## СООБЩЕНИヶ ОБЬЕАИНЕННОГО ИНСТИТУТА <br> คАЕРНЫX ИСС^ЕАОВАНИЙ

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\end{aligned}
$$

R.Lednicky

THE PROBLEM OF THE $X^{\circ} / 958 /$ SPIN.
Part 2. Production and Decay Correlations

E2 - 10522

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THE PROBLEM OF THE $\mathrm{X}^{\circ} / 958$ / SPIN.<br>Part 2. Production and Decay Correlations

Ледницки•Р.
К вопросу о спине X 〒958)Часть II. Корреляции между образованием и распвдом

Анлизируются коррелянии между обраэованием и распадом $\mathrm{X}^{\circ}$-мезона, обсуждается вопрос обрезания согласно мегоду Эдейра.Показано, что отсутствие анизотропий в распределениях Эдеира в ревкиии $\mathrm{K}^{-} \mathrm{p} \rightarrow \mathrm{X}^{\circ} \Lambda$ при 1,75 ГэВ/с $\left(\cos \theta_{\text {c.m. }}>0.6\right)$ и при 4,16 ГэВ/с $\left(\cos \theta_{c . m}>0,995\right)$ не противоречит гипотезе о псевдотензорном $\mathrm{X}^{\circ}$-мезоне,

Работа выполнена в Лаборвтории высоких энергий ОИЯИ.


Lednický R.
E2 - 10522
The Problem of the $\mathrm{X}^{\circ}(958)$. Spin. Part II. Production and Decay Correlations

The $X^{n}$ production and decay cotrelations have been analyzed, and the question of the Adair cut is discussed. In particular, it is shown that the lack of anisotropies in the Adair distributions for the reaction $\mathrm{K}^{-} \mathrm{p}-\mathrm{X}^{\circ} \mathrm{I}$ at $1.75 \mathrm{GeV} / \mathrm{c}\left(\cos \theta_{c, m .}>0.6\right)$ and at $4.16 \mathrm{GeV} / \mathrm{c}$ ( $\cos \theta_{\text {c.m. }} \times 0.995$ ) is not in contradiction with the spin-2 $X^{\circ}$ assignment.

The investigation has been performed at the Laboratory of High Energies, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1977

1. The question of the $x^{\circ}-m e s o n ~ s p i n$,
still remains open, $J^{P}\left(X^{\circ}\right)=0^{-}$or $2^{-/ 1 /}$ In particular, according to a relativistic analysis of the $X^{\circ} \rightarrow \eta \pi \pi$ and $X^{\circ} \rightarrow \gamma \pi^{+} \pi^{-}$ decays $/ 1,2 /$ both the $0^{-}$and $2^{-}$hypotheses equally well agree with world Dalitz plot data. In the present paper we analyze the $X^{\circ}$ production and decay correlations and discuss the question of the Adair cut.

It is well-known that possible $\mathrm{X}^{\circ}$-meson spin effects could be the most pronounced in the $X^{\circ}$ production and decay correlations. Such correlations were studied in the reaction $K^{-} p \rightarrow X^{\circ} \Lambda$ in many experiments at Brookhaven and Berkeley with incident beam momenta over a range of $2-5 \mathrm{GeV} / \mathrm{c}^{/ 3-6 /}$. For a long period of time no deviations from isotropy were observed in the distributions of the angle $\theta$ between the $K^{-}$beam momentum and the $X^{\circ}$ decay analyzers. Since a spin zero particle must decay isotropically, this fact was interpreted as a strong argument supporting the $0^{-}$hypothesis. However, the statistics in all these experiments were insufficient to perform the Adair analysis. Only in 1973 the statistics in the experiment at $2.18 \mathrm{GeV} / \mathrm{c} / 7,8 /$ were rich enough to study the correlations in the almost collinear events, $x=\cos \theta_{\text {c.m. }}>0.98$, critical for solution of the $X^{\circ}$-meson spin
problem ${ }^{/ 9}$ i*. Deviations $^{\text {from }}$ isotropy were observed in the Adair distributions with the decay analyzers along the normal $\hat{\vec{n}}$ to the $\mathrm{X}^{\circ} \rightarrow \eta \pi \pi \quad$ decay plane, and the $\eta$-meson ( $\gamma$-quantum) momentum $\vec{k}$ in the $\mathrm{X}^{\circ} \rightarrow \eta \pi \pi\left(\gamma \pi^{+} \pi^{-}\right)$ decay. The corresponding polar-equatorial ratios $\mathrm{P} / \mathrm{E}=\mathrm{N}(|\cos \theta| \geq 0.5) / \mathrm{N}(|\cos \theta|<0.5)$ shown in Table 1 have a probability (in a $\chi^{2}$ sense) of a small fraction of aper cent to be in agreement with isotropy ${ }^{1 / 8 \%}$ and agree well with pseudotensor predictions. However, the near threshold experiment at $1.75 \mathrm{GeV} / \mathrm{c}^{/ 10 /}$ finished in 1974, did not support these anisotropies. Note that near threshold of the reaction $K^{-} p \rightarrow X^{\circ} \Lambda$ the $X^{\circ}$-meson spin projections $\pm 2$ on the c.m.s. beam direction $\vec{K}$ are damped, i.e., the $X^{\circ} \operatorname{spin}$ alignment and corresponding anisotropies should appear at not too small production angles. Therefore, it may seem quite natural to interpret the lack of anisotropies in this experiment as a strong argument against a nonzero $X^{\circ}$ spin/10, $11 /$. The absence of anisotropies in the Adair distributions for very forward produced $\mathrm{X}^{\circ}$ s ( $\mathrm{x}>0.995$ ) in the reaction $K^{-} p \rightarrow X^{\circ} \Lambda$ at $4.16 \mathrm{GeV} / \mathrm{c}$ has been recently reported at the Tbilisi Conference $/ 12$. According to the authors, their high statistics sample unambiguously excluded the $2^{-}$asignment, thus definitely establishing the $X^{\circ}$ spin parity as $0^{-}$.

