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THE PROBLEM OF THE X ° (958) SPIN.

Part III. Multidimensional Analysis

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Ледницки Р. **E2 - 10567**

К вопросу о спине $X^\circ(958)$. Часть III. Многомерный анализ

Обсуждается вопрос о спине $X^\circ(958)$ -мезона. В частях I-й и 2-й мы показали, что статистики имеющихся данных по X°-мезону пока недостаточны для решения этого вопроса с помощью стандартных методов, таких как анализы диаграмм Далица и одномерных распределений Эдейра. В настоящей работе развит многомерный формализм, который используется при анализе Монте-Карло событий реакции $K^- p \to X^\circ \Lambda$. На этом основании отношение функций правдоподобия гипотез $J^P(X^\circ) = 0^-$ и 2^- для имеющихся данных ожидается меньше чем $10^{-5} - 10^{-2}$ (в предположении, что гипотеза 2^- верна). Таким образом многомерная подгонка имеющихся данных по реакции $K^- p \to X^\circ \Lambda$ могла бы существенно прояснить проблему спина X°-мезона.

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Lednický R. **E2 - 10567**

The Problem of the $X^\circ(958)$ Spin.
Part III. Multidimensional Analysis

The multidimensional formalism has been developed and applied to an analysis of the Monte-Carlo events of the reaction $K^- p \to X^\circ \Lambda$. It is shown that a similar fit of the available data could yield the likelihood ratio of the 0^- and 2^- hypotheses less than $10^{-5} - 10^{-2}$ providing the pseudotensor hypothesis is valid.

The investigation has been performed at the Laboratory of High Energies, JINR.

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1. The question of the X°-meson spin is still far from its final solution, $J^P/X^\circ/=0^-$ or 2^- [1]. In particular, both these spin parity assignments equally well agree with world Dalitz plot data [1,2]. Moreover, the Adair analyses of X° production and decay correlations, due to insufficient statistics, give no definite answer as well [3]. In ref. [4] we have stressed a significance of multidimensional analysis for a more reliable separation of the X° spin alternatives as compared to the standard Adair analysis. The general formalism was developed incorporating, however, the simplest non-relativistic description of X° 3-particle decays. In the present paper we set out analogous results obtained with the help of relativistic decay matrix elements discussed in ref. [2]. These results have been already applied to a multidimensional analysis of Monte-Carlo events of the reaction $K^-p \to X^\circ \Lambda$ in ref. [1]. The estimates of the likelihood ratios $P(0^-/2^-)$ of the 0^- and 2^- hypotheses have been obtained for real events at 2.18 and 1.75 GeV/c [5,6]. Here we analyze also the recent data at 4.16 GeV/c [7] reported at the Tbilisi Conference. It is shown that the available data on the reaction $K^-p \to X^\circ \Lambda$ could yield $P(0^-/2^-) < 10^{-5}-10^{-2}$ providing the 2^- assignment is valid.

2. Below we discuss the reaction $K^- p \to X^\circ \Lambda$. The generalization of the formalism to any reaction is straightforward. In principle, the method applied consists of a combination of a convenient decay description in terms of cartesian tensors[8] with the well-known multipole analysis. This results in an essential simplification of usual tiresome calculations (see, e.g., ref.[9]).

The differential cross section of the reaction $K^- p \to X^\circ \Lambda$ /$X^\circ \to 1,\ldots,\ell$, $\Lambda \to p\pi^-$ / can be expressed through the joint spin density matrix elements in the production /$\rho^{nn'}_{mm'}$/ and spin density matrix elements in the X° and Λ decays /$r^1_{m'm}$ and $r^2_{n'n}$ / determined in the coordinate systems $x_1 y_1 z_1$ and $x_2 y_2 z_2$ in the X° and Λ rest frames, respectively:

$$d\sigma = \Sigma r^1_{m'm}\, r^2_{n'n}\, \rho^{nn'}_{mm'}(x)dx\, d_\ell(X^\circ; 1\ldots\ell)d_2(\Lambda; p\pi^-), \qquad (1)$$

where $x = \cos\theta_{c.m.}$ and $d_\ell(a;1\ldots\ell)$ are usual Lorentz invariant decay phase space elements. With the aid of vectors in the X° and Λ decays, the coordinate systems $\xi_1 \eta_1 \zeta_1$ and $\xi_2 \eta_2 \zeta_2$ can be fixed. Let us denote the Euler angles of the rotations $x_i y_i z_i \to \xi_i \eta_i \zeta_i$ by $\Omega_i = \phi_i \theta_i \psi_i$, $i = 1,2$. Note that ϕ_i , θ_i are the ζ_i azimuthal and polar angles in the $x_i y_i z_i$ system and $\pi - \psi_i$ is the z_i azimuthal angle in the $\xi_i \eta_i \zeta_i$ system. The phase space elements in the decays $a \to 12$ and $a \to 123$ can then be written in the form

