^{+},\theta_{3}^{-},\xi_{1}^{-},\xi_{2}^{-},\xi_{3}^{+}) = \\ &= C|\mathbf{Y}_{12}|^{\frac{1}{2}|(\mathbf{d}_{3}-\mathbf{d}_{1}-\mathbf{d}_{2})} |\mathbf{Y}_{23}|^{\frac{1}{2}(\mathbf{d}_{1}-\mathbf{d}_{2}-\mathbf{d}_{3})} |\mathbf{Y}_{13}|^{\frac{1}{2}|(\mathbf{d}_{2}-\mathbf{d}_{1}-\mathbf{d}_{3})} \times \\ &\times (\mathbf{1}-\mathbf{1}6i\theta_{3}^{-}\gamma^{\circ}\xi_{1}^{-})^{-\frac{1}{2}\cdot(\mathbf{d}_{1}^{-},\frac{3}{2}\cdot\mathbf{z})} (\mathbf{1}-\mathbf{1}6i\theta_{3}^{-}\gamma^{\circ}\xi_{2}^{-})^{-\frac{1}{2}\cdot(\mathbf{d}_{2}^{-},\frac{3}{2}\cdot\mathbf{z})} \times \\ &\times (\mathbf{1}-\mathbf{1}6i\xi_{3}^{+}\gamma^{\circ}\theta_{1}^{+})^{\frac{1}{2}\cdot(\mathbf{d}_{2}-\mathbf{d}_{3}^{-},\frac{3}{2}\cdot\mathbf{z})} (\mathbf{1}-\mathbf{1}6i\xi_{3}^{+}\gamma^{\circ}\theta_{2}^{+})^{\frac{1}{2}\cdot(\mathbf{d}_{1}-\mathbf{d}_{3}^{-},\frac{3}{2}\cdot\mathbf{z})} \times \\ &\times (\mathbf{1}-\mathbf{1}6i\xi_{3}^{+}\gamma^{\circ}\theta_{1}^{+})^{\frac{1}{2}\cdot(\mathbf{d}_{2}-\mathbf{d}_{3}^{-},\frac{3}{2}\cdot\mathbf{z})} (\mathbf{1}-\mathbf{1}6i\xi_{3}^{+}\gamma^{\circ}\theta_{2}^{+})^{\frac{1}{2}\cdot(\mathbf{d}_{1}-\mathbf{d}_{3}^{-},\frac{3}{2}\cdot\mathbf{z})} \times \\ &\times \mathbf{h}(\mathbf{z}_{13}^{-},\xi_{1}^{-},\xi_{3}^{+},\theta_{3}^{-})^{-\frac{1}{2}\cdot(\mathbf{d}_{2}^{-},\frac{3}{2}\cdot\mathbf{z})} \times \\ &\times \mathbf{h}(\mathbf{z}_{23}^{-},\xi_{2}^{-},\xi_{3}^{+},\theta_{3}^{-})^{-\frac{1}{2}\cdot(\mathbf{d}_{2}^{-},\frac{3}{2}\cdot\mathbf{z})} \times \\ &\times \mathbf{h}(\mathbf{z}_{23}^{-},\xi_{2}^{-},\xi_{3}^{+},\theta_{3}^{-})^{-\frac{1}{2}\cdot(\mathbf{d}_{2}^{-},\frac{3}{2}\cdot\mathbf{z})} \times \\ &Y_{13\mu} = \mathbf{x}_{1\mu} - \mathbf{x}_{3\mu} - \mathbf{8}i\theta_{3}^{-}\gamma^{\rho}\gamma_{\mu}\theta_{1}^{+}, \qquad \mathbf{Y}_{12\mu} = \mathbf{Y}_{13\mu} - \mathbf{Y}_{23\mu}, \\ &\mathbf{Y}_{23\mu} = \mathbf{x}_{2\mu} - \mathbf{x}_{3\mu} - \mathbf{8}i\theta_{3}^{-}\gamma^{\rho}\gamma_{\mu}\theta_{2}^{+}, \\ &\mathbf{Z}_{13\mu} = (\mathbf{1}-\mathbf{1}6i\xi_{3}^{+}\gamma^{\rho}\theta_{1}^{+})^{1/2} \quad \mathbf{Y}_{13\nu}\Lambda_{\mu}^{\nu}(\xi_{3}^{+},\theta_{1}^{+}), \\ &\mathbf{Z}_{23\mu} = (\mathbf{1}-\mathbf{1}6i\xi_{3}^{+}\gamma^{\rho}\theta_{2}^{+})^{1/2} \quad \mathbf{Y}_{23\nu}\Lambda_{\nu}^{\nu}(\xi_{3}^{-},\theta_{2}^{+}), \end{split}$$