We show in the present paper that the lack of anisotropies in the Adair distri~ butions at $1.75 \mathrm{GeV} / \mathrm{c}$ or even an indication

[^0]Table 1
The number of polar events ( P ), the number of equatorial events ( E ) and the results of the $2^{-}$fits for the Adair distributions discussed in the text; $N_{\sigma}$ is the number of standard deviations, the respective entries differ from equal numbers of $P$ and $E$ (isotropic distribution)

| Brperiment $\dot{K}_{p} \rightarrow Y^{0}(958) \wedge$ | $\begin{aligned} & 2.18 \operatorname{cov} / c^{7 T, 8 T} \\ & x>0.98 \\ & P_{T}<101 \mathrm{HeV} / \mathrm{c} \end{aligned}$ | $\begin{aligned} & 2^{\text {¹ }}+ \\ & \rho_{11}=0.13^{\ddagger} 0.39 \\ & \rho_{22}=0.05 \pm 0.12 \end{aligned}$ | $\begin{cases}1.75 \mathrm{GoV} / \mathrm{c} / 10 / \mathrm{L} & 2^{-1 t} \\ \mathrm{I}>0.6 & \rho_{11=0.01 \pm 0.45} \\ P_{T}<197 \mathrm{MoV} / \mathrm{c} & \rho_{22}=0.34 \pm 0.14\end{cases}$ |  |  | $\left\{\begin{array}{c} 4.16 \mathrm{GeV} / \mathrm{c} / 12 / \\ \mathrm{x}>0.995 \\ \mathrm{P}_{\mathrm{T}}<108 \mathrm{MeV} / \mathrm{c} \end{array}\right.$ |  |  | $\begin{aligned} & 2^{-} \quad 1 \text { it } \\ & \rho_{11}=0.13^{ \pm} 0.20 \\ & \rho_{22}=0.21 \pm 0.07 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x^{0}$ decal Decay mode analyzer |  | P/E | P $\quad$ B | $\mathrm{N}_{\sigma}$ | P/E | P | R | $\mathrm{N}_{\sigma}$ | P/B |
| $\hat{n}$ | $23 \quad 43 \quad 2.6$ | 23/43 | $34 \quad 24$ |  | 33/25 | 28 | 28 | 0 | 27/29 |
| $\eta \pi \pi \quad \hat{\eta}(\vec{k})$ | $\begin{array}{lll}39 & 27 & 1.5\end{array}$ | 39/27 | $24 \quad 34$ |  | 25/33 |  | 26 |  | 28/28 |
| $\hat{\pi} \hat{\pi}(\vec{q})$ | - | 36/30 |  | - | 23/35 |  | 30 |  | 27/29 |
| $\hat{n}$ | - | 16/11 |  | - | 19/23 |  |  | - | 1.02 |
| $\gamma^{+\pi^{+} \pi^{-} \quad \hat{\gamma}(\vec{k})}$ | $7 \pm 420 \pm 41.6 \%$ | 9/18 | $22 \quad 20$ | 0.3 | 24/18 |  |  | - | 0.96 |
| 敉( $(\vec{q})$ | - | 16/11 |  |  | 20/22 |  |  | - | 1.02 |

*These are background - subtracted numbers.


Fig. 1. Polar-equatorial asymmetries for different decay analyzers (a) in the $X^{\circ} \rightarrow \eta \pi \pi$ decay and (b) in the $\mathrm{X}^{\circ} \rightarrow \gamma \pi^{+} \pi^{-}$decay ( $620<$ $<\mathrm{m}<880 \mathrm{MeV} / \mathrm{c}^{2}$ ) vs the $\rho_{22}$ spin density matrix element of the $x^{n}-$ meson. The bounds shown correspond to extreme $\rho_{00}$ values; the rise of $\rho_{00}$ from $\rho_{00}=0$ to $\rho_{00}^{\max }=1-2 \rho_{22}$ is indicated by the arrows. The experimental values have been calculated from the data of Table 1.
of an opposite character of the $\mathrm{P}-\mathrm{E}$ asymmetries at 1.75 and $2.18 \mathrm{GeV} / \mathrm{c}$ (see Fig. 1) are not in contradiction with the pseudotensor $\mathrm{X}^{\circ}$-meson and, even more, the last fact can be considered as a hint against the $0^{-}$assignment $/ 1 /$, although statistically insufficiently grounded. Moreover, although the preliminary data at $4.16 \mathrm{GeV} / \mathrm{c}$ indeed yield an argument against the spin-2 hypothesis, they are not yet able to exclude this possibility.
2. The distribution over the cosine of the angle $\theta$ between the production and decay $X^{\circ}$ spin analyzers is uniform in the case of zero $X^{\circ} \operatorname{spin}$. If the $X^{\circ} \operatorname{spin}$ is 2 , this distribution has the following general form/13/

