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A HIGH ENERGY REPRESENTATION IN THE RELATIVISTIC HAMILTONIAN THEORY

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A HIGH ENERGY REPRESENTATION IN THE RELATIVISTIC HAMILTONIAN THEORY

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Высокоэнергетическое представление в релятивистской гамильтоновой теории

Получены представления для амплитуды рассеяния при высоких энергиях путем суммирования диаграмм ковариантной гамильтоновой формулировки квантовой теории поля.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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A High Energy Representation in the Relativistic Hamiltonian Theory

Representations for the scattering amplitude at high energies are obtained by summing the diagrams of the covariant Hamiltonian formulation of QFI and the diagrams of the three-dimensional formulation of QFT on the light cone.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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I. INTRODUCTION

Basing on high-energy representation for the scattering amplitude it is possible to describe a number of properties of the hadron interaction at high energies (see reviews /1,2/). The so-called eikonal representation for the two-particle amplitudes was obtained by summing the perturbation theory series in the four-dimensional Feynman-Dyson formalism /3/, on the basis of the quasipotential equation (QPE) /1/ and by the functional integration method in quantum field theory (QFT) /2/.

In paper /4/ the high energy representation for the scattering amplitude was derived in the framework of the QPE /5/ using Fourier-analysis on the three-parametric group of horospherical shiftings which is embedded as a subgroup in the Lorentz group /6,7/. This representation has the form

$$T(s,t) = -2is\int e^{-i\tilde{\Delta}\tilde{\varrho}} d^2\tilde{\varrho}\left\{\int d^2\tilde{\varrho}_1 \langle\tilde{\varrho}_1|\hat{P}_z \exp\left[\frac{i}{2s}\int \hat{V}_s(z)dz\right]|\tilde{\varrho}\rangle - 1\right\}, \quad \tilde{\Delta}^2 = -t, \qquad (1.1)$$

where $\hat{P}_z$ is an ordering operator, in which instead of the usual θ-functions the step functions

$$\hat{\theta}(z) = \frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{e^{iaz}}{e^{a}-1-i\varepsilon}da$$

of the finite-difference analysis /7,8/ are used; and $\hat{V}_s(z)$ is a quasipotential operator in the space of "state vectors" $|\tilde{\varrho}\rangle$ with two-dimensional vector $\tilde{\varrho} = (\varrho_1, \varrho_2)$, being the analogue of the impact parameter. Formula (1.1) is a direct relativistic generalization of the eikonal representation of the non-relativistic quantum mechanics /9/.

The aim of this paper is to derive the generalized eikonal representation (1.1) for the scattering amplitude at high energies by summing the diagrams of the covariant Hamiltonian formulation of QFT /10/ and diagrams of the three-dimensional formulation of QFT on the light cone /11/. We consider the interaction Lagrangian $\mathcal{L}(x)=g:\psi^2(x)\phi(x):$, where $\psi(x)$ is the "scalar nucleon" with mass M and $\phi(x)$ is the scalar meson with mass m.

2. A GENERALIZED EIKONAL REPRESENTATION IN THE RELATIVISTIC HAMILTONIAN THEORY

First we demonstrate how the usual eikonal representation is derived in the framework of the three-dimensional formulation of QFT by summing the generalized ladder diagrams, describing the scattering of two high energy "nucleons", with the help of the variational derivatives method (see Fig.1). To this end let us

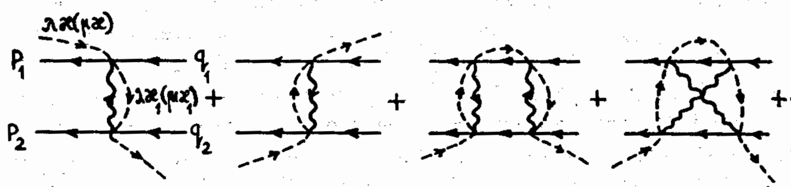

Fig.1

recall the basic rules for constructing matrix elements. Suppose that all vertices of a given Feynman diagram are numbered. Then the continuous dotted line of quasiparticles must connect all vertices and be oriented along the increasing vertex number. Internal solid lines of the physical particles are oriented in the opposite direction-along the decreasing vertex number. To each internal dotted line with the 4-momentum $\lambda\chi$ $(\lambda^2=\lambda_0^2-\vec{\lambda}^2=1, \lambda^0>0)$ there corresponds the factor $\frac{1}{2\pi}\frac{1}{\chi-i\varepsilon}$, and to each internal "nucleon" (meson) line with the 4-momentum k - the function $\Delta^{(+)}(k)=\Theta(k^0)\times\delta(k^2-M^2)$ $(\mathcal{D}^{(+)}(k)=\Theta(k^0)\delta(k^2-m^2))$.