$$d_2(a;12) = (k/4m_a)\, d\phi\, d\cos\theta$$

$$d_3(a;123) = (kq/8m_a)\, dm_{23}\, d\cos\delta\, d\phi\, d\cos\theta\, d\psi, \qquad (2)$$

where $\vec{k}=\vec{p}_1^{(a)}$ is the momentum of particle 1 in the a-rest frame, $\vec{q}=\vec{p}_2^{(23)}$ is the momentum of particle 2 in the c.m.s. of particles 2,3; m_{23} is the effective mass of particles 2,3, and δ is the angle between the vectors $\vec{k}$ and $\vec{q}$. The decay spin density matrix elements are determined through the X° and Λ decay amplitudes $A^i_{[\lambda]}(m_i) = \langle p_1\lambda_1 \ldots p_\ell\lambda_\ell | A^i | J_i m_i \rangle$:

$$r^i_{m_i' m_i} = \sum_{[\lambda]} A^{i*}_{[\lambda]}(m_i') A^i_{[\lambda]}(m_i), \tag{3}$$

where $[\lambda]$ are the helicities of decay particles. Under the rotation $x_i y_i z_i \to \xi_i \eta_i \zeta_i$ the decay amplitudes are transformed with the aid of the D-functions according to the law [10]

$$A^i_{[\lambda]}(m_i) = \sum_{\mu_i} A^{i'}_{[\lambda]}(\mu_i) D^{J_i *}_{m_i \mu_i}(\Omega_i), \tag{4}$$

where μ_i are now the J_i projections on the ζ_i axis, $J_1 = 2(0)$ and $J_2=1/2$. Using this transformation, the Ω_i dependence of the distribution (1) can explicitly be calculated [10]

$$d\sigma = \sum (2L_1+1)(2L_2+1) t^{L_2 M_2 *}_{L_1 M_1}(x) T^1_{L_1 N_1} T^2_{L_2 N_2} \times \\ \times D^{L_1 *}_{M_1 N_1}(\Omega_1) D^{L_2 *}_{M_2 N_2}(\Omega_2)\, dx\, d_\ell(X^\circ; 1 \ldots \ell) d_2(\Lambda; p\pi^-). \tag{5}$$

In this calculation, the products of the D-functions have been simplified with the help of the well-known coupling rule. The production and decay multipole parameters thus arising are expressed through the spin density matrix elements by means of the Clebsch-Gordan coefficients

$$t^{L_2 M_2 *}_{L_1 M_1} = \sum_{[m]} \rho^{m_2 m_2'}_{m_1 m_1'} (J_1 m_1' L_1 M_1 | J_1 m_1)(J_2 m_2' L_2 M_2 | J_2 m_2), \qquad (6)$$

$$T^{i}_{L_i N_i} = \sum_{[\mu]} r^{i}_{\mu_i' \mu_i} (J_i \mu_i' L_i N_i | J_i \mu_i). \qquad (7)$$

Hermiticity of the ρ- and r-matrices implies the relations:

$$t^{L_2 -M_2 *}_{L_1 -M_1} = (-1)^{M_1 + M_2} t^{L_2 M_2}_{L_1 M_1}, \quad T^{i *}_{L_i N_i} = (-1)^{N_i} T^{i}_{L_i -N_i}. \qquad (8)$$

Additional symmetry relations between the multipole parameters, following from parity conservation, depend on coordinate systems. We choose the z_1-axis along the beam momentum $\vec{K}$ in the overall c.m.s. and $z_2 = -z_1$. Such a choice is convenient for the Adair analysis. We direct the y_i-axes along the normal to the production plane $y_1 = y_2 = \vec{K} \times \vec{P}$ and fix the x_i-axes so that both coordinate systems should be right-hand. Parity conservation in the production process then yields the relation

$$t^{L_2 -M_2}_{L_1 -M_1} = (-1)^{L_1 + M_1 + L_2 + M_2} t^{L_2 M_2}_{L_1 M_1}, \qquad (9)$$

which also holds in any systems with z-axes lying in the production plane. In a two-particle decay $a \to 12$ /$\Lambda \to p\pi^-$, $X^\circ \to \gamma\gamma$/ we direct the ζ-axis along the momentum $\vec{p}_1^{(a)}$. The other axes are not important since the decay amplitudes cannot depend on the rotation around the ζ-axis (assuming the final spins are not measured). Therefore all the nondiagonal r-matrix elements vanish, implying $T_{LN} = 0$ if $N \neq 0$, so that we can put $\psi_i = 0$. In the three-particle decays $X^\circ \to \eta\pi\pi$ and $X^\circ \to \gamma\pi^+\pi^-$ we choose the ζ-axis along the normal $\hat{n}$ to the decay plane and the

ξ -axis along the η -meson (photon) momentum $\vec{k}$ in the X^0 rest frame. The η -axis is fixed by the requirement that the coordinate system is right-hand. Parity conservation in the X^0 decays gives particularly simple relations in these coordinate systems, i.e.,

$$T^{1}_{L_1 N_1} = 0 \quad \text{for odd } L_1 \text{ or } N_1 . \qquad (10)$$

In the $\Lambda \to p\pi^-$ decay, parity is not conserved thus making possible an asymmetric Λ decay. The Λ asymmetry parameter $\alpha_\Lambda = 0.646$ is related to the normalized multipole parameters $/T^2_{00} = 1/$ by the relation $\alpha_\Lambda = \sqrt{3}\, T^2_{10}$.