$$
W(\cos \theta)=\frac{1}{2}\left[1+\frac{10}{7} c_{2} d_{2} P_{2}(\cos \theta)+\frac{18}{7} c_{4} d_{4} P_{4}(\cos \theta)\right],
$$

where $P_{L}(z)$ are the Legendre polynomials. The quantities $c_{L}$ are determined by the production mechanism only. Choosing the production analyzer ( $z$-axis) in the $X^{c}$ production plane (say, along the c.m.s. beam momentum $K$ ), these quantities can be expressed through the $X^{\circ}$ spin density matrix elements in the form

$$
\begin{align*}
& \mathrm{c}_{2}=\rho_{00}+\frac{1}{2}\left(\rho_{-1-1}+\rho_{11}\right)-\left(\rho_{-2-2}+\rho_{22}\right)  \tag{2}\\
& \mathrm{c}_{4}=\rho_{00}-\frac{2}{3}\left(\rho_{-1-1}+\rho_{11}\right)+\frac{1}{6}\left(\rho_{-2-2}+\rho_{22}\right),
\end{align*}
$$

Note that $\rho_{-m-m}=\rho_{m m}$ due to parity conservation in the produc mion process. The quantities $d_{L}$ depend on the $X^{\circ}$ decay mechanism only. They are also determined by the formulae (2) with the elements $\rho_{m m}$ replaced
by the spin density matrix elements $r_{m m}$ in the $X^{\circ}$ decay averaged over the decay phase space and normalized; $m$ is now the $X^{\circ}$ spin projection on the decay analyzer $\vec{v}$. Note that the elements $r_{m}$ m are defined through the $x^{\prime \prime}$ decay amp1itudes $A_{[\lambda]}(m)=\left\langle p_{1} \lambda_{1} \cdots p_{\ell} \lambda_{\ell}\right| \hat{A}|m\rangle$

$$
\begin{equation*}
r_{m^{\prime} m}=\sum_{[\lambda]} A_{\lambda \lambda]}^{\left(m^{\prime}\right) A_{[\lambda]}(m)} \tag{3}
\end{equation*}
$$

where [ $\lambda$ ] are the helicities of decay particles.

The $X^{\circ}$ spin will most clearly manifest itself in the distribution (1) if the $\mathrm{X}^{\circ}$ production and decay analyzers are chosen in such a way that the corresponding quantities $c_{1}$ and $d_{L}$ achieve maximal absolute values. Note that these quantities are limited due to the normalization condition $\operatorname{Sp} \hat{\rho}=1$, i.e.,

$$
\begin{equation*}
\left|c_{2}\right| \leq 1, \quad-\frac{2}{3} \leq c_{4} \leq 1, \tag{4}
\end{equation*}
$$

with the same inequalities valid for the $d_{\text {L }}$ values. We have calculated the decay elements $d_{\text {f }}$ in ref. ${ }^{/ 13 /}$ and analyzed the ques tion of the best decay analyzer in ref./14/ using, however, simplest nonrelativistic matrix elements for the $X^{\circ}$-meson 3 -particle decays. Here we recalculate the $d_{L}$ values for the 3 -particle $X^{\circ}$ decays using the results of the relativistic decay analysis performed in ref:/2/.

The matrix element of the $X^{\circ} \rightarrow \eta \pi \pi$ decay is supposed to be bilinear and quadrulinear in the 4 -momenta of the $\eta$-meson ( $\mathrm{k}_{\mu}$ ) and of the pions ( $\mathrm{p}_{1,2 \mu}$ )

$$
\begin{equation*}
\mathrm{A}_{\mu \nu}=\left(1+\mathrm{a}_{\mathrm{I}} \mathrm{~m}^{2} / \mathrm{m}_{\mathrm{X}}^{2}\right) \mathrm{k}_{\mu} \mathrm{k}+\left(\mathrm{a}_{2}+\mathrm{a}_{3} \dot{\mathrm{~m}}^{2} / \mathrm{m}_{X}^{2}\right) \overrightarrow{\mathrm{q}}_{\mu} \tilde{\mathrm{q}}_{\nu}+2 \mathrm{a}_{4}\left(\overrightarrow{\mathrm{k}} \overrightarrow{\mathrm{q}}_{\mathrm{x}} \mathrm{~m}_{\mathrm{x}} \mathrm{~m}_{\mu} \tilde{\mathrm{q}}_{\nu},\right. \tag{5}
\end{equation*}
$$

where $m\left(m_{x}\right)$ is the dipion ( $\left.x^{n}-m e s o n\right)$ mass, $\tilde{\mathrm{q}}_{\mu}=\left(\mathrm{p}_{1 \mu}-\mathrm{p}_{2 \mu}\right) / 2$ and $\mathrm{a}_{\mathrm{i}}$ are free parameters. The $\mathrm{X}^{\circ} \rightarrow \gamma \pi^{+} \pi^{-}$decay amplitude is a mixture of the M1-, E2- and M3-transitions