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while in the case 
$$z = 4$$
, we have  

$$\Gamma(x_{1}, x_{2}, x_{3}, \theta_{1}^{+}, \theta_{2}^{+}, \theta_{3}^{-}, \xi_{1}^{-}, \xi_{2}^{-}, \xi_{3}^{+}) =$$

$$=\Gamma_{0}(x_{1}, x_{2}, x_{3}, \theta_{1}^{+}, \theta_{2}^{+}, \theta_{3}^{-}, \xi_{1}^{-}, \xi_{2}^{-}, \xi_{3}^{+}) \times$$

$$\times(\xi_{12}^{-}, \gamma^{\circ}\xi_{12}^{-})(\theta_{12}^{+}, \gamma^{\circ}\theta_{12}^{+} + 2\theta_{12}^{+}, \gamma^{\circ}\hat{x}_{12} \times x_{12}^{-} -$$

$$(2.30)$$

$$-x_{12}^{2}, \chi_{12}^{-}, \gamma^{\circ}\chi_{12}^{-}),$$

$$\xi_{12}^{-} = \frac{1}{2}(\xi_{1}^{-} - \xi_{2}^{-}),$$

$$\chi_{12}^{-} = \frac{1}{2}(\xi_{1}^{-} - \xi_{2}^{-}),$$

$$\chi_{12}^{-} = \frac{1}{2}(\theta_{1}^{+} - \theta_{2}^{+}), / x_{12}^{-} \frac{1}{2}(x_{1}^{-} - x_{2}^{-}),$$
where  $\Gamma_{0}$  is determined by formula (2.29).  
b) Three-point function of one scalar  
and two spinor superfields of the kind  

$$<0|\Phi(x_{1}, \theta_{1}^{+}, \xi_{1}^{-})\Psi_{a}^{+}(x_{2}, \theta_{2}^{+}, \xi_{2}^{-})\widetilde{\Psi}_{\beta}^{-}(x_{3}, \theta_{3}^{-}, \xi_{3}^{+})|0>.$$
(2.31)  
In the case  $z = 0$ , we have  

$$\Gamma_{0\alpha\beta}^{+-}(x_{1}, x_{2}, x_{3}, \theta_{1}^{+}, \xi_{1}^{-}, \theta_{2}^{+}, \xi_{2}^{-}, \theta_{3}^{-}, \xi_{3}^{+}) =$$

í

 $= C |Y_{12}^2|^{1/2(d_3^{-d_1^{-d_2^2}}} |Y_{13}^2|^{1/2(d_2^{-d_1^{-d_3^2}})} |Y_{23}^2|^{1/2(d_1^{-d_2^{-d_3^{-1}}})} \times$ 

•

$$\times (1 - 16i\theta_{3}^{-}\gamma^{\rho}\xi_{1}^{-})^{-\frac{1}{2}(d_{1} - \frac{3}{2}z_{1})} (1 - 16i\theta_{3}^{-}\gamma^{\rho}\xi_{2}^{-})^{-\frac{1}{2}(d_{2} - \frac{3}{2}z_{2})} \times \\ \times (1 - 16i\xi_{3}^{+}\gamma^{\rho}\theta_{1}^{+})^{-\frac{1}{2}(d_{2} - d_{3} - \frac{3}{2}z_{1})} \times \\ \times (1 - 16i\xi_{3}^{+}\gamma^{\rho}\theta_{2}^{-})^{-\frac{1}{2}(d_{1} - d_{3} - 1 - \frac{3}{2}z_{2})} (2 \cdot 32) \times \\ \times (1 - 16i\xi_{3}^{+}\gamma^{\rho}(\theta_{1}^{+}-\theta_{2}^{+}))^{-\frac{1}{2}(d_{3} - d_{1} - d_{2})} h(z_{13}, \xi_{1}^{-}, \xi_{3}^{+}, \theta_{3}^{-})^{-\frac{1}{2}(d_{1} - \frac{3}{2}z_{1})} \times \\ \times h(z_{23}, \xi_{2}^{-}, \xi_{3}^{+}, \theta_{3}^{-})^{-\frac{1}{2}(d_{2}^{-} - \frac{3}{2}z_{2})} [\hat{Y}_{23}\gamma^{\rho}(1 + i\gamma_{5})]_{\alpha\beta} ,$$

while in the case z=4, we have

$$\Gamma_{\alpha\beta}^{+-}(\mathbf{x}_{1},\mathbf{x}_{2},\mathbf{x}_{3},\theta_{1}^{+},\theta_{2}^{+},\theta_{3}^{-},\xi_{1}^{-},\xi_{2}^{-},\xi_{3}^{-}) =$$