Symmetry relations (8)-(10) allow one to rewrite the distribution (5) in a more convenient form /4/

$$d\sigma = \sum_{L=0,2,4} (2L+1)\{ t^{00}_{L0}(x)[T_{L0}\, d^{L}_{00}(\theta_1) + 2 \sum_{N=2,..L} \mathrm{Re}B_{LN}\, d^{L}_{0N}(\theta_1)] +$$

$$+ 2 \sum_{M=1,...L} t^{10}_{LM}(x)[T_{L0} \cos M\phi_1\, d^{L}_{M0}(\theta_1) +$$

$$+ \sum_{N=2,...L} (\mathrm{Re}B_{LN} \cos M\phi_1\, d^{L+}_{MN}(\theta_1) - \mathrm{Im}B_{LN} \sin M\phi_1\, d^{L-}_{MN}(\theta_1))] +$$

$$+ 2\sqrt{3}\, \alpha_\Lambda \sum_{M=1,..L} t^{10}_{LM}(x) \cos\theta_2 [T_{L0} \sin M\phi_1\, d^{L}_{M0}(\theta_1) +$$

$$+ \sum_{N=2,..L} (\mathrm{Re}B_{LN} \sin M\phi_1\, d^{L+}_{MN}(\theta_1) + \mathrm{Im}B_{LN} \cos M\phi_1\, d^{L-}_{MN}(\theta_1))] -$$

$$- \sqrt{6}\, \alpha_\Lambda \sum_{M=0,\pm1,...\pm L} \mathrm{Im}\, t^{11}_{LM}(x) \sin\theta_2 [T_{L0} \sin(M\phi_1 + \phi_2)\, d^{L}_{M0}(\theta_1) +$$

$$+ \sum_{N=2,...L} (\mathrm{Re}B_{LN} \sin(M\phi_1 + \phi_2)\, d^{L+}_{MN}(\theta_1) +$$

$$+ \mathrm{Im}\, B_{LN} \cos(M\phi_1 + \phi_2)\, d^{L-}_{MN}(\theta_1))]\}\, dx\, d\Omega_2\, d_\ell(X^\circ; 1 \dots \ell), \qquad (11)$$

where $d^{L\pm}_{MN} = d^{L}_{MN} \pm d^{L}_{M-N}$ and $B_{LN} = T_{LN} e^{iN\psi_1}$. Here and from now on we put $T^1_{LN} = T_{LN}$. It is seen that the distribution (11) contains 30 multipole parameters, i.e., 9 parameters t^{00}_{LM}, $L=0,2,4$, $M=0,1,\dots L$, 6 parameters $\mathrm{Im}\, t^{10}_{LM}$ $L=2,4$, $M=1,\dots L$, and 15 parameters $\mathrm{Im}\, t^{11}_{LM}$ $L=0,2,4$, $M=0, \pm 1, \dots \pm L$*. These parameters are independent in the sense of the symmetry relations, but, being bilinear products of 10 complex amplitudes, they depend on 18 real parameters only (the common phase and the overall normalization factor are not included). In the collinear case $/|x| = 1/$ only two independent amplitudes survive, so that out of 29 normalized $/t^{00}_{00} = 1/$ multipole parameters only 4 ones can be different from zero. They can be expressed through two parameters, e.g., ρ_{00} and the phase ϵ of the two amplitudes:

$$t^{00}_{20} = -\frac{1}{2}\sqrt{\frac{2}{7}}(1+\rho_{00}), \quad t^{00}_{40} = \frac{1}{3}\sqrt{\frac{2}{7}}(5\rho_{00} - 2),$$

$$t^{11}_{21} = -i\sqrt{\frac{1}{21}}\sqrt{\rho_{00}(1-\rho_{00})}\sin\epsilon, \quad t^{11}_{41} = -\sqrt{\frac{10}{3}}\, t^{11}_{21}. \qquad (12)$$

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*Integration over the Λ decay angles in eq. (11) gives the distribution in the reaction $\pi^- p \to X^\circ n$ containing 9 elements t^{00}_{LM} only. Integration over the X° Dalitz plot variables and over the angle ψ yields the result of Berman and Jacobb /11/ (see their eq. (27) and multiply $R_0 = r_{00}$ by the lost factor 1/2).

