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THE PROBLEM OF THE X° /958/ SPIN.

Part 1. Dalitz Plot Analysis

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Ледницки Р. **E2 - 10521**

К вопросу о спине $X^{\circ}[958]$. Часть 1. Анализ диаграмм Далица

На основании феноменологического анализа распадов $X^{\circ} \to \eta\pi\pi$ и $X^{\circ} \to \gamma\pi^{+}\pi^{-}$ с помощью релятивистских матричных элементов делается заключение о том, что гипотезы 0^{-} и 2^{-} для спин-четности $X^{\circ}[958]$ - мезона одинаково хорошо согласуются с мировыми данными по диаграммам Далица.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

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The Problem of the $X^{\circ}[958]$ Spin. Part I.Dalitz Plot Analayis

Basing on the relativistic description of the $X^{\circ} \to \eta\pi\pi$ and $X^{\circ} \to \gamma\pi^{+}\pi^{-}$ decays, we conclude that possible 0^{-} and 2^{-} X°-spin parity hypotheses equally well agree with world Dalitz plot data.

The investigation has been performed at the Laboratory of High Energies, JINR.

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The $X^{\circ}(958)$-meson was observed in 1964 in the reaction $K^{-}p \to K^{\circ}\Lambda$ /1/ and, basing on the Dalitz plot analysis of the $X^{\circ} \to \eta\pi\pi$ decay, it was claimed to be pseudoscalar. It was cleared up in 1967 by Ogievetsky, Tybor and Zaslavsky that the X°-meson pseudoscalarity was not proved and that the 2^{-} spin parity hypothesis also well agreed with experimental decay data /2/. Since that the question has been discussed many times and now, according to the latter edition of the Review of Particle Properties /3/, the pseudoscalar hypothesis is again supposed to be established: "The Dalitz plot analyses favour spin 0, but cannot rule out spin 2. The indication of anisotropy in the decay of very forward produced η' /KALBFLEISCH 73/ has not been confirmed by BALTAY 74 thus favouring strongly spin 0". It should be stressed that the 0^{-} hypothesis is more natural in comparison to the 2^{-} one: it is simpler and, besides, the X°-meson is the nearest (lightest) candidate for the ninth pseudoscalar meson; it is even called the η'-meson *.

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* Note, however, that there exist symmetry formulae predicting the η' mass around 1.5 GeV /4/.

We could probably be satisfied by the present experimental proofs of the X° -meson pseudoscalarity if for several times the proofs were not found wrong. Despite the fact the "minimum complexity" arguments beforehand prompt the answer, it seems to us that the important question of the X° -meson spin parity should be and, of course, can be resolved experimentally. We have analyzed the experimental situation in ref. /5/ and have found that this question still remains open, i.e., $J^P(X^\circ) = 0^-$ or 2^-. In the present paper we give the details concerning the Dalitz plot analysis. Basing on the relativistic description of the $X^\circ \to \eta\pi\pi$ and $X^\circ \to \eta\pi^+\pi^-$ decays, we conclude that both the 0^- and 2^- hypotheses equally well agree with world Dalitz plot data.

The $X^\circ \to \eta\pi\pi$ decay. Belief in the X° -meson pseudoscalarity is partly based on the Dalitz plot analysis of this decay with the help of the simplest nonrelativistic matrix elements /6/. Of course, as it is often remarked, more complicated matrix elements could improve the Dalitz plot description in the pseudotensor case. Below we show that for this purpose a mixture of bilinear and quadrulinear in 4-momenta matrix elements is required, i.e., the amplitude $\ell_\eta, \ell_{\pi\pi} = 2,2$ should be taken into account.

Let us start from the pseudoscalar case. The 4-momenta, we have at our disposal, are those of the η-meson (k_μ) and of the pions ($p_{1,2\mu}$). The mixture of bilinear and quadrulinear matrix element constructed out of these 4-momenta can be written in the form

$$A = 1 + a_1 m^2/m_X^2 + a_2 m^4/m_X^4 + a_3 k^2 q^2 \cos^2\delta/m^2 m_X^2 , \quad (1)$$