$$= \Gamma_{0\alpha\beta}^{+-}(\mathbf{x}_{1},\mathbf{x}_{2},\mathbf{x}_{3},\theta_{1}^{+},\theta_{2}^{+},\theta_{3}^{-},\xi_{1}^{-},\xi_{2}^{-},\xi_{3}^{-}) \times$$

$$\times (\xi_{12}^{-}\gamma^{\circ}\xi_{12}^{-})(\theta_{12}^{+}\gamma^{\circ}\theta_{12}^{+} + 2\theta_{12}^{+}\gamma^{\circ}\hat{\mathbf{x}}_{12}\chi_{12}^{-} +$$

$$+ \mathbf{x}_{12}^{2}\chi_{12}^{-}\gamma^{\circ}\chi_{12}^{-}).$$

3. In this section we find the two- and three-point functions constructed from  $\Phi_{A}(\mathbf{x},\eta^{+},\xi^{-})$  . (In the case when these functions are constructed from the conjugated fields only, all the results remain the same with the substitution  $\eta^+ \rightarrow \eta^-, \xi^- \rightarrow \xi^+$ ).

Let us denote by

$$\Delta_{AB}(x_{1}, x_{2}, \eta_{1}^{+}, \eta_{2}^{+}, \xi_{1}^{-}, \xi_{2}^{-}) =$$

$$= \langle 0 | \Phi_{A}(x_{1}, \eta_{1}^{+}, \xi_{1}^{-}) \Phi_{B}(x_{2}, \eta_{2}^{+}, \xi_{2}^{-}) | 0 \rangle$$
and
$$(3.1)$$

· <u>\*</u> . .

$$\Gamma_{ABC}(x_{1}, x_{2}, x_{3}, \eta_{1}^{+}, \eta_{2}^{+}, \eta_{3}^{+}, \xi_{1}^{-}, \xi_{2}^{-}, \xi_{3}^{-}) = (3.2)$$

$$= \langle 0 | \Phi_{A}(x_{1}, \eta_{1}^{+}, \xi_{1}^{-}) \Phi_{B}(x_{2}, \eta_{2}^{+}, \xi_{2}^{-}) \Phi_{C}(x_{3}, \eta_{3}^{+}, \xi_{3}^{-}) | 0 \rangle$$

the two-and three-point functions, respectivelv.

Performing the infinitesimal transformations and taking into account the invariance of the vacuum state with respect to the superalgebra, we obtain a system of differential equationd for the functions  $\Delta_{AB}$  and  $\Gamma_{ABC}$ 

$$[L_{1} + L_{2}]\Delta_{AB}(x_{1}, x_{2}, \eta_{1}^{+}, \eta_{2}^{+}, \xi_{1}^{-}, \xi_{2}^{-}) = 0$$

(3.3)

 $[L_{1}+L_{2}+L_{3}]\Gamma_{ABC}(\mathbf{x}_{1},\mathbf{x}_{2},\mathbf{x}_{3},\eta_{1}^{+},\eta_{2}^{+},\eta_{3}^{+},\xi_{1}^{-},\xi_{2}^{-},\xi_{3}^{-}) = 0,$ 

where L, is any arbitrary generator of the algebra (1.1).

Consider, first the two-point function  $\Delta_{AB}$  . As a result of S<sup>+</sup>-invariance

$$[i(\gamma^{\circ}\frac{\partial}{\partial \eta_{1}^{+}})_{a}+i(\gamma^{\circ}\frac{\partial}{\partial \eta_{2}^{+}})_{a}]\Delta_{AB}(x_{1},x_{2},\eta_{1}^{+},\eta_{2}^{+},\xi_{1}^{-},\xi_{2}^{-})=0 \quad (3.4)$$

and T<sup>-</sup>-invariance

$$[i(\gamma^{\circ}\frac{\partial}{\partial\xi_{1}})_{\alpha} + i(\gamma^{\circ}\frac{\partial}{\partial\xi_{2}})_{\alpha}]\Delta_{AB}(x_{1},x_{2},\eta_{1}^{+},\eta_{2}^{+},\xi_{1}^{-},\xi_{2}) = 0 (3.5)$$

it has to be a function of  $\eta_{12}^+ = \eta_1^+ - \eta_2^+$  and  $\xi_{\overline{12}}^- = \xi_{\overline{1}}^- - \xi_{\overline{2}}^-$ . The condition for  $P_{\mu}$ -invariance

$$[i\partial_{\mu_{1}} + i\partial_{\mu_{2}} + i\xi_{1} \gamma^{\circ} \gamma_{\mu} \gamma^{\circ} \frac{\partial}{\partial \eta^{+}} + i\xi_{2} \gamma^{\circ} \gamma_{\mu} \gamma^{\circ} \frac{\partial}{\partial \eta^{+}}]\Delta_{AB} = 0.$$
(3.6)

gives:

$$\Delta_{AB} = \Delta_{AB} (z_{12}, u_{12}^+, \xi_{12}^-) \quad z_{12} = x_1 - x_2, \ u_{12}^+ = \eta_{12}^+ - \hat{x}_1 \xi_{12}^-.$$

Further we restrict ourselves to the case of scalar and spinor superfields.

