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M.I.Shirokov

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Широков М.И.

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Новое определение закона распада

Рассматривается эволюция во времени нестабильных состояний. Показана ограниченность обычного определения закона распада. Предложено более общее определение. Обсуждается поведение закона распада при больших и малых временах.

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Shirokov M.I.

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New Definition of the Decay Law

Time evolution of unstable states is considered. The usual definition of the decay law is shown to be of a limited application. A more general definition is proposed. The decay law behaviour at large and small times is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. The following general statements about the quantum-mechanical decay law are known:

1) The time dependence of the decay law which is bound by an exponential $\exp(-at)$ as $t \to \infty$ is inconsistent with the boundedness from below of the spectrum of the total Hamiltonian H (which describes the decay dynamics)[1-4].

2) The decay law time derivative vanishes at $t=0$ if the energy expectation value $\langle\psi_u, H\psi_u\rangle$ of the unstable state ψ_u is finite[5,6].

The statement generated a series of papers in which it was stressed that the process of the decay observation must change the observed decay law. Various theoretical methods for describing this process lead to an exponential asymptotics, see, e.g., ref.[7-12]. However, suppose that unstable particles propagate in vacuum after their production, and their observation begins only when the asymptotical decay regime has been set in. According to 1) it is nonexponential and one must observe more undecayed particles than one could expect if the asymptotics were exponential[13]. Therefore the papers[7-12] do not exclude the possibility of the experimental verification of the statement 1). A relevant experiment is discussed in ref. [14].

It was shown in ref. [10,15] that the phenomenon 2) may be observable in the following sense: the observed decay is delayed if decay measurements are sufficiently frequent.

2. The purpose of this note is to show that statements 1) and 2) are true (as mathematical theorems) only for a restricted class of the physical decay models. Two important suppositions must hold for these models (the authors cited above do not mention them when formulating 1) and 2)). If the suppositions are wrong, one cannot use the usual definition of the decay law. We suggest a new definition (the usual one turns out to be its particular case). Its application to a solvable decay model provides results which can be considered as counterexamples to 1) and 2) as <u>physical</u> statements.

3. The first supposition a) looks very natural: the total space of states in the decay problem is $\mathcal{H} = \mathcal{H}_u \oplus \mathcal{H}_d$, where $\mathcal{H}_u$ and $\mathcal{H}_d$ are spanned by the vectors describing the unstable particle u and its decay product d, resp. In each instant of time the system reveals itself either as u or as d. The decay is realized due to a loss of the probability from $\mathcal{H}_u$ into $\mathcal{H}_d$ (ref. [3]). So the decay law $P(t)$ is defined as the survival probability. If only one vector ψ_u enters into $\mathcal{H}_u$ then

$$P(t) = |\langle\psi_u, \exp(-iHt)\psi_u\rangle|^2. \tag{1}$$

Of course $\mathcal{H}_u$ is not one-dimensional if u is a <u>particle</u>: e.g., u may be in states with different momenta p. Let its state be a packet, which can be described in terms of the field theory as

$$\psi_u = a_f^+ \Omega_0, \; a_f^+ = \int d^3p f(p) a_p^+, \; \int d^3p |f(p)|^2 = 1. \qquad (2)$$

Here Ω_0 is the state without particles and a_p^+ is the creation operator. If (2) is the initial state then $\exp(-iHt)\psi_u$ changes not only because of the decay itself but simply because of the packet diffusion. The latter is not the decay. So we must define $P(t)$ as the transition probability from ψ_u to all possible u states (i.e., from ψ_u to $\mathcal{H}_u$)

$$P(t) = \int d^3p |\langle a_p^+\Omega_0, U(t)\psi_u\rangle|^2, \; U(t) \equiv \exp(-iHt). \qquad (3)$$

We call (3) the usual definition of the decay law. Other definitions are possible which either are equivalent to (3))(e.g.,the definition in terms of the u Green function) or somewhat generalize it (e.g., when u state is described by a density matrix).

