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CONFINEMENT POTENTIAL PRODUCED BY INDEFINITE METRIC MULTIPOLE FIELDS OF INFINITE ORDER

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Потенциал запирания, созданный мультипольными полями бесконечного порядка с индефинитной метрикой

В рамках теории мультипольных полей N -го порядка рассматривается возможность получения некоего потенциала запирания частиц. В пределе $N \to \infty$ получены статические потенциалы запирания различных видов.

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Confinement Potential Produced by Indefinite Metric Multipole Fields of Infinite Order

The possibility of obtaining some type of confinement of particles is discussed in a field theory with multipole fields of the N-th order. The limiting case $N \to \infty$ yields various forms of static confinement potentials.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. The data of the charmonium spectrum seem to fit to experiments quite well[1,2,3] when a confinement potential between c and $\bar{c}$ is supposed in the form:

$$V(r) = \frac{1}{r}(\alpha + \delta r + K r^2). \tag{1}$$

For example ref.[2] gives the values of α, δ, K as

$$\alpha = -0.2, \quad K = -\frac{\alpha}{a^2}, \quad a = 0.2 \text{ fm}.$$

The value of δ is irrelevant (actually $\delta = 0$ is taken), it is an additive constant only to the total energy. A great variety of theoretical considerations predicts the form (1). We quote two rather conservative field theoretical approaches only [4,5] (similar to ours [6]), where other references can be found.

In a previous work[6] we wanted to call the attention to multipole fields as possible sources of some kind of particle confinement. There the free Lagrangian has been proposed to possess the form

$$L_0 = -\frac{1}{2}(\partial_\mu B \partial_\mu B + m^2 B) - (\partial_\mu A \partial_\mu C + m^2 AC) - \lambda AB, \tag{2}$$

and it has been shown that the fields A,B,C interacting via an interaction Lagrangian L_I in the form

$$L_I = \bar{c}c\phi, \quad \phi = g_1 A + g_2 B + g_3 C$$

give the potential

$$V(r) = \frac{e^{-mr}}{r}(\alpha + \delta r + K r^2), \tag{3}$$

where α, δ, K are given functions of g_i, m, and λ. Actually the propagator

$$\tilde{D}(k) = \frac{1}{m^2-k^2}\left(\alpha + 2m\left(\delta - \frac{K}{m}\right)\frac{1}{m^2-k^2} + 8m^2 K \frac{1}{(m^2-k^2)^2}\right)$$

leads to the form (3). Equation (1) follows from eq. (3) in the limit

$$m \to 0, \quad \frac{\lambda}{m^2} = \text{const}., \quad g_i \sim \frac{m_c^2}{m^2} \to +\infty,$$

where m_c is the quark mass.

2. Multipole fields seem to provide another possibility to obtain a set of static potentials we want to describe here.

A multipole field of an N+1 order satisfying the field equations

$$(\Box - m^2)\phi_0 = 0,$$
$$(\Box - m^2)\phi_1 = \lambda\phi_0,$$
$$\vdots$$
$$(\Box - m^2)\phi_N = \lambda\phi_{N-1},$$

gives the Green function for ϕ

$$\phi = \sum_{s=0}^{N} g_s \phi_s \tag{4}$$

in the form [7]:

$$D(x) = \sum_{s=0}^{N} \alpha_s \lambda^s \left(\frac{\partial}{\partial m^2}\right)^s \Delta(x; m^2), \tag{5}$$

where a_s is quadratic in coupling constants g_i. A comparison with the formulae of Ref. /7/ gives immediately

$$a_s = \frac{\epsilon}{s!} \sum_{n \geq s}^{N} g_n g_{N+s-n} , \quad \epsilon = \pm 1 .$$

Introducing

$$A_s = a_s \left(\frac{\lambda}{m^2}\right)^s .$$

D(x) becomes

$$D(x) = \sum_{s=0}^{N} A_s (m^2)^s \left(\frac{\partial}{\partial m^2}\right)^s \Delta(x; m^2). \tag{6}$$

From (6) it is easy to calculate the potential V in an interaction $L_I = \bar{c} c \phi$ with the result:

$$V(r) = \frac{1}{r} \sum_{s=0}^{N} A_s \sum_{n=0}^{\infty} \frac{n}{2}\left(\frac{n}{2} - 1\right) \ldots \left(\frac{n}{2} - (s-1)\right) \frac{(-mr)^n}{n!} =$$

$$= \frac{1}{r} \sum_n B(n) (-mr)^n ,$$

$$B(n) = \sum_{s=0}^{N} \frac{A_s}{n!} \frac{n}{2}\left(\frac{n}{2} - 1\right) \ldots \left(\frac{n}{2} - (s-1)\right), \tag{7}$$

i.e.,

$$B(0) = A_0$$

$$B(1) = A_0 + \frac{1}{2} A_1 + \frac{1}{2}\left(\frac{1}{2} - 1\right) A_2 + \ldots + \frac{1}{2}\left(\frac{1}{2} - 1\right) \ldots \left(\frac{1}{2} - (N-1)\right) A_N ,$$

$$B(r) = \frac{1}{2} (A_0 + A_1),$$

If $N \to \infty$ a new possibility opens up. Suppose we wish to reproduce the Coulomb potential, then

$$B(0) = \alpha, \quad B(n) = 0, \quad n \geq 1. \tag{8}$$

This gives an infinite number of linear equations for A_i from which A_i can be calculated to any given order in "mr". (Det $A_i^{(N)} \neq 0$).

It is interesting to compare (8) with the requirement that the Fourier transform $\tilde{D}(k)$ of (6) should be equal to $-k^{-2}$. Eq. (6) gives

$$\tilde{D}(k) = \sum_s A_s (m^2)^s \frac{(-1)^s s!}{(m^2 - k^2)^{s+1}} = -\frac{1}{k^2} \sum_n C(n) (\frac{m^2}{k^2})^n ,$$

$$C(n) = \sum_s A_s \frac{n!}{(n-s)!} .$$

The prescription

$$C(0) = \alpha , \quad C(n) = 0, \quad n \geq 1.$$

reproduces (8) for even n. In fact

$$\frac{C(n)}{B(2n)} = \frac{n!}{n(n+1) \ldots 2n} .$$

The confinement potential (1) requires

$$B(0) = \alpha , \quad -mB(1) = \delta , \quad m^2 B(2) = K, \quad B(n) = 0, \quad n \geq 3.$$

from which A_i can be determined to any given order. The mass m can be chosen arbitrarily, a small value of it may be useful only for practical calculations.

Having obtained the set A_i in such a way, one can substitute that into Eq. (6) and start with $D(x)$ a relativistic calculation.

3. Although we wished to call the attention to multipole fields as a field theo-

retical explanation of particle confinement, D obtained in such a way can also be considered purely phenomenologically.

To multipole fields Lorentz or inner indices can also be attached.

Since multipole field theories imply a quantization with indefinite metric, a complete field theory as proposed above requires an artificial unitarization [6]. One of the simplest ways is to take the principal value integral in $\tilde{D}(k)$ instead of the usual "$i\epsilon$" prescription.

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