A. Two-point function of scalar superfields

$$\Delta(\mathbf{z}_{12},\mathbf{u}_{12}^{+},\boldsymbol{\xi}_{12}^{-}) = \langle 0 | \Phi(\mathbf{x}_{1},\boldsymbol{\eta}_{1}^{+},\boldsymbol{\xi}_{1}^{-}) \Phi(\mathbf{x}_{2},\boldsymbol{\eta}_{2}^{+},\boldsymbol{\xi}_{2}^{-}) | 0 \rangle.$$

From the equation for II-invariance

$$\left[-z_{1}-z_{2}+u_{12}^{+}\frac{\partial}{\partial u_{12}^{+}}+\xi_{12}^{-}\frac{\partial}{\partial \xi_{12}^{-}}\right]\Delta = 0 \qquad (3.8)$$

it follows that the expansion of the function  $\Delta$  in powers of the spinor variables contains no zero degree term, with respect to these variables \*,  $z = z_1 + z_2$  takes the values z = 2,4, both cases should be treated separately.

a) 
$$z = z_1 + z_2 = 2$$
.

The most general form of a Lorentz-invariant scalar two-point function  $\Delta(z_{12}, u_{12}^+, \xi_{12}^-)$  is

$$\Delta = A(z_{12}^{2})u_{12}^{+}\gamma^{\circ}u_{12}^{+} + B(z_{12}^{2})u_{12}^{+}\gamma^{\circ}\hat{z}_{12}^{-}\xi_{12}^{-} + C(z_{12}^{2})\xi_{12}^{-}\gamma^{\circ}\xi_{12}^{-},$$

Now we make use of the equation for  $S^-$ -in-variance:

$$[8(\gamma^{\nu}\eta_{1}^{+})_{a} \partial_{\nu}^{1} + 8(\gamma^{\nu}\eta_{2}^{+})_{a} \partial_{\nu}^{2} - 8(\gamma^{\nu}\hat{x}_{1}\xi_{1}^{-})_{a} \partial_{\nu}^{1} - 8(\gamma^{\nu}\hat{x}_{2}\xi_{2}^{-})_{a} \partial_{\nu}^{2} - (3.9) - \xi_{1a}^{-}(8d_{1}^{-}-12z_{1}^{-}) - \xi_{2a}^{-}(8d_{2}^{-}-12z_{2}^{-}) - 16\xi_{1a}^{-}\eta_{1}^{+} \frac{\partial}{\partial\eta_{1}^{+}} - (3.9) - 16\xi_{2a}^{-}\eta_{2}^{+} \frac{\partial}{\partial\eta_{1}^{+}} - 16\xi_{2a}^{-}\eta_{2}^{-} \frac{\partial}{\partial\xi_{2}^{-}}] \Delta = 0$$

\*This is also valid in the general case of n-point function constructed from the superfields only (or from the conjugated ones only) with arbitrary Lorentz structure. which together with  $K_{\mu}$ -invariance:

$$\{i[2x_{\mu}^{1}x_{\nu}^{1} - x_{1}^{2}g_{\mu\nu}]\partial_{1}^{\nu} + i[2x_{\mu}^{2}x_{\nu}^{2} - x_{2}^{2}g_{\mu\nu}]\partial_{2}^{\nu} + + 2ix_{\mu}^{1}d_{1} + 2ix_{\mu}^{2}d_{2} - i\eta_{1}^{+}\gamma^{\circ}\gamma_{\mu}\gamma^{\circ}\frac{\partial}{\partial\xi_{1}^{-}} - - i\eta_{2}^{+}\gamma^{\circ}\gamma_{\mu}\gamma^{\circ}\frac{\partial}{\partial\xi_{2}^{-}}\}\Delta = 0$$

$$(3.10)$$

gives that the function  $\Lambda$  does not depend explicitly on  $\xi_{12}^-$ , and that  $d_1 - d_2 = -1 d_1 = 3/2z_1$ . The D-invariance condition