The second supposition b) can be stated as follows: ψ_u can be expanded in terms of the H eigenfunctions ϕ_E which belong to the continuous part of the H spectrum: $\psi_u = \int dE c(E) \phi_E$. Then (1) turns into the equation:

$$P(t) = |\int dE |c(E)|^2 \exp(-iEt)|^2, \qquad (4)$$

which was used in [1] and [4] to prove 1) and 2). Approaches using the resolvent operator need the following modification of the above formulation: ψ_u is in the H domain [15].

4. Relativistic local field theories are examples of theories for which a) and b) may

be wrong. The usual way to describe unstable particle state is to take, as ψ_u, a suitable eigenfunction of a part H_0 of the total Hamiltonian H (or a superposition of such eigenfunctions). H_0 may contain not only the free part of H but also its strong interaction part if the decay is due to the weak interaction. Local interaction Hamiltonians always have terms with creation operators only. Therefore such processes as $\Omega_0 \to \bar{u} + d$ ($\bar{u}$ is u antiparticle) or $u \to u + d + \bar{u}$ are possible along with $u \to d$. They are "virtual" but nevertheless turn out to be important for our subject, see below the concluding sect.6.

We see that unstable particles may be also in states not belonging to $\mathcal{H}_u$. So the definition (3) does not give their total number at the moment t. Analogously the observation of the decay products at the moment t does not guarantee that u is not present at this moment.

If ψ_u is taken as stated above then its expansion in ϕ_E turns out to be impossible in all local models where the expansion possibility can be investigated, see, e.g., ref.[16]*.

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*The supposition b) may fail even if the expansion is made possible (e.g.,by introducing a suitable formfactor).In this case it usually contains such a normalizable vector ϕ_E (belonging to a discrete H eigenvalue)as the physical vacuum Ω.Just such a situation arises in the model discussed below in sect.6. As a consequence the r.h.s. of (4) will acquire the time independent summand $(|c(E_\Omega)|^2)^2$, i.e., according to (3) the decay stops at large times.

5. We conclude that (3) is not valid for any theory of the decay process. We propose another definition of the decay law

$$N(t) = \langle U(t)\psi_u, \hat{N} U(t)\psi_u \rangle - \langle U(t)\Omega_0, \hat{N} U(t)\Omega_0 \rangle . \qquad (5)$$

Here $\hat{N}$ is the operator of the unstable particle number and Ω_0 is its no-particle eigenvector $\hat{N}\Omega_0 = 0$. $N(t)$ is the average number of unstable particles at the moment t minus "theoretical background" of these particles which can be present even if there were no unstable particles initially (e.g., the "background" may be not zero if the state Ω_0 is nonstationary). As the first consequence of the "background" subtraction, $N(t)$ vanishes identically if the unstable particles were absent initially (i.e., if $\psi_u = \Omega$).

We shall give another formulation of our definition before (and in order of) commenting it. Let us consider the probability to find, at the moment t, at least one unstable particle (i.e., one, two, etc.) in any state and irrespective of any other particles (e.g., decay products) which may accompany it. This inclusive quantity may be written out as

$$\dot{I}(t) = S_m \int d^3p \, |\langle a_p^+\phi_m, U\psi_u \rangle|^2 , \quad U \equiv \exp(-iHt) . \qquad (6)$$

Here ϕ_m's denote states, which form a complete set: $S_m \phi_m \phi_m^* = 1$., (S_m denotes the corresponding summation and integration). It contains Ω_0, one-particle states $a_p^+\Omega_0$, two unstable particles, decay products and so on. By $a_p^+\phi_m$ we denote a state which differs from ϕ_m by the presence of one more unstable particle. Of course $\langle a_p^+\phi_m, U\psi_u \rangle = 0$ if ψ_u

cannot pass to the state $a_p^+\phi_m$. For example, the process $\pi^- \to \pi^- + \nu$ is forbidden (even as a virtual one) by the lepton number conservation law. Rewriting (6) as