Note that in the case of zero X°-meson spin there is only one nontrivial multipole parameter, proportional to the Λ polarization P_Λ along the normal to the production plane (y-axis), i.e., $\mathrm{Im}\,t^{11} = \sqrt{\frac{2}{3}}\mathrm{Im}\,\rho^{+-} = -\sqrt{\frac{1}{6}}P_\Lambda$.

Until now we have not specified the decay multipole parameters T_{LN} defined in eqs. (3) and (7) through the decay amplitudes $A(m_\zeta)$. The X° decay amplitudes have been determined in ref. /2/ in the tensor formalism allowing one to form the simplest amplitudes satisfying the requirement of the Lorentz invariance. These two descriptions are connected by the relations, well-known for the vector amplitudes: $A(\pm 1) = (\mp A_1 - iA_2)/\sqrt{2}$, $A(0) = A_3$. In the spin-2 case we have the relations

$$
\begin{aligned}
A(\pm 2) &= \frac{1}{2}(A_{11} - A_{22}) \pm \frac{i}{2}(A_{12} + A_{21}) \\
A(\pm 1) &= \mp\frac{1}{2}(A_{13} + A_{31}) - \frac{i}{2}(A_{23} + A_{32}) \qquad (13) \\
A(0) &= \sqrt{\frac{1}{6}}(2A_{33} - A_{11} - A_{22}),
\end{aligned}
$$

which automatically pick out a symmetric and zero trace part of the matrix $\{A_{ij}\}$. Below, with the help of these relations, we express the quantities T_{LN} through the X° decay parameters determined in ref. /2/. We have performed similar calculations in ref. /4/ using, however, the simplest nonrelativistic amplitudes for the three-particle X° decays. Compare also with ref. /12/ where different normalization factors for the multipole parameters have been used.

The $X^\circ \to \gamma\gamma$ decay. The decay amplitude is unambiguously determined by the requirement of the Bose-symmetry and by the transversality of γ-quanta, i.e.,

$$A_{ij} = \hat{\vec{k}}_i [\vec{e}^{\,(1)} \vec{e}^{\,(2)}]_j , \qquad (14)$$

where $\vec{e}^{\,(1,2)}$ are the γ polarization vectors and $\hat{\vec{k}} = (0,0,1)$. It is seen that only $A_{33} \neq 0$, i.e., only $r_{00} \neq 0$ so that, according to (7), we have

$$T_{00} = r_{00}, \quad T_{20} = -T_{40} = -\sqrt{\frac{2}{7}}\, r_{00} . \qquad (15)$$

The $X^\circ \to \eta\pi\pi$ decay. The decay amplitude is given by

$$A_{ij} = w_0 \hat{k}_i \hat{k}_j + w_2 \hat{q}_i \hat{q}_j + w_4 \hat{k}_i \hat{q}_j \cos\delta , \qquad (16)$$

where the quantities w_L, depending on $m_{\pi\pi}$ and $\cos^2\delta$, have been determined in ref. /2/. In the chosen coordinate system $\xi_I \eta_I \zeta_I$, we have $\hat{k} = (1,0,0)$ and $\hat{q} = (\cos\delta, \sin\delta, 0)$. Using again eqs. (13), we obtain the following expressions for the decay multipole parameters:

$$T_{00} = \frac{2}{3} [\,|w_0|^2 + |w_2|^2 + \frac{1}{4} |w_4|^2 \cos^2\delta (3 + \cos^2\delta) +$$

$$+ \mathrm{Re}\, w_0^* w_2 (3\cos^2\delta - 1) + 2\,\mathrm{Re}\, w_4^* (w_0 + w_2) \cos^2\delta]$$

$$T_{20} = \frac{1}{3} \sqrt{\frac{2}{7}} [\,|w_0|^2 + |w_2|^2 + \frac{1}{2} |w_4|^2 \cos^2\delta (3 - \cos^2\delta) +$$

$$+ 2\,\mathrm{Re}\, w_0^* w_2 (3\cos^2\delta - 2) + 2\,\mathrm{Re}\, w_4^* (w_0 + w_2) \cos^2\delta]$$

$$T_{22} = -\sqrt{\frac{1}{21}}\{|w_0|^2 + \mathrm{Re}\,w_0^*(w_2 + w_4\cos^2\delta) + [\mathrm{Re}\,w_4^*(w_0 + w_2) +$$

$$+ |w_4|^2\cos^2\delta]e^{i\delta}\cos\delta + [\mathrm{Re}\,w_2^*(w_0 + w_4\cos^2\delta) + |w_2|^2]e^{2i\delta}\}$$

$$T_{40} = \frac{1}{4}\sqrt{\frac{2}{7}}\,[|w_0|^2 + |w_2|^2 + \frac{1}{3}|w_4|^2(1+2\cos^2\delta)\cos^2\delta +$$