$$[z_{12}^{\nu}\partial_{\nu}^{12} + d_1 + d_2 + \frac{1}{2}u_{12}^{+}\frac{\partial}{\partial u_{12}^{+}}]\Delta(z_{12}^{+}, u_{12}^{+}) = 0 \quad (3.11)$$
  
leads to the following solution for the function

$$\Delta(z_{12}, u_{12}^{+}) = C|z_{12}^{2}|^{-1/2(d+1)} u_{12}^{+} \gamma^{\circ} u_{12}^{+}, d_{1} - d_{2}^{-1} d_{1} = 3/2z_{1}$$

$$z_{1} + z_{2}^{-2} 2 \qquad (3.12)$$

In the special case when  $d_1 = 3/2z_1$ ,  $d_2 = 3/2z_2$  $d = d_1 + d_2 = 3$ 

$$\Delta(\mathbf{z}_{12},\mathbf{u}_{12}^+) = C\delta^4(\mathbf{z}_{12})\mathbf{u}_{12}^+\gamma^\circ\mathbf{u}_{12}^+. \qquad (3.13)$$

b) 
$$z = z_1 + z_2 = 4$$
.

In this case the most general form of a Lorentz-invariant function is

$$\Delta(z_{12}, u_{12}^+, \xi_{12}^-) = A(z_{12}^2) u_{12}^+ \gamma^{\circ} u_{12}^+ \xi_{12}^- \gamma^{\circ} \xi_{12}^-. \quad (3.14)$$

The equation for  $S^-$ -invariance is identically satisfied by this function, and  $K_{\mu}$  and D -invariance lead to the following results:

$$C|z_{12}^{2}|^{-d}u_{12}^{+}\gamma^{\circ}u_{12}^{+}\xi_{12}^{-}\gamma^{\circ}\xi_{12}^{-}, d_{1}=d_{2}=d$$

$$\Delta(z_{12},u_{12}^{+},\xi_{12}^{-})=\{0 \quad d_{1}\neq d_{2}$$

$$C\delta(z_{12})u_{12}^{+}\gamma^{\circ}u_{12}^{+}\xi_{12}^{-}\gamma^{\circ}\xi_{12}^{-}, d_{1}+d_{2}=4.$$

B. Two-point function for spinor superfields (spin 1/2)

 $\begin{array}{l} \Delta_{\alpha\beta}(z_{12}\,,u_{12}^{+},\xi_{12}^{-})=<0|\Phi_{\alpha}(x_{1},\eta_{1}^{+},\xi_{1}^{-})\Phi_{\beta}(x_{2},\eta_{2}^{+},\xi_{2}^{-})|0>. \\ (3.16)\\ \text{In this case the number } z=z_{1}+z_{2} \text{ takes the values } z=1,2,3,4. \text{ But the odd values } z==1,3 \text{ have to be excluded because otherwise one has to construct spinor coefficient functions in the expansion of } \Delta_{\alpha\beta} \text{ form the } 4-\text{vector } z_{12u}^{-}. \end{array}$ 

a)  $z = z_1 + z_2 = 2$ 

It can directly be verified that the equations for  $S_{\alpha}^{-}$  and  $K_{\mu}$  invariance admit only a trivial solution.

b)  $z = z_1 + z_2 = 4$ 

The  $S_{\alpha}^{-}$ ,  $K_{\mu}$  and D - invariance equations lead to the following result

$$\Delta_{\alpha\beta}(z_{12}, u_{12}^{+}, \xi_{12}^{-}) = [C_{1}(\gamma^{\circ}(1 - i\gamma_{5}))_{\alpha\beta} + C_{2}(\gamma^{\circ}(1 + i\gamma_{5}))_{\alpha\beta}] \times \delta^{4}(z_{12})u_{12}^{+}\gamma^{\circ}u_{12}^{+}\xi^{-}\gamma^{\circ}\xi_{12}^{-}, \quad d_{1} + d_{2} = 4.$$
(3.17)

Consider now the 3- point function. Analogously  $S_a^+$ -invariance

$$\begin{bmatrix} i(\gamma^{\circ} \frac{\partial}{\partial \eta_{1}^{+}})_{a} + i(\gamma^{\circ} \frac{\partial}{\partial \eta_{2}^{+}})_{a} + i(\gamma^{\circ} \frac{\partial}{\partial \eta_{3}^{+}})_{a} \end{bmatrix} \Gamma_{ABC} = 0 \quad (3.18)$$
  