where m is the dipion mass; m_X is the X° -meson mass; $\vec{k}$ is the η-meson momentum in the X° rest frame $(k = |\vec{k}|)$, $\vec{q}$ is the π momentum in the dipion rest frame $(q = |\vec{q}|)$, and δ is the angle between the vectors $\vec{k}$ and $\vec{q}$; a_i are free parameters. Note that C -parity conservation requires symmetry under spatial interchange of the two pions. Further, $a_2 = a_3 = 0$ if only bilinear in momenta terms contribute. In this case formula /1/ coincides with the usual "linear" matrix element /7/ $A + 1 + \alpha Y$, where Y is the y -coordinate on the triangle Dalitz-Fabri plot, $\alpha \simeq -a_1/(3a_1 + 22)$. As indicated in Table 1, FIT 1-3, a good description of the world (unsubtracted) Dalitz plot data /6-11/ can be achieved with a bilinear in momenta matrix element; $a_1 = 2.8 \pm 1.0$ yields the slope parameter $\alpha = -0.09 \pm 0.03$, i.e., the same value as obtained in ref. /11/. These fits are not sensitive to the imaginary part of the parameter a_1. In FIT 4 we take into account the s-wave final state $\pi\pi$-interaction by multiplying the $\ell_{\pi\pi}=0$ decay amplitude by the Omnès function $f_0(m)$. The s-wave $\pi\pi$ phase shift is described by the $\epsilon(700)$ resonance with a width $\Gamma_\epsilon = 500$ MeV, i.e., /12/,

$$f_0(m) = (m_\epsilon^2 - 4m_\pi^2)/(m_\epsilon^2 - m^2 - i m_\epsilon \gamma_\epsilon), \gamma_\epsilon = \Gamma_\epsilon (q/q_\epsilon)^\ell m_\epsilon/m, \quad (2)$$

where $\ell=0$. However, the fit is not improved.

The matrix element of the pseudotensor X° decay, containing bilinear and quadrulinear in momenta terms, takes the form

$$A_{\mu\nu} = C_1 k_\mu k_\nu + C_2 \tilde{q}_\mu \tilde{q}_\nu + C_3 (kq/m_X m) k_\mu \tilde{q}_\nu , \quad (3)$$

Table 1

The results of fitting the m-/10,11/ and $\cos\delta$ -/6-10/ distributions in the $X^\circ \to \eta\pi^+\pi^-$ decay

FIT No.	J^P	a_1	a_2	a_3	a_4	χ^2/ND	CL %
1	0^-	0	0	0	-	29.8/11	0.1 */
2		2.8±1.0	"	"	-	14.8/10	14 */
3		"	"	"	-	15.9/14	32
4		3.2±1.1	"	"	-	17.7/14	23 **/
5	2^-	0	-1.8±0.1	0	-32.1±2.4	14.1/9	12
6		"	-1.9±0.1	"	-29.8±1.4	16.5/13	22
7		"	-2.2±0.2	"	-20.9±6.3+i(20.1±6.3)	14.1/12	29
8		"	-2.1±0.3	"	-19.3±7.5+i(32.4±7.3)	13.2/12	36 **/
9		"	-2.1±0.4+i(-0.5±1.6)	"	-22.6±10.2+i(22.6±8.5)	13.8/11	25
10		"	-1.6±0.6+i(1.4±0.6)	"	-10.4±10.2+i(27.0±6.1)	10.3/11	50 **/
11		5.5±14	-2.6±2.5	-1.1±26	-49.6±56	13.6/11	26
12		-7.4±4.9	-4.3±1.2	18.6±2.5	9.5±12.5+i(10.9±10.9)	9.8/10	46
13		-7.5±7.3	-3.2±1.0	13.5±12.3	7.5±25.0+i(12.6±21.7)	10.4/10	42 **/
14		0	-0.1±0.3+i(2.8±0.1)	0	0	77/13	$<10^{-3}$

*Only the m-distribution is fitted.
**The final state $\pi\pi$ -interaction taken into account.

where $\tilde{q}_\mu = (p_{1\mu} - p_{2\mu})/2$ and the quantities C_i can be parametrized as follows

$$C_1 = 1 + a_1 m^2/m_X^2,\ C_2 = a_2 + a_3 m^2/m_X^2,\ C_3 = 2a_4. \quad (4)$$

Note that $a_1 = a_3 = a_4 = 0$ provided the X° decay matrix element is bilinear in 4-momenta of the decay particles; $a_2 = -4$ if the decay amplitide further satisfies the Adler self-consistency condition, i.e., if it takes the form/2/ $A_{\mu\nu} = p_{1\mu} p_{2\nu}$. Introducing the unit vectors $\hat{\vec{k}}$ and $\hat{\vec{q}}$ along the momenta $\vec{k}$ and $\vec{q}$, respectively, the matrix element/3/ can be rewritten in a more convenient form

$$A_{ij} = w_0 \hat{k}_i \hat{k}_j + w_2 \hat{q}_i \hat{q}_j + w_4 \hat{k}_i \hat{q}_j \cos\delta, \quad (5)$$

where the quantities w_L are given by the expressions

$$w_0 = k^2 [C_1 + (kq\cos\delta/(\omega+m)m)^2 (C_2 + C_3(\omega+m)/m_X)], \quad (6)$$