$$+ \frac{2}{3}\mathrm{Re}\,w_0^* w_2(1+2\cos^2\delta) + 2\,\mathrm{Re}\,w_4^*(w_0 + w_2)\cos^2\delta] \qquad (17)$$

$$T_{42} = \frac{1}{2}\sqrt{\frac{5}{3}}\,T_{22}$$

$$T_{44} = \frac{\sqrt{5}}{12}(|w_0|^2 + |w_2|^2 e^{4i\delta} + |w_4|^2 e^{2i\delta}\cos^2\delta +$$

$$+ 2e^{2i\delta}\,\mathrm{Re}\,w_0^* w_2 + 2e^{i\delta}\,\mathrm{Re}\,w_0^* w_4\cos\delta +$$

$$+ 2e^{3i\delta}\,\mathrm{Re}\,w_{2.}^* w_4\cos\delta).$$

The $X^\circ \to \gamma\pi^+\pi^-$ decay. It is well-known that the pions in this decay are mainly produced in the p-wave ρ°-state. Therefore the decay amplitude is given by

$$A_{ij} = G_1[\hat{k}e]_i\,\hat{q}_j + G_2[\hat{k}\hat{q}]_i e_j + G_3[\hat{k}e]_i\hat{k}_j\,, \qquad (18)$$

where the quantities G_i, depending on the di-pion mass, have been determined in ref. /2/ Summing over the γ helicities $/\sum_\lambda e_i^{\lambda *} e_j^\lambda = \delta_{ij} - \hat{k}_i\hat{k}_j/$ in the r-matrix elements, we obtain the following formulae for the parameters T_{LN} in the $\xi_1\eta_1\zeta_1$ system:

$$T_{00} = \left(\frac{1}{6}|G_1+2G_2|^2 + \frac{1}{2}|G_1+G_2|^2 + \frac{1}{2}|G_1|^2\right)\sin^2\delta + $$
$$+ |G_1+G_3|^2\cos^2\delta$$

$$T_{20} = -\sqrt{\frac{2}{7}}\left[\left(\frac{1}{6}|G_1+2G_2|^2 + \frac{1}{4}|G_1+G_2|^2 - \frac{1}{2}|G_1|^2\right)\sin^2\delta - \right.$$
$$\left. - \frac{1}{4}|G_1+G_3|^2\cos^2\delta\right]$$

$$T_{22} = \frac{1}{4}\sqrt{\frac{1}{21}}\{\mathrm{Re}(G_1+G_3)^*(2G_1+7G_2-3G_3) + \tag{19}$$
$$+[3|G_2-G_3|^2 - 4\,\mathrm{Re}\,G_3^*(G_1+2G_2)]\sin^2\delta -$$
$$- e^{2i\delta}\,\mathrm{Re}(G_1+G_3)^*(5G_1+7G_2)\}$$

$$T_{40} = \sqrt{\frac{2}{7}}\left[\left(\frac{1}{6}|G_1+2G_2|^2 - \frac{1}{3}|G_1+G_2|^2 + \frac{1}{12}|G_1|^2\right)\sin^2\delta - \right.$$
$$\left. - \frac{1}{4}|G_1+G_3|^2\cos^2\delta\right]$$

$$T_{42} = \frac{1}{12}\sqrt{\frac{5}{7}}[2|G_1+G_3|^2\cos^2\delta + 2(|G_1|^2 - |G_2|^2)\sin^2\delta +$$
$$+ (e^{2i\delta}-1)\,\mathrm{Re}\,G_1^*(G_1+G_3)]$$

$$T_{44} = -\frac{\sqrt{5}}{12}[\mathrm{Re}\,G_3^*(G_1+G_3\cos^2\delta) + e^{2i\delta}\,\mathrm{Re}\,G_1^*(G_1+G_3)].$$

3. Let us now discuss briefly the $\cos\theta_l$ - distribution, which is currently used in the Adair analysis (see also ref. [3]). The

desired distribution is obtained by integrating over the phase space, except $\cos\theta_1$ in eq. (11):

$$W(\cos\theta_1) = \frac{1}{2}[1 + \frac{10}{7} c_2 d_2 d^2_{00}(\theta_1) + \frac{18}{7} c_4 d_4 d^4_{00}(\theta_1)]. \qquad (20)$$