T<sup>-</sup> -invariance

$$\left[i(\gamma^{\circ}\frac{\partial}{\partial\xi_{1}^{-}})_{a}+i(\gamma^{\circ}\frac{\partial}{\partial\xi_{2}^{-}})_{a}+i(\gamma^{\circ}\frac{\partial}{\partial\xi_{3}^{-}})_{a}\right]\Gamma = 0 \quad (3.19)$$

and  $P_{\mu}$  -invariance

$$\begin{bmatrix} i\partial_{\mu_{1}} + i\partial_{\mu_{2}} + i\partial_{\mu_{3}} + i\xi_{1}\overline{\gamma}^{\circ}\gamma_{\mu}\gamma^{\circ}\frac{\partial}{\partial\eta_{1}^{+}} + i\xi_{2}\overline{\gamma}^{\circ}\gamma_{\mu}\gamma^{\circ}\frac{\partial}{\partial\eta_{2}^{+}} + i\xi_{3}\overline{\gamma}^{\circ}\gamma_{\mu}\gamma^{\circ}\frac{\partial}{\partial\eta_{3}^{+}} \end{bmatrix} \Gamma_{ABC} = 0$$

lead to the following dependence for the function  $\Gamma_{ABC}$  from the variables  $x_i$ ,  $\eta_i^+$ ,  $\xi_i^-$ , i=1,2,3

$$\Gamma_{ABC} = \Gamma_{ABC} (z_{12}, z_{23}, u_{12}^{+}, u_{23}^{+}, \xi_{12}^{-}, \xi_{23}^{-}),$$

$$z_{12} = x_{1} - x_{2}, \quad u_{12}^{+} = \eta_{12}^{+} - \hat{x}_{1}\xi_{12}^{-},$$

$$\eta_{12}^{+} = \eta_{1}^{+} - \eta_{2}^{+}, \quad \xi_{12}^{-} = \xi_{1}^{-} - \xi_{2}^{-},$$

$$z_{23} = x_{2} - x_{3}, \quad u_{23}^{+} = \eta_{23}^{+} - \hat{x}_{3}\xi_{23}^{-},$$

$$\eta_{23}^{+} = \eta_{2}^{+} + \eta_{3}^{+}, \quad \xi_{23}^{-} = \xi_{2}^{-} - \xi_{3}^{-}.$$
(3.21)

A. Three-point function of scalar superfields It follows from II-invariance that  $z=z_1+z_2+z_3$  takes the values z = 2,4,6,8.a) z = 2

From  $S_a^-$  and  $K_{\mu}$ -invariance it follows that the function  $\Gamma$  does not depend explicitly on  $\xi_{12}^-$  and  $\xi_{23}^-$ . And the equations for  $S_a^-$ ,  $K_{\mu}$  and D-invariance have a nontrivial solution

 $\Gamma(z_{12}, z_{23}, u_{12}^+, u_{23}^+, \xi_{12}^-, \xi_{23}^-) = C(z_{12}^2)^{d_3 - 2} (z_{23}^2)^{d_1 - 2} \times (3.22) \times (z_{13}^2)^{d_2 - 2} \{ -\frac{1}{2} z_{23}^2 u_{12}^+ \gamma^{\circ} u_{12}^+ - \frac{1}{2} z_{12}^2 u_{23}^2 \gamma^{\circ} u_{23}^+ + u_{12}^+ \gamma^{\circ} \hat{z}_{12} \hat{z}_{23}^2 u_{23}^+ \hat{z}_{33}^+ \}$ 

only if

$$d_{1} + d_{2} + d_{3} = 3$$

$$d_{i} = 3/2 z_{i}, i = 1,2,3.$$

$$b) z = 4$$

$$\Gamma(z_{12}, z_{23}, u_{12}^{+}, u_{23}^{+}, \xi_{12}^{-}, \xi_{23}^{-}) = C |z_{12}^{2}|^{-1/2} (d_{1} + d_{2} - d_{3})$$

$$\times |z_{23}^{2}|^{-1/2} (d_{2}^{+} d_{3}^{-} d_{1})|z_{13}^{2}|^{-1/2} (d_{1} + d_{3}^{-} d_{2}^{+2})$$

$$\times u_{12}^{+} \gamma^{\circ} u_{12}^{+} u_{23}^{+} \gamma^{\circ} u_{23}^{+} ,$$