$$
\begin{align*}
\mathrm{A} \cdot \mathrm{X} & =\left\{g_{1}[\mathrm{P}, \mathrm{k}, \tilde{\mathrm{q}} \cdot \mathrm{X}, \mathrm{e}\}+\mathrm{g}_{2} \mid \ddot{\mathrm{q}}, \mathrm{k}, \mathrm{k} \cdot \mathrm{X}, \mathrm{e}\right\rceil+  \tag{6}\\
& \left.\left.+\mathrm{g}_{3} \mathrm{k} \cdot \mathrm{X} \cdot \mathrm{k} \mid \ddot{\mathrm{q}}, \mathrm{k}, \mathrm{P}, \mathrm{e}\right] / \mathrm{m}_{\mathrm{X}}^{2}\right\} \mathrm{f}(\mathrm{~m}),
\end{align*}
$$

where $P_{\mu}$ and $X_{\mu I^{\prime}}\left(k_{\mu}\right.$ and $\left.\mathrm{e}_{\mu}\right)$ are the $X^{\circ}(\gamma-$ quantum) 4 -momentum and polarization tensor, $[\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}]=\epsilon_{\mu \nu \rho \sigma} \quad \mathrm{a}_{\mu} \mathrm{b}_{\nu}, \mathrm{c}_{\rho} \mathrm{d}_{\sigma} \quad$ and $\mathrm{f}(\mathrm{m}) \quad$ is the $\rho^{\circ}$-meson propagator. The mixing parameters $g_{i}$ are assumed to be independent of the dipion mass ( $\left.g_{1}-1\right)$. There are three natural decay analyzers in the $X^{n} \rightarrow \eta \pi^{+1} \pi^{-}\left(X^{n} \rightarrow \gamma^{\prime} \pi^{-}\right)$ decay: normal $\overrightarrow{\vec{n}}$ to the $x^{\circ}$ decay plane, $\eta^{-}$ meson ( $\gamma$-quantum) momentum $\vec{k}$ in the $X^{\prime}$ rest frame and $\pi^{+}$-meson momentum $\vec{q}$ in the dipion rest frame. The corresponding decay elements $d_{1}$. and extreme values of $d_{2}$ are presented in Table 2 and Fig. 2. The calculations were performed with different sets of the decay parameters a; and $q$; obtained in the fits of world Dalitz plot data, see Table 1 (FIT 6-13) and Table 2 (FIT 4-5) in ref./2/As is seen from Table 2, the values of the elements $d_{1}$. are quite stable with respect to different fits. Later on we use the divalues obtained with the parameters of FIT $7\left(a_{1}=a_{3}=0\right.$, $a_{2}=-2.2 \pm 0.2, \quad a_{4}=-20.9 \pm 6.3+i(20.1 \pm 6.3) \quad$ and FIT $4 \quad\left(g_{2}=2.3 \pm 0.3, g_{3}=0\right)$ for the $X^{\rho} \rightarrow \eta \pi \pi$ and $X^{\circ} \rightarrow \gamma \pi^{+} \pi^{-}$decays, respectively.

At last, in the $X^{\circ} \rightarrow \gamma \gamma$ decay, the only natural decay analyzer is the $\gamma$-momentum $\vec{k}$ in the $X^{\circ}$ rest frame. The decay matrix element is unambiguously determined by the requirement of the Bose-symmetry and

## Table 2

The values of the quantities $d_{2}$ and $d_{4}$ for different $X^{\circ}$ decay analyzers $\vec{v}$ and different sets of the decay parameters (see Tables 1,2 in ref. ${ }^{2 /}$ ). The $d_{L}$ values in the $\mathrm{X}^{\circ} \rightarrow y \pi^{+} \pi^{-}$decay have been calculated with the dipion mass in the $\rho^{\circ}-$ region, $620-880 \mathrm{MeV} / \mathrm{c}^{2}$. Note that in the $X^{\circ} \rightarrow \eta \pi \pi$ decay $d_{2}^{\text {max }}=0.86, \vec{v}^{\text {max }} \perp \overrightarrow{\mathrm{n}} \quad$ and $\vec{v}^{\text {min }}=\hat{\vec{n}}$. The extreme $d_{2}$ values in the $\mathrm{X}^{\circ} \rightarrow \gamma \pi^{+} \pi^{-}$decay are displayed in Fig. 2.