$$w_2 = q^2 C_2,$$

$$w_4 = k^2 q^2 [2C_2 + C_3(\omega+m)/m_X]/(\omega+m)m,$$

where ω is the dipion energy in the X°-meson rest frame. Note that the quantities $w_0 + w_4/3$, $w_2 + w_4/3$ and w_4 represent essential parts of the amplitudes with $\ell_\eta, \ell_{\pi\pi} = (2,0)$, $(0,2)$ and $(2,2)$. The Dalitz plot distribution, proportional to the square of the matrix element/5/ averaged over the X° spin projections, is of the form

$$dN/kq\,dm\,d\cos\delta = |w_0|^2 + |w_2|^2 + \frac{1}{4}|w_4|^2\cos^2\delta(3+\cos^2\delta) + \tag{7}$$
$$+ \mathrm{Re}\,w_0^* w_2(3\cos^2\delta - 1) + 2\mathrm{Re}\,w_4^*(w_0+w_2)\cos^2\delta.$$

The parameters a_i, determined by fitting the m- and $\cos\delta$-distributions, are presented in Table 1, FIT 5-14. Note that large errors of the parameters in FIT 11-13 arise from strong correlations, especially between the parameters $a_1(a_3)$ and a_2. In FIT 8, 10, 13 we take into account the final state $\pi\pi$-interaction as well, making replacements

$$k^2C_1 \to (k^2C_1 + w_4/3)f_0(m) - w_4/3,\ w_2 \to (w_2 + w_4/3)f_2(m) - w_4/3 \tag{8}$$

in eqs. (6), with the function $f_0(m)$ defined in eq. (2) and the function $f_2(m)$ given by a similar formula (with $\ell = 2$) assuming the d-wave $\pi\pi$ phase shift is described by the f(1271) resonance with a width $\Gamma_f =$ = 180 MeV. This slightly improves the FITS 7,9 but not the FIT 12. The following conclusions can be drawn:

(1) The bilinear in momenta matrix element $(a_1 = a_3 = a_4 = 0)$ is not consistent with the experimental data (see FIT 14).

(2) The quadrulinear in momenta terms essentially contribute to the amplitude w_4. However, the value of this amplitude seems to be not excessive if we take into account that even a small natural contribution, coming from a bilinear matrix element (w_4 depends on the parameter a_2) due to Lorentz transformation to the dipion rest frame, composes 10-30% of $\mathrm{Re}\,w_4$.

(3) The fitted value of the parameter a_2 is not far from -4 which is predicted by the simplest (bilinear) matrix element satisfying the Adler self-consistency condition /2/.

(4) The matrix element, obtained by fitting the m-distribution only (see FIT 5), predicts almost a uniform $\cos\delta$-distribution in agreement with the experimental data.

The $X^\circ \to \gamma\pi^+\pi^-$ decay. It is well-known that the pions in this decay are mainly produced in the p-wave ρ°-meson state /3/. Consequently, the X° decay matrix element should be linear in the relative four-momentum of the two pions $\tilde{q}_\mu$ and, of course, in the γ polarization vector e_μ. Besides, it can contain the 4-momenta of the γ-quantum (k_μ) and of the X°-meson (P_μ). Gauge invariance further implies that under transformation $e_\mu \to e_\mu + k_\mu$ the matrix element must remain invariant.

The pseudoscalar X° decay is then described by the only matrix element, i.e., M1-transition (dipole)

$$A = g_1[\vec{q}\vec{k}]\vec{e}f(m), \tag{9}$$

where $\vec{q}$ is the π^+ momentum in the dipion rest frame, $\vec{k}$ is the γ momentum in the X° rest frame, and $f(m)$ is the Omnès function similar to that in eq. (2) (with $\ell = 1$) due to the ρ° dominance in the p-wave $\pi^+\pi^-$ phase shift ($m_\rho = 770$ MeV, $\Gamma_\rho = 150$ MeV). We put $g_1 = 1$ though g_1 can depend on the dipion mass. The Dalitz plot distribution is obtained by squaring the amplitude (9) and summing over the γ polarizations

Table 2

The results of fitting the m- and $\cos\delta$ - distributions [6,7,13] in the $X^\circ \to \gamma\pi^+\pi^-$ decay

FIT No	J^P	Values of free parameters g_2	g_3	χ^2/ND	CL %
1	0^-	-	-	53.3/45	19 */
2	"			66.3/57	19
3	2^-	1.9 ± 0.6	0	47.6/44	33 */
4	"	2.3 ± 0.3	0	64.6/56	19
5	"	2.5 ± 0.4	-3.4 ± 68	63.3/55	21

*Only the m-distribution is fitted.

$$dN/kqdmd\cos\delta = |g_1 f(m)|^2 q^2 k^2 \sin^2\delta, \qquad (10)$$

where δ is the angle between the vectors $\vec{k}$ and $\vec{q}$. As is indicated in Table 2, FIT 1,2, the world data [6,7,13] are well described by this distribution.