$$2d_{1} = 3z_{1}, 2d_{3} = 3z_{3},$$

c) 
$$z = 6$$
  
 $\Gamma(z_{12}, z_{23}, u_{12}^+, u_{23}^+, \xi_{12}^-, \xi_{23}^-) = C_1 |z_{12}^2|^{-1/2(d_1 + d_2 - d_3 - 1)} \times |z_{23}^2|^{-1/2(d_2 + d_3 - d_1 + 1)} |z_{13}^2|^{-1/2(d_1 + d_3 - d_2 + 1)} \times |z_{23}^2|^{-1/2(d_1 + d_2 - d_3 + 1)} |z_{23}^2|^{-1/2(d_2 + d_3 - d_1 - 1)} \times |z_{23}^2|^{-1/2(d_2 + d_3 - d_1 - 1)} \times |z_{13}^2|^{-1/2(d_1 + d_2 - d_3 + 1)} |z_{23}^2|^{-1/2(d_2 + d_3 - d_1 - 1)} \times |z_{13}^2|^{-1/2(d_1 + d_3 - d_2 + 1)} u_{12}^+ y^\circ u_{12}^+ \xi_{23}^- y^\circ \xi_{23}^- u_{23}^+ y^\circ u_{23}^+, d_1 = 3/2 z_1, d_3 = 3/2 z_3, d_1 = 3/2 z_3, d_2 = 0.$   
d)  $z = 8$   
 $\Gamma(z_{12}, z_{23}, u_{12}^+, u_{23}^+, \xi_{12}^-, \xi_{23}^-) = C |z_{12}^2|^{-1/2(d_1 + d_2 - d_3)} \times |z_{23}^2|^{-1/2(d_2 + d_3 - d_1)} |z_{13}^2|^{-1/2(d_1 + d_3 - d_2)} \times |z_{23}^2|^{-1/2(d_2 + d_3 - d_1)} |z_{13}^2|^{-1/2(d_1 + d_3 - d_2)} \times |u_1^+ y^\circ u_{12}^+ u_{23}^+ y^\circ u_{23}^+ \xi_{12}^- y^\circ \xi_{12}^- \xi_{23}^- y^\circ \xi_{23}^- .$ 

B. Three point function of spinor superfields (spin 1/2).

From the condtion for  $\Pi$ -invariance it follows that  $z = z_1 + z_2 + z_3$  should take the values z = 1, 2, 3, 4, 5, 6, 7, 8. However, it can be seen immediately that  $\Gamma_{ABC} \equiv 0$  for even z,

if

as for these values it is impossible to construct the corresponding expansion of  $\Gamma_{ABC}$ in powers of the spinor variable. But it can be verified by direct calculation that  $\Gamma_{ABC} = 0$ also for the odd values of  $\Gamma$  z, because the equations for  $S_{\alpha}$  and  $K_{\mu}$  invariance have no other solution but the trivial one.

### Appendix

Here we give the formulae for the global transformations in terms of the "physical" variables (see ref.  $^{3/}$ ). We begin with the transformations of the variables.

a. Global transformation with the parameter  $\beta_{\overline{a}}$ , corresponding to the generator  $\overline{S_{a}}$ 

 $\begin{aligned} \mathbf{x}_{\mu} &\to \mathbf{x}_{\mu} + 8\mathbf{i}\beta^{-}\gamma^{\circ}\gamma_{\mu}\theta^{+}, \\ \boldsymbol{\xi}_{a}^{-} &\to \boldsymbol{\xi}_{a}^{-} + 8\mathbf{i}\boldsymbol{\xi}^{-}\gamma^{\circ}\boldsymbol{\xi}^{-}\beta_{a}^{-}, \\ \boldsymbol{\theta}_{a}^{+} &\to \boldsymbol{\theta}_{a}^{+}. \end{aligned}$ 

b. Global transformation with the parameter  $\beta_{\alpha}^{+}$  corresponding to the generator  $T_{\alpha}^{+}$ 

$$\begin{aligned} \mathbf{x}_{\mu} &\to (\mathbf{1} + \mathbf{16i}\beta^{+}\gamma^{\circ}\theta^{+})^{1/2} \mathbf{x}_{\nu}\Lambda^{\nu}_{\mu}(\beta^{+},\theta^{+}) = \mathbf{y}_{\mu}, \\ \theta^{+}_{a} &\to \theta^{+}_{a} + \mathbf{8i}\theta^{+}\gamma^{\circ}\theta^{+}\beta^{+}_{a}, \\ \xi^{-}_{a} &\to (\mathbf{1} + \mathbf{16i}\beta^{+}\gamma^{\circ}\theta^{+})^{-1} (\xi^{-}_{a} - \mathbf{8i}\xi^{-}\gamma^{\circ}\xi^{-}(\hat{\mathbf{y}}\beta^{+})_{a}), \end{aligned}$$

where

$$\Lambda^{\nu}_{\mu}(\beta^{+},\theta^{+}) = \delta^{\nu}_{\mu} + 16i\beta^{+}\gamma^{\circ}\sigma^{\nu}_{\mu}\theta^{+} + 48\delta^{\nu}_{\mu}\beta^{+}\gamma^{\circ}\beta^{+}\theta^{+}\gamma^{\circ}\theta^{+}.$$