| $\mathrm{X}^{0}$ decay Fit mode $\mathrm{N}^{\circ}$ | $\mathrm{x}^{0} \text { decay apalyzer }$ |  |  |  | $\hat{q}$ |  | Number of fitted paramoters | $\begin{aligned} & \text { CL } \\ & \% \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | -0.77 | 0.26 | 0.58 | 0.23 | 0.58 | 0.76 | 2 | 22 |
| 7 | -0.77 | 0.27 | 0.57 | 0.30 | 0.60 | 0.69 | 3 | 29 |
| 8 | -0.80 | 0.25 | 0.59 | 0.32 | 0.55 | 0.67 | 3 | $36^{*}$ |
| 9 | -0.77 | 0.26 | 0.56 | 0.30 | 0.60 | 0.68 | 4 | 25 |
| 10 | -0.77 | 0.26 | 0.60 | 0.34 | 0.56 | 0.65 | 4 | 50*/ |
| 11 | -0.79 | 0.25 | 0.59 | 0.23 | 0.55 | 0.76 | 4 | 26 |
| 12 | -0.61 | 0.33 | 0.21 | 0.31 | 0.81 | 0.66 | 5 | 46 |
| 13 | -0.71 | 0.29 | 0.31 | 0.33 | 0.77 | 0.64 | 5 | 42\% |
| 4 | 0.44 | 0.04 | -0.88 | 0.11 | 0.40 | 0 | 1 | 19 |
| 5 | 0.33 | 0.00 | -0.89 | 0.13 | 0.46 | 0 | 2 | 21 |

*Final state $\pi n$-interaction is taken into account.
by the $\gamma$-quantum transversality

$$
\begin{equation*}
A_{i j}=k_{j}\left[\vec{e}^{(1)} \vec{e}^{(2)}\right]_{j} \tag{7}
\end{equation*}
$$

i.e., only the $r_{00}$ element is different from zero. This leads to the maximal possible $d_{L}$ values $d_{2}=d_{4}=1$ thus making the


Fig. 2. The decay coefficients $d_{L}$ vs the mixing parameter $g_{2}\left(g_{3}=0\right)$ in the $X^{\circ} \rightarrow \gamma \pi^{+} \pi^{-}$ decay ( $620<m<880 \mathrm{MeV} / \mathrm{c}^{2}$ ). The vertical lines indicate the corridor $\mathrm{g}_{2}=2.3 \pm 0.3 / 2 /$. The $g_{2}$ dependence of the $\rho_{00}$ element (helicity frame) of the $\rho^{\circ}-m e s o n$ in the $X^{\circ} \rightarrow \gamma \rho^{\circ}$ decay is presented as well. The value $\rho_{00}=0.04 \pm 0.04$ has been fitted from the $\cos \delta$-data of refs. $/ 3,7,13 /$. Note that $\rho_{00}=0$ if the $X^{\circ}$ is a pseudoscalar meson.
$X^{\circ} \rightarrow \gamma \gamma$ decay especially useful for the $\mathrm{X}^{\circ}$-meson spin determination/13,15/.

The quantities $c_{L}$ can vanish in the case when there is no diagonal $X^{\circ}$ spin alignment,
i.e., if $\rho_{\mathrm{mm}_{\mathrm{m}}=1 / 5}$ for all $\mathrm{m}=0, \pm 1, \pm 2$ But in the forward $X^{\circ}$ production process $K^{-} p \rightarrow X^{\circ} \Lambda$ $\left(\pi^{-} p \rightarrow X^{n} n\right)$ or at threshold of this reaction, the $X^{\prime \prime}$ spin projections $\pm 2$ on the c.m.s.. beam momentum $\overrightarrow{\mathrm{K}}$ ( z -axis) are forbidden, i.e., $\rho_{22}=0$. Consequently,

$$
\begin{equation*}
c_{2}=\frac{1}{2}\left(1+\rho_{00}\right), \quad c_{4}=\frac{1}{3}\left(5 \rho_{00}-2\right), \tag{8}
\end{equation*}
$$

so that the anisotropies should be presented in the distribution (1) for an arbitrary ${ }^{\prime}{ }_{0}$ ) value ( $c_{2} \geq 1 / 2$ ). This is illustrated by $\mathrm{Fig}_{\mathrm{i}} .1$, where (based on the $\mathrm{d}_{\mathrm{L}}$ estimates in Table 2 and on the inequalities $0^{\prime}-\mu_{00}{ }^{\prime}\left(1-2 \rho_{22}\right)$ the $2^{-}$predictions for the $\mathrm{P}-\mathrm{E}$ asymmetries in the Adair distribution (1),

$$
\begin{equation*}
(\mathrm{P}-\mathrm{F})^{\prime}(\mathrm{P}+\mathrm{E})=\frac{15}{8}\left(<\mathrm{P}_{2}>-\frac{9}{16}<\mathrm{P}_{4}>\right), \therefore \mathrm{P}_{\mathrm{L}}>=\frac{2}{7} \mathrm{c}_{\mathrm{I}} \mathrm{~d}_{\mathrm{L}} \tag{9}
\end{equation*}
$$