The coefficients d_L and c_L are given by

$$d_L = \mp\sqrt{\frac{7}{2}} \int T_{L0} d_\ell(X^\circ;1\dots\ell) / \int T_{00} d_\ell(X^\circ;1..\ell),$$
$$(21)$$
$$c_L = \mp\sqrt{\frac{7}{2}} \int t^{00}_{L0}(x)dx / \int t^{00}_{00}(x)dx,$$

with the upper (lower) sign corresponding to $L = 2(4)$. In the $X^\circ \to \gamma\gamma$ decay, we have $d_2 = d_4 = 1$. The multipole parameters T_{L0}, given in eqs. (17) and (19), allow us to calculate the coefficients d_L in the distributions of the normal to the $X^\circ \to \eta\pi\pi$ and $X^\circ \to \gamma\pi^+\pi^-$ decay planes, respectively. In order to get the coefficients d_L for another decay analyzer (ζ -axis), e.g., $\hat{k}$ or $\hat{q}$, the parameters T_{L0} should be calculated in a coordinate system with ζ-axis along the new analyzer (the ξ- and η -directions are inessential). Note that the Dalitz plot density $T_{00} = \mathrm{Sp}\hat{r}$ is, of course, the same in any coordinate system. The d_L values obtained in such calculations can be found in Table 2 of ref.[3].

It should be pointed out that in the collinear production process $K^-p \to X^\circ\Lambda / \pi^-p \to X^\circ n/$, or at threshold of this reaction, we have $\rho_{22} = 0$ so that the coefficient c_2 should be positive for an arbitrary ρ_{00} value $/c_2 \geq 1/2/$ contrary to the coefficient c_4 vanishing if $\rho_{00} = 2/5$. Moreover, e.g., in the annihilation $\bar{p}p \to \rho^\circ X^\circ$ at rest, $c_4 = 0$ for an arbitrary production angle[13,14]. Therefore spin alignment effects in the distribution (20) can be often enhanced with the help of the decay analyzers corresponding to extreme values of the coefficient d_2, i.e., of the quantity T_{20}. We have shown in ref.[12] that T_{20} achieves extreme values provided the decay analyzer $\vec{v}$ coincides either with the normal to the decay plane or lies in this plane, i.e., $\vec{v} = \hat{n}$ or $\vec{v} = \vec{v}_0$. In our standard coordinate system: $v_0 = (\cos\alpha, \sin\alpha, 0)$. The angle α and the extreme T_{20} values are given by

$$e^{2i\alpha} = \pm T_{22} / |T_{22}| \tag{22}$$

$$T_{20}^{ext} = \begin{cases} -\frac{1}{2}(T_{20} \pm \sqrt{6}T_{22}) \text{ if } \pm T_{20} < \sqrt{\frac{2}{3}}|T_{22}|, \vec{v} = \vec{v}_0 \\ T_{20} \qquad\qquad \text{if } \pm T_{20} \geq \sqrt{\frac{2}{3}}|T_{22}|, \vec{v} = \hat{n}. \end{cases} \tag{23}$$

The upper (lower) sign in eqs. (22), (23) corresponds to the maximal (minimal) T_{20} value. For the $X^\circ \to \eta\pi\pi$ decay, we have[3/] $\vec{v}^{min} = \hat{n}$ and $\vec{v}^{max} = \vec{v}_0 / d_2^{min} \cong -0.77$ and $d_2^{max} \cong 0.86$/. The $X^\circ \to \gamma\pi^+\pi^-$ decay turns out to be more complicated. We only note that extreme d_2 values /~ −0.95, −0.59/ are close to the coefficients d_2 corresponding to the analyzers $\hat{k}$ and $\hat{n}/\hat{q}$/, see Fig. 2 of ref.[3].

It should be stressed that neither the "best analyzers" nor even all natural X° decay analyzers[14] have been used in experimental Adair analyses (see Table 1 of ref.[3]).

4. We have shown in ref.[1] that there is a good chance to increase the confidence level of arguments in favour of or against the pseudotensor X° assignment with the help of available experimental data only. For this purpose a likelihood fit of the multi-dimensional X° production and decay correlations should be done. In some way, such a fit corresponds to the use of the best decay analyzers in the one-dimensional Adair analyses. There is also a possibility to use the method of moments to look for non-zero multipole parameters[4,15]. We recall that out of 29 normalized parameters $t^{L_2 M_2}_{L_1 M_1}$ in the reaction $K^- p \to X^\circ \Lambda$, only that, describing the transverse Λ polarization, can survive if the 0^- assignment is valid. Note, however, that in such an analysis the X° spin can remain unnoticeable since spin alignment effects can be spread among many moments, while in the likelihood fit these effects are accumulated in the likelihood ratio.