Superfield depending on these variables transforms as follows

$$e^{i\beta^{-}\gamma^{\circ}S^{-}} f_{(d,z)} (x,\theta^{+},\xi^{-}) = (1+16i\beta^{-}\gamma^{\circ}\xi^{-})^{-} \frac{1}{2} (d-\frac{3}{2}z) \times$$

$$\times f_{(d,z)} (x_{\mu} + 8i\beta^{-}\gamma^{\circ}\gamma_{\mu}\theta^{+},\theta_{\alpha}^{+},\xi_{\alpha}^{-} + 8i\xi^{-}\gamma^{\circ}\xi^{-}\beta_{\alpha}^{-}) ,$$

$$e^{i\beta^{+}\gamma^{\circ}T^{+}} f_{(d,z)} (x_{\mu},\theta_{\alpha}^{+},\xi_{\alpha}^{-}) = (1+16i\beta^{+}\gamma^{\circ}\theta^{+}) \frac{1}{2} (d+\frac{3}{2}z) \times$$

$$\times (1+16i\beta^{+}\gamma^{\circ}\hat{y}\xi^{-})^{-} \frac{1}{2} (d-\frac{3}{2}z) f_{(d,z)} (y_{\mu},\theta_{\alpha}^{+} + 8i\theta^{+}\gamma^{\circ}\theta^{+}\beta_{\alpha}^{+},$$

$$(1+16i\beta^{+}\gamma^{\circ}\theta^{+})^{-1} [\xi_{\alpha}^{-} - 8i\xi^{-}\gamma^{\circ}\xi^{-} (\hat{y}\beta^{+})_{\alpha}]) .$$

The finite dimensional parts of these transformations have the following form:

$$(e^{i\beta^{+}\gamma^{\circ}S^{-}})_{AB} = \mathcal{U}_{AB}(\beta^{-},\xi^{-})e^{i\beta^{-}\gamma^{\circ}S^{\prime}},$$
$$(e^{i\beta^{+}\gamma^{\circ}T^{\prime}})_{AC} = \mathcal{U}_{AB}(\beta^{+},\theta^{+})\mathcal{K}_{BC}(\mathbf{y},\mathbf{c})e^{-i\beta^{+}\gamma^{\circ}T^{\prime}},$$

where  $S'_a$  and  $T'_a^+$  are the differential parts of the corresponding generators and the matrices  $U_{AB}$  and  $K_{BC}$  are defined as follows:

×'

$$\begin{aligned} & \mathfrak{U}_{AB} (\beta^{-}, \xi^{-}) = \delta_{AB}^{+} 8(\Sigma_{\mu\nu})_{AB} \beta^{-} \gamma^{\circ} \sigma^{\mu\nu} \xi^{-} - \\ & -16q(q+2) \frac{1}{2} (1 - i\gamma_{5})_{AB} \beta^{-} \gamma^{\circ} \beta^{-} \xi^{-} \gamma^{\circ} \xi^{-} , \\ & \mathfrak{U}_{AB} (\beta^{+}, \theta^{+}) = \delta_{AB}^{+} 8(\Sigma_{\mu\nu})_{AB} \beta^{+} \gamma^{\circ} \sigma^{\mu\nu} \theta^{+} - \\ & -16p(p+2) \frac{1}{2} (1 + i\gamma_{5})_{AB} \beta^{+} \gamma^{\circ} \beta^{+} \theta^{+} \gamma^{\circ} \theta^{+} , \end{aligned}$$

$$\begin{aligned} & \chi_{\{a_{p}\}\{a_{p}^{*}\}\{\beta_{q}\}\{\beta_{q}^{*}\}} - (y_{\mu}, \beta^{+}y^{\circ}\gamma_{\nu}\xi^{-}) = \frac{1}{2}(1+i\gamma_{5})_{\{a_{p}\}\{a_{p}^{*}\}} \times \\ & \times (1+16i\beta^{+}y^{\circ}\gamma^{\nu}\xi^{-}y_{\nu})^{q/2} \prod_{j=1}^{q} \{\frac{1}{2}(1-i\gamma_{5})\beta_{j}\beta_{j}^{*}\beta_{j}^{*} + \\ & + 8iy^{\mu}\beta_{\gamma}\gamma^{\circ}\gamma^{\nu}\xi^{-}[\gamma_{\nu}\gamma_{\mu}, \frac{1}{2}(1-i\gamma_{5})]\beta_{j}\beta_{j}^{*}\beta_{j}^{*}\}, \end{aligned}$$

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