are displayed vs $\rho_{22}$. They fit well with the $P-E$ asymmetries at $2.18 \mathrm{GeV} / \mathrm{c}$ ( $x(0.98)^{17}$, at $1.75 \mathrm{GeV} / \mathrm{c}(x>0.6)^{10 /}$ and at $4.16 \mathrm{GeV} / \mathrm{c}(\mathrm{x}: 0.995)^{/ 12 /}$, the corresponding $\rho .2$ values being equal to $0.05 \pm 0.12$, $0.34 \pm 0.14$ and $0.21 \pm 0.07$, respectively, see also Table 1 . The $P-E$ asymmetries obtained in the latter two experiments also agree well with the isotropic distribution contrary to the asymmetries at $2.18 \mathrm{GeV} / \mathrm{c}$ which have a confidence level only a small fraction of a per cent to be in agreement with isotropy $/ 8 /$. It should be stressed in this context that, even if all the Adair distributions would be consistent with isotropy, the $2^{-}$hypothesis could not yet be excluded, while it is not justified that the Adair cut is sufficient to make $\rho_{22} \ll 1 / 5$.
3. Let us now discuss the question of the Adair cut. It is pointed out in ref. that the cut $x>0.6$ used in the Adair analysis at $1.75 \mathrm{GeV} / \mathrm{c}$ is sufficient to ensure the Adair condition $\rho_{22} \gg 1 / 5$ assuming that only s-and $p$-waves are present in the final state of the reaction $K p \rightarrow X^{\circ} \Lambda$. The dominance of the lowest orbital momentum waves in this experiment is indicated by the $\cos \theta$ c.m. distribution which can be well fitted by a second order polynomial in $x=\cos \theta$ (.om., see the solid curve $W(x)$ in Fig. 3. Unfortunately, the statistics in this experiment are not rich enough to draw an unambiguous conclusion, e.g., the dashed curve $W(x)$ containing a large contribution from the waves with $\ell$ up to 4 also well describes the $x$-distribution in Fig. 3. Note that the $X^{\circ} \quad c . m . s . m o m e n t u m ~(P=243 \mathrm{MeV} / \mathrm{c})$ is not small enough to make centrifugal barriers effective for suppression of the amplitudes with $\ell \geq 2$ as compared to a strong $p$-wave amplitude. In fact, we do not need many waves in order to obtain a large $\rho_{22}$ value for $x>0.6$. Below we show that even only amplitudes with $\ell \leq 2(4)$ can give $\rho_{22} \simeq 0.2(0.3) \quad$ in the interval $x>0.6$. Besides, a simultaneous good description of the $x$-distribution can be achieved in the case with $P \leq 4$. First we recall that the $\rho_{22}$ element is a bilinear product of the amplitudes with $\ell_{z} \geq 1$, i.e., $\rho_{22}(x)=0$ if $\ell^{\max }=0$ and generally

$$
\begin{equation*}
\rho_{22}(x)=F_{n}(x) \sin ^{2} \theta_{\text {c.m. }}, \tag{10}
\end{equation*}
$$

where $F_{n}(x)$ is a polynomial in $x$ of the order $n \stackrel{n}{=} 2 \ell^{\max }-2$. The function $\rho_{22}(\mathrm{x})$ is $1 \mathrm{imi}-$


Fig. 3. The $\cos \theta_{\text {c.m. }}{ }^{-}$ distribution in the ${ }_{\left(\mathrm{X}^{\circ} \rightarrow \eta \pi^{+} \pi^{-}, \mathrm{K}^{-} \mathrm{p}^{+}{ }_{\pi^{-}} \mathrm{X}^{\wedge} \Lambda\right.}$ at $1.75 \mathrm{GeV} / \mathrm{c}^{/ 10 /}$. The solid line $W(x)$ is the second order polynomial fit to this distribution. Curves 1 and 2 are maximal estimates for $p^{m a x}=1$ and 2 , respectively $\left(<\rho_{22}\right\rangle=\frac{1}{5}\langle W$ : is $\mathrm{re}-$ quired). The dashed curves $2 \rho_{22}(x)$ and $W(x)$ correspond to a fit of the $x$-distribution wi.th $\ell^{\text {max }}=4 \quad\left(\bar{\mu}_{22}(x \times 0.6)=\right.$ $=0.3$ is required).
Events/0.1 $x=\cos \theta_{c m}$
ted by the positivity condition

$$
\begin{equation*}
2 \rho_{22}(\mathrm{x}) \leq \operatorname{Sp} \hat{\rho}=\mathrm{W}(\mathrm{x}) \tag{11}
\end{equation*}
$$

and by the fact that no anisotropies are seen in the decay angular distributions averaged over all production angles, i.e.,

$$
\begin{equation*}
<\rho_{\mathrm{mm}}>=\frac{1}{5}<\mathrm{H} . \tag{12}
\end{equation*}
$$

where $\langle\rho\rangle=\int_{-1}^{1} \rho \mathrm{dx}$. For $\ell^{\max }=1$ we then have
$2 \rho_{22}(\mathrm{x})=0.3\langle\mathrm{~W}\rangle \sin ^{2} 0_{\text {c.m. }}$, curve 1 in Fig. 3, yielding the averaged normalized value $\bar{\rho}_{22}(\mathrm{x}>0.6)=0.05 \ll 1 / 5\left(\bar{\rho}_{22}=\int \rho_{22} \mathrm{dx} / \int \mathrm{Wdx}\right)$. For $\ell^{\max }=2$ the function $\rho_{22}(\mathrm{x})$ cannot be determined unambiguously. In Fig. 3 we show curve 2 for the function $2 \rho_{22}(x)$, normalized by the condition (12), yielding the maximal value of the element $\rho_{22}$ in the interval $x>0.6$, $\bar{\rho}_{22}=0.18=1 / 5$. This curve also satisfies the positivity condition (11). A large value of $\bar{\rho}_{22}(x>0.6)$ and at the same time a good description of the $x$-distribution can be achieved with $\ell^{\text {max }}=4$, see dahsed curves $W(x)$ and $2 \rho_{22}(x)$ in Fig. 3. In this quite approximate fit with an essentially reduced number of possible free parameters, we $c_{1} \operatorname{aim} \bar{\rho}_{22}(x>0.6)=0.3$. The corresponding $p_{T}^{2}$-dependence ( $p_{T}$ is the $X^{\circ}$ perpendicular momentum) of the $\bar{\rho}_{22}$ value in the interval $\left(0, p_{\mathrm{T}}\right)$ displayed in Fig. 4 , indicates that the Adair condition $\left(\rho_{22} \ll 1 / 5\right)$ would be fulfilled in this case only for extremely forward produced $\mathrm{X}^{\circ}$-mesons.