A multidimensional fit of the reaction $K^- p \to X^\circ_{\to \eta \pi^+ \pi^-} \Lambda$ has been recently performed at 4.16 GeV/c[7]. The authors claim that they unambiguously excluded the 2^- assignment, thus definitely establishing the X° pseudo-scalarity. However, there are several shortcomings in their analysis which make the conclusion less significant. First, the 2^- decay amplitude with $\ell_\eta = \ell_{\pi\pi} = 2$, required to describe satisfactorily world $X^\circ \to \eta\pi\pi$ Dalitz plot data (see ref.[2]), has not been taken into account. Second, an essentially larger number of parameters in the 2^- fit, as compared to the 0^- one, makes a comparison of the fits difficult. It is convenient for such a comparison to integrate the distribution (11) over the Λ decay angles and over the azimuthal angle ϕ in the $X^\circ \to \eta\pi\pi$ decay (see eq. (24)) and to fit only two parameters ρ_{00} and ρ_{22}. At last the multidimensional fit has not been performed for the events satisfying the Adair cut, i.e., for the events with the X° perpendicular momentum p_T at least less than 100 MeV/c [3]. To neglect the X° production amplitudes with $J_z = \pm 2$ is incorrect in their fit of the events with $x > 0.99$ ($p_T < 153$ MeV/c).

Below we set forth in more detail than in ref.[1] a method to estimate the likelihood ratio $P(0^-/2^-)$ based on a multidimensional analysis of Monte-Carlo events. At first we have generated collinear events ($x = 1$) of the reaction $K^- p \to X^\circ \Lambda$ according to the density (11) integrated over the Λ decay angles, i.e., we have put

$$\underset{\to \eta\pi\pi}{}$$

$$W_{2^-} = T_{00} + \sum_{L=2,4} (2L+1)\, t^{00}_{L0} \left[T_{L0}\, d^{L}_{00}(\theta_1) + 2 \sum_{N=2,..L} \mathrm{Re}\, B_{LN}\, d^{L}_{0N}(\theta_1) \right], \quad (24)$$

with the decay parameters given in Table 2, FIT 7 of ref. [2]. The density W_{0^-} was approximated by the Dalitz plot distribution obtained in ref.[2] see Table 2, FIT 3 herein. The ρ_{00} dependence of $\log P(0^-/2^-)$ is shown in Fig. 1a for $N = 100$ collinear events ($\rho_{22} = 0$). Note that $\log P$ and its dispersion σ^2 are proportional to the number of events N. Thus we see that in an analogous 4-dimensional ($m_{\pi\pi}$, $\cos\delta$, $\cos\theta$, ψ) fit of 66 real events at 2.18 GeV/c ($x > 0.98$) the $0^-/2^-$ likelihood ratio can be expected

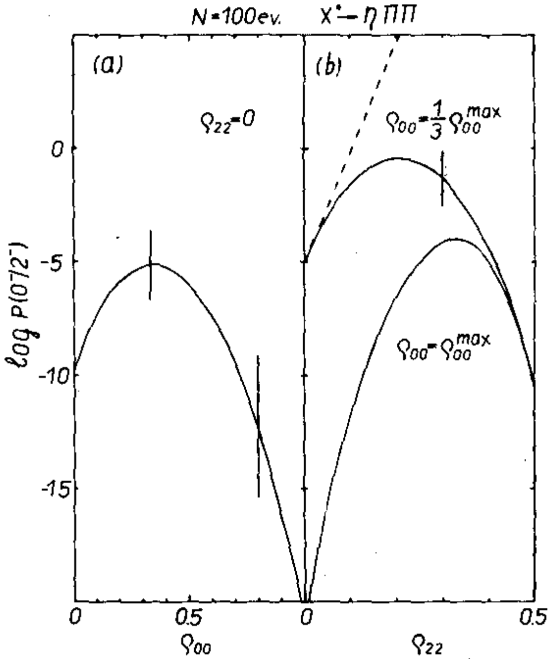

Fig. 1. The log of the $0^-/2^-$ likelihood ratio vs ρ_{00} and ρ_{22} X°- meson spin density matrix elements for N=100 events of the reaction $K^- p \to \underset{\to\eta\pi\pi}{X^\circ} \Lambda$ generated according to the density (24): (a) $\rho_{22}=0$, (b) The upper bound (full line) corresponds to $\rho_{00}=\frac{1}{3}\rho_{00}^{max}$ and the lower one to $\rho_{00}^{max}=1-2\rho_{22}$. The dashed curve shows $\log P$ for the events generated with a given ρ_{22} and $\rho_{00}=\frac{1}{3}\rho_{00}^{max}$ but fitted with fixed $\rho_{22}=0$. The error bars are shown in several characteristic points.

less than 10^{-3} * provided that $\rho_{22} \simeq 0$ (the fitted value is /3/ $\rho_{22} = 0.05 \pm 0.12$). Note that the upper limit $\log P(0^-/2^-) = -3.4 \pm 1.3$ is achieved at $\rho_{00} = 1/3$. This upper estimate can be compared with the value 4.2 ± 2.2 expected for $N = 66$ events (in the fit with fixed $\rho_{22} = 0$) if the 0^- hypothesis is valid. In order to demonstrate the usefulness of the multidimensional fit, we have analyzed the one-dimensional Adair distribution of the normal $\hat{n}$ to the $X^\circ \to \eta\pi\pi$ decay plane for $N = 66$ events generated with $\rho_{22} = 0$ and $\rho_{00} = 1/3$ and got $\log P(0^-/2^-) = -1.8 \pm 0.9$ (compare to the value -3.4 ± 1.3 obtained in the 4-dimensional analysis).