In the case of the pseudoscalar X° decay there are three independent matrix elements corresponding to the M1-, E2- and M3-transitions (dipole, quadrupole and octupole) [7]

$$A\cdot X = \{g_1 [P,k,\tilde{q}\cdot X, e] + g_2[\tilde{q},k,k\cdot X,e] + g_3 k\cdot X\cdot k[\tilde{q},k,P,e]/m_X^2\} f(m), \qquad (11)$$

where $X_{\mu\nu}$ is the X° polarization tensor, $[a,b,c,d] = \epsilon_{\mu\nu\rho\sigma} a_\mu b_\nu c_\rho d_\sigma$ and g_i are mixing parameters. The first two terms represent some mixture of the M1- and E2-transitions and the third term is proportional to the octupole contribution M3. In the X° rest frame we then have

$$A_{ij} = G_1 [\hat{\vec{k}}\vec{e}]_i \hat{q}_j + G_2[\hat{\vec{k}}\hat{\vec{q}}]_i e_j + G_3[\hat{\vec{k}}\vec{e}]_i \hat{k}_j , \qquad (12)$$

where $\hat{\vec{k}}$ and $\hat{\vec{q}}$ are the unit vectors along the momenta $\vec{k}$ and $\vec{q}$ and the quantities G_i depend on the mixing parameters g_i:

$$G_1 = (g_1 - g_2 k/m_X - g_3 k^2/m_X^2) kqf(m),$$

$$G_2 = g_1 kqf(m) - G_1, \qquad (13)$$

$$G_3 = \{[g_2(1 + m/m_X) - g_1](m - m_X + k)/m + g_3 k^2/m_X^2\} kqf(m).$$

Again, we assume the parameters g_i to be independent of the dipion mass and put $g_1 = 1$. In our earlier papers [14,15] we have neglected the octupole contribution ($g_3 = 0$) and, besides, we have supposed: G_2/G_1 = const and $G_3/G_1 = 0$. Such assumptions are reasonable provided that only events with the dipion mass near the ρ° mass are analyzed, i.e., if $k^2/m_X^2 \ll 1$. The Dalitz plot distribution has been calculated in ref. [7] for the following two particular cases: $g_1 = 1$, $g_2 = a$, $g_3 = 0$ and $g_1 = g_2 = 0$, $g_3 = 1$ *. For the general case we have

$$dN/kqdmd\cos\delta = \left(\frac{1}{6}|G_1 + 2G_2|^2 + \frac{1}{2}|G_1 + G_2|^2 + \frac{1}{2}|G_1|^2\right)\sin^2\delta + |G_1 + G_3|^2 \cos^2\delta. \qquad (14)$$

* In ref. [7] the γ momentum k_γ has been calculated in the dipion rest frame, i.e., $k_\gamma = km/m_X$.

As is indicated in Table 2, FIT 3-5, the experimental data well agree with this distribution. In particular, note the following:

(1) The fits are not very sensitive to the octupole contribution g_3 and to the imaginary part of the mixing parameter g_2 as well.

(2) The fitted value of the parameter g_2 is not far from the value $g_2 = g_1 = 1$ predicted by the simplest matrix element satisfying the Adler self-consistency condition (a rather strong violation of this condition can be expected due to a large energy release in the $X^\circ \to \gamma \pi^+ \pi^-$ decay). Note that a similar matrix element ($g_2 = g_1 = 1$, $g_3 = 0$) was suggested earlier /2/ from another consideration.

(3) The matrix elements, obtained by fitting the m-distribution only, predict almost a zero value of the ρ_{00} spin density matrix element of the ρ°-meson in the $X^\circ \to \gamma \rho^\circ$ decay, i.e., $W(\cos\delta)$ close to $\sin^2\delta$. This is in agreement with the experimental $\cos\delta$-distribution.

In conclusion, we point out that both the pseudoscalar and pseudotensor matrix elements equally well describe the world Dalitz plot data on the $X^\circ \to \eta\pi\pi$ and $X^\circ \to \gamma \pi^+ \pi^-$ decays. The "minimum complexity" argument in favour of the more simple 0^- hypothesis is sufficiently weakened by the fact that the values of the 2^- amplitudes are found to be essentially real and not far from the simplest relativistic matrix elements satisfying the Adler self-consistency condition. We thus see that the X°-meson spin can be established only by studying the X°-production and decay correlation, desirably for very forward produced X°s (Adair analysis).

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