Let us now discuss some other considerations concerning the Adair condition. Usually $\mathrm{p}_{\mathrm{T}} \ll \mathrm{R}^{-1}-200 \mathrm{MeV} / \mathrm{C}$ is required, where $R$ is an effective radius of $\sim 1 \mathrm{fm}$. This inequality follows from the quasiclassical relation $\left\langle\ell_{z}\right\rangle-\operatorname{Rp}_{T}$ and from the fact that the element $\rho_{22}$ is a bilinear product of the amplitudes with $\ell_{z} \geq 1$. This can be written in a more quantitative form, e.g., in a rather general absorption model of Dar, Watts and Weiskopf/16/According to this model, at high energies and small values of $p_{T}$, the $p_{T}$ dependence of the helicity ampli-
tudes of quasi-two-body reactions is almost completely determined by the absorption effects, i.e., very approximately,

$$
\begin{equation*}
T_{[\lambda]} \sim C_{[\lambda]} J_{\mathbf{n}}\left(R \sqrt{t^{\prime}}\right), \tag{13}
\end{equation*}
$$

where $t^{\prime}=|t|-|t|_{\text {min }}(t$ is the 4 -momentum transfer) and $J_{n}(y)$ is the Bessel function of the $n$-th order; $n$ is the helicity change, $n=\left|\lambda_{X} \lambda^{-\lambda} \Lambda_{p}^{+\lambda}\right|$ for the reaction $K^{-} p \rightarrow X^{\circ} \Lambda$.
The constants $G_{[\lambda]}$ depending on the helicity configurations can be estimated, e.g., from the one-particle exchange diagrams*. It follows from eq. (13) that the $x$-distribution for $x$ close to 1 can be approximated as

$$
\begin{equation*}
W(x)=J_{0}(y)^{2}+a J_{1}(y)^{2}, \quad y=R \sqrt{t^{\prime}}, \tag{14}
\end{equation*}
$$

where $a \geq 0$ is some parameter. Besides, the function $\frac{1}{2}$ aj ${ }_{1}(y)^{2}$ gives an upper estimate for the element $\rho_{22}$. We have fitted the $t^{\prime}$ distribution at $2.18 \mathrm{GeV} / \mathrm{c}$ by the formula (14) and achieved a good description for $t^{\prime}<0.2(\mathrm{GeV} / \mathrm{c})^{2}$ with the parameters $\mathrm{R}=1.3 \pm 0.1 \mathrm{fm}$ and $\mathrm{a}=3.0 \pm 0.7$. The corresponding $\mathrm{p}_{\mathrm{T}}^{2}$ dependence of the upper $\rho_{22}$ estimate in the interval ( $0, \mathrm{p}$ T) is shown in Fig. 4, curve 2. A similar fit of the $1.7 \overline{5 \mathrm{GeV}} / \mathrm{c}$ data $\left.\mathrm{t}^{\prime}<0.2(\mathrm{GeV} / \mathrm{c})^{2}\right)$ yields $R=1.0 \pm 0.5 \mathrm{fm}, a=2.1 \pm 1.7$ and the upper

[^1]

Fig. 4. The estimates of the normalized $\rho_{22}$ elements in the intervals ( $0, \mathrm{p}_{\mathrm{T}}$ ) vs $\mathrm{p}_{\mathrm{T}}^{2}$ Curve 1 has been obtained from a fit of the $x$-distribution at $1.75 \mathrm{GeV} / \mathrm{c}$ with $\ell^{\max }=4$ providing $\bar{\rho}_{22}(x>0.6)=0.3$. Curve 2 is an upper $\rho_{22}$ estimate following from a fit of the 't' -distribution at $2.18 \mathrm{GeV} / \mathrm{c}$ to the formula (14).
$\rho_{22}$ estimate compatible with curve 2 as well. Of course, these estimates cannot be considered too seriously at so small primary momenta.