The Monte-Carlo calculations have been also performed for noncollinear events ($\rho_{22} \neq 0$ is allowed) with the density W_2 - given again by eq. (24). This formula now follows from the distribution (11) after integration over the Λ decay angles and over the azimuthal angle ϕ in the $X^\circ \to \eta\pi\pi$ decay as well. The ρ_{22} dependence of the upper and lower $\log P$ bounds (corresponding to $\rho_{00} = \frac{1}{3}\rho_{00}^{max}$ and $\rho_{00}^{max} \simeq 1 - 2\rho_{22}$, respectively) is shown in Fig. 1b for $N = 100$ events. As indicated in this figure, an analogous 4-dimensional $(m_{\pi\pi}, \cos\delta, \cos\theta, \psi)$ fit of 58 real events at 1.75 GeV/c $(x > 0.6)$ can yield $P(0^-/2^-) \sim 10^{-1}$ provided $\rho_{22} \simeq 0.3$ (the fitted value is /3/ $\rho_{22} = 0.34 \pm 0.14$).

* Taking into account the Λ decay information, this upper estimate is about 10 times smaller in the best case if in (12) $|\sin\epsilon| = 1$.

The dashed curve in Fig. 1b shows the $\log P$ for the events generated according to the density (24) with a given ρ_{22} and $\rho_{00} = \frac{1}{3}\rho_{00}^{max}$ but fitted with fixed $\rho_{22} = 0$. This curve indicates that the value $\log P = 2.6$ obtained under the collinear assumption ($x = 1$, i.e., $J_z \neq \pm 2$) for $N = 110$ events with $x > 0.99$ ($p_T < 153$ MeV/c) at 4.16 GeV/c [7] can be still explained by the 2^- hypothesis. For this purpose $\rho_{22} \gtrsim 0.15$ is required which is consistent with the upper ρ_{22} estimate ~ 0.23 in this p_T -interval (see Fig. 4 in ref. [3]). The likelihood fit of 56 more collinear events with $x > 0.995$ ($p_T < 108$ MeV/c) has not been performed in ref. [7]. However, the lack of anisotropies in the Adair distributions for these events[7] indicates that $P(0^-/2^-)$ is ~ 1 or $10^2 - 10^3$ provided that $\rho_{22} = 1/5$ or 0, respectively. The fact that the fitted value $\rho_{22} = 0.21 \pm 0.07$ is one standard deviation higher than the upper ρ_{22} estimate in the interval $p_T < 108$ MeV/c [3] can be interpreted as an argument against the 2^- assignment (see ref. [3]).

We have also analyzed the decay $X^\circ \to \gamma \pi^+ \pi^-$ and obtaind the estimate $P(0^-/2^-) \leq 10^{-1}$ for available experimental data. A relatively high value of this estimate is caused by an essential background in this decay channel[5].

Therefore, if the 2^- hypothesis is valid, the multidimensional fit of available data on the reaction $K^- p \to X^\circ \Lambda$ could yield the $0^-/2^-$ likelihood ratio less than $10^{-5} - 10^{-2}$ depending on the interpretation of the 4.16 GeV/c data. It seems to us that such an analysis can throw light on the X° - meson spin problem.

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Note added in proof

After the present work was completed Dr. Holmgreen was so kind as to send us the revised version of ref. [7] (CERN/EP/PHYS 77-11). The authors performed the likelihood fit in the interval $x > 0.995$ (p_T < < 108 MeV/c) and obtained $\ln P = 5$ $(\log P = 2.2)$ for 90 weighted events (~ 79 real events) assuming $J_z \neq \pm 2$. The dashed curve in Fig. 1b indicates that this $\log P$ value can be still explained by the 2^- hypothesis if $\rho_{22} \gtrsim 0.15$. In fact, the ρ_{22} value deduced from their P-E ratios is 0.19 ± 0.06 as compared with the upper estimate ~ 0.14 in this p_T -interval [3]. In our opinion it would be very desirable to look for the anisotropies in more collinear events, say with $p_T \lesssim$ $\lesssim$ 80 MeV/c. We expect the ρ_{22} value and the number of events in this interval to be ~2 times less than in the interval p_T<108 MeV/c. [3] Note that the authors claim to find the $\ell_\eta = \ell_{\pi\pi} = 2$ decay amplitude to be negligible if $X^\circ(958)$ were 2^-. It looks somewhat surprising because this amplitude turns out to be necessary to describe the world $X^\circ \to \eta\pi\pi$ Dalitz plot data [2]. This amplitude may influence considerably the fits of the noncollinear events (see Table 2 in the revised version of ref. [7]).

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