$$\begin{aligned} I(t) &= S_m \int d^3p \langle U\psi_u, a_p^+\phi_m \rangle \langle a_p^+\phi_m, U\psi_u \rangle = \\ &= S_m \int d^3p \langle a_p U\psi_u, \phi_m \rangle \langle \phi_m, a_p U\psi_u \rangle = \\ &= \langle U\psi_u, \int d^3p\, a_p^+ a_p U\psi_u \rangle \end{aligned} \tag{7}$$

we see that $I(t)$ is the average number of unstable particles in the state $U\psi_u$. So, we have

$$N(t) = S_m \int d^3p |\langle a_p^+\phi_m, U\psi_u \rangle|^2 - S_n \int d^3p |\langle a_p^+\phi_n, U\Omega_0 \rangle|^2 . \tag{8}$$

In the case of the decay $\pi^- \to \mu^- + \bar{\nu}$ the states $a_p^+\phi_m$ really reduce to π^-, $\pi^-\pi^+\bar{\nu}\mu^-$, etc., while $a_p^+\phi_n$ reduce to $\pi^-\mu^+\nu$, etc.

Let the supposition a) be true, i.e., the physical dynamics is such that in S_m only the element $\langle a_p^+\Omega_0, U\psi_u \rangle$ is not zero. Let, in addition, the "background" be absent (this will be the case when, e.g., Ω_0 is stationary and coincides with the physical vacuum Ω, so that supposition b) turns out to be true). Then the new definition (8) or (5) turns into (3) as in its particular case.

Let us stress that the new definition properly takes into account the virtual transitions, discussed above in sect.4.

The calculation of $N(t)$ can be reduced to the evaluation of the expectation values of the Heisenberg operator $\hat{N}(t) = U^+\hat{N}U$ in the states ψ_u and Ω_0. Thus one can avoid the explicit calculation of the evolution operator $U(t)$. By this remark we conclude the general discussion of the new difinition.

6. For our purpose declared in sect.2 it will be sufficient to apply the new decay law definition (5) to a particular solvable model, for which the suppositions a) and b) are not true. This was done by the author in ref./17/. The model describes the charged particle which is in an oscillatory potential and interacts with photons in the dipole approximation. The particle excited state ψ_u (one-phonon state) is described by a corresponding eigenvector of the free Hamiltonian H_0. The phonon may pass into photons. The exact calculation of the phonon number at the moment t according to (5) reveals that $N(t)$ is bounded by an exponential when $t \to \infty$, though the total Hamiltonian spectrum is bounded from below. More exactly, one can represent $N(t)$ at all times by the expression $\exp(-\Gamma t)(1+\alpha(t))$, where $\alpha(t) \ll 1$ and oscillates with large frequency $\gg \Gamma$. Because of $\alpha(t)$ the derivative dN/dt at $t=0$ is equal to -2Γ, but not to $-\Gamma$ and <u>not to zero</u> (note that a few oscillations later, we have already $dN/dt \cong$ $\cong -\Gamma$). This result holds equally well for two variants of the model formfactor: the one corresponding to <u>infinite</u> $\langle\psi_u, H\psi_u\rangle$ (the formfactor $g(k)=\mu/\sqrt{k^2+\mu^2}$ chosen in/17/) and the other corresponding to finite $\langle\psi_u, H\psi_u\rangle$ (the formfactor $g(k)=\mu^2/(k^2+\mu^2)$). So both assertions 1) and 2) are wrong for the discussed decay model.

Let us stress that this result was obtained using the new decay law definition (5), which takes into account the transitions of the type $u \to u + \bar{u} + d$, $\Omega_0 \to u + \bar{d}$. They do not conserve "energy" if the term means the H_0 eigenvalue. Therefore their probabilities are small compared to the probabilities of the "energy" conserving processes, i.e., $u \to u$ and $u \to d$. Nevertheless, these "virtual" transitions turn out to be important. Indeed, if one neglects them, then (8) turns into (3) and nonexponential asymptotics follows in contradiction with the exact result stated above.

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