The experimental $\rho_{22}$ values at $1.75 \mathrm{GeV} / \mathrm{c}$ ( $p_{T}<197 \mathrm{MeV} / \mathrm{c}$ ) and at $2.18 \mathrm{GeV} / \mathrm{c} \quad\left(\mathrm{p}_{\mathrm{T}}<\right.$ $<101 \mathrm{MeV} / \mathrm{c}$ ) are not in contradiction with
curve 2 , while the $\rho_{22}$ value at $4.16 \mathrm{GeV} / \mathrm{c}$ ( $\mathrm{p}_{\mathrm{T}}<108 \mathrm{MeV} / \mathrm{c}$ ) is one standard deviation higher than the upper estimate according to curve 2. The latter fact can be interpreted as an argument in favour of the $\mathrm{X}^{\circ}$-meson pseudoscalarity (assuming that curve 2 is near the true upper $\rho_{22}$ bound at $4.16 \mathrm{GeV} / \mathrm{c}$ ). However, we should take care of this point. Assuming, e.g., that unnatural parity exchange dominates in the $\mathrm{X}^{\circ}$ production amplitude, a rapid increase of the $\rho_{22}$ element (at small $p_{T}$ ) can be obtained with increasing primary momentum. Note that a strong energy dependence of the $\rho_{22}$ element may be also indicated by the absence of anisotropies in LBL data for $\mathrm{x}>0.98^{/ 8 /}$ (these data come mainly from the $2.65 \mathrm{GeV} / \mathrm{c}$ exposure; the correspodning $\mathrm{p}_{\mathrm{T}} \mathrm{s}$ are less than $136 \mathrm{MeV} / \mathrm{c}$ ).

We thus see that the large $\rho_{22}$-values obtained in the $2^{-}$fits of the Adair distributions at $1.75 \mathrm{GeV} / \mathrm{C} \quad\left(\mathrm{p}_{\mathrm{T}}<195 \mathrm{MeV} / \mathrm{c}\right)$ and at $4.16 \mathrm{GeV} / \mathrm{c} \quad\left(\mathrm{p}_{\mathrm{T}}<108 \mathrm{MeV} / \mathrm{c}\right.$ (see Fig. 1) apparently do not yield conclusive arguments against the spin-2 assignment. Sometimes the absence of anisotropies in the production and decay correlations averaged over all production angles is also used as an argument against a nonzero $\mathrm{X}^{\circ}$ spin because such a situation seems un1ikely in an incident $\mathrm{K}^{-}$momentum range of $2: 5 \mathrm{GeV} / \mathrm{c}^{/ 6 /}$. However, this argument is model-dependent and evidently not conclusive as well.

Therefore the $X^{\circ}$ spin parity analyses performed up to now give no definite answer to this problem. Both $0^{-}$and $2^{-}$hypotheses are still possible. It is seen that the eventual solution of this question requires
a comprehensive study of the $X^{\circ}$ production and decay correlations in different kinematical regions (especially near threshold) and in different reactions $1 \times 3 /$ Probably, the most suitable and relatively simple experiment would be a study of the Adair distribution in the reaction $\pi^{-} p \rightarrow \mathrm{X}^{\circ} \mathrm{n}^{/ 15 /}$.

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## REFERENCES

1. Lednický R. JINR, B5-2-9753, Dubna, 1976; Czech.J.Phys., 1976, B26, p. 1242.
2. Lednický R. JINR, E2-10521, Dubna, 1977.
3. Rittenberg A. Ph.D.Thesis. UCRL Report No. UCRL-18863, Rerkeley, 1969.
4. Aguilar-Benitez M. e.a. Phys.Rev., 1972, D6, p. 29.
5. Danburg J.S. e.a. Experimental Meson Spectroscopy, 1972, ed. by A.H.Rosenfeld and K.W.Lai (American Institute of Phys., New York, 1972), p. 91.
6. Jacobs S. e.a. Phys.Rev., 1973, D8, p. 18.
7. Danburg J.S. e.a. Phys. Rev., 1973, D8, p. 3744 .
8. Kalbfleisch G.R. e.a. Phys.Rev.Lett., 1973, 31, p. 333.
9. Giler S., Klosinski I., Lefik W., Tybor W. Acta Phys.Polon., 1970, A37, p. 475.
10. Baltay C. e.a. Phys. Rev., 1974, D9, p. 2999.
11. Particle Data Group, Review of Particle Properties, Rev.Mod.Phys., 1976, 48, No. 2, Part 2.
12. Amsterdam - CERN - Nijmegen - Oxford Collaboration, "A Measurement of the $\eta^{\prime}$ Spin Parity", CERN/EP/PHYS 76-37, Paper submitted to the International Conf. on High Energy Phys., Tbilisi, 15-21 July 1976.
13. Lednický R., Ogievetsky V.I. Zaslavsky A.N. JINR, E2-7666, Dubna, 1974; YaF, 1974, 20, p. 203.
14. Lednický R. JINR, E2-7801, Dubna, 1974.
15. Lednický R., Shafranov M.D. JINR, E1-8452, Dubna, 1974; YaF, 1975, 22, p. 837 .
16. Dar A., Watts T.L., Weisskopf V.F. Nucl.Phys., 1969, B13, p.477.

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[^0]:    *Compare with ref. ${ }^{5 /}$ where the $\mathrm{X}^{\circ}$-meson was claimed to be pseudoscalar based on the same data but with a much weaker Adair cut.

[^1]:    *At low energies, besides $t$-channel exchanges, s-channel effects may be important. Note that the c.m.s. energy at $1.75 \mathrm{GeV} / \mathrm{c}$ is only 30 MeV higher than the strong $\mathrm{K}^{-} \mathrm{p}$ resonance $\Lambda(2100) 7 / 2^{-}$.