The sum of diagrams of Fig.1 gives the following expression for the amplitude on the energy shell (i.e. when $p_1+p_2=q_1+q_2$)

$$(2\pi)^4 T(p_1,p_2;q_1,q_2)\delta(p_1+p_2-q_1-q_2)=$$
$$=\lim_{p_i^2,q_i^2\to M^2}\prod_{i=1}^{2}(p_i^2-M^2)(q_i^2-M^2)\mathcal{K}_\varphi\mathcal{K}_A G(p_2,q_2|\chi_2)G(p_1,q_1|\chi_1)\Big|_{\chi_1=\chi_2=0}, \qquad (2.1)$$

where $G(p,q|\chi)=\int dx\,dx' e^{ipx-iqx'}G(x,x'|\chi)$ is the Fourier-transform of the one-particle Green function of nucleon in the external field $\chi(x)\equiv A(x)\varphi(x)$, which satisfies the equation

$$[\partial_x^2-M^2-g\chi(x)]G(x,x'|\chi)=\delta(x-x').$$

The operators $\mathcal{K}_A$ and $\mathcal{K}_\varphi$, in which there enter derivative operators over external fields A_i (i = 1,2) and φ, have the form

$$\mathcal{K}_A=\exp\Big\{-ig^2\int du\,dv\Big[\theta(\lambda u-\lambda v)\overset{(-)}{\mathcal{D}}(u-v)-\theta(\lambda v-\lambda u)\overset{(+)}{\mathcal{D}}(u-v)\Big]\frac{\delta^2}{\delta A_1(u)\delta A_2(v)}\Big\},$$

$$\mathcal{K}_\varphi=\int dz\frac{\delta}{\delta\varphi(z)}\exp\Big\{\int\theta(\lambda z-\lambda z_1)\frac{\delta}{\delta\varphi(z_1)}dz_1\Big\},$$

where $\mathcal{D}^{(\mp)}(x)=\pm i(2\pi)^{-3}\int e^{\mp ikx}\mathcal{D}^{(+)}(k)dk$ is the negative (positive)-frequency part of the Pauli-Jordan commutator function. The operator $\mathcal{K}_\varphi$ appears due to the presence of quasiparticles in the theory.

The validity of the formula (2.1) can be verified by perturbation expansion using the following formal properties of Θ-function:

$$
\begin{aligned}
&1)\ \Theta(x-x_1)\Theta(x-x_2)=\Theta(x-x_1)\Theta(x_1-x_2)+\Theta(x-x_2)\Theta(x_2-x_1);\\
&2)\ \Theta(x_1-x_2)\Theta(x_2-x_3)\cdots\Theta(x_n-x_1)=0, \quad n\geq 2;\\
&3)\ \Theta^n(x)=\Theta(x), \quad n\geq 1;\\
&4)\ \sum_{\substack{\text{over all } n!\\ \text{permutations}}}\Theta(x_{i_1}-x_{i_2})\Theta(x_{i_2}-x_{i_3})\cdots\Theta(x_{i_{n-1}}-x_{i_n})=1.
\end{aligned}
\qquad (2.2)
$$

Let us compare (2.1) with the analogous formula of refs./3/which is obtained by summing the generalized ladder diagrams of the 4-dimensional Feynman-Dyson formalism (see Fig.2):

$$(2\pi)^4 T(p_1,p_2;q_1,q_2)\delta(p_1+p_2-q_1-q_2)=$$
$$=\lim_{p_i^2,q_i^2\to M^2}\prod_{i=1}^{2}(p_i^2-M^2)(q_i^2-M^2)\mathcal{K}G(p_2,q_2|A_2)G(p_1,q_1|A_1)\Big|_{A_1=A_2=0}, \qquad (2.3)$$

where

$$\mathcal{K}=\exp\Big\{-ig^2\int\overset{c}{\mathcal{D}}(u-v)\frac{\delta^2}{\delta A_1(u)\delta A_2(v)}du\,dv\Big\}$$

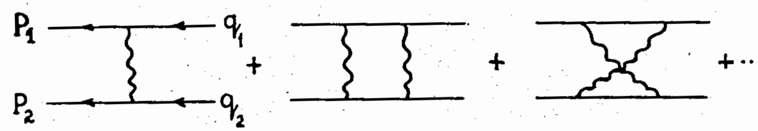

Fig.2

For the physical external lines the relativistic Hamiltonian scheme coincides with the Feynman one, the action of the operator $\mathcal{K}_{\varphi}$ leads to the multiplication by unity and the chain of diagrams of Fig.1 coincides with the one given in Fig.2, i.e., the relation (2.1) gives the same result as (2.3).

In the $\sum k_i k_j = 0$ approximation /2,3/ for the amplitude $T(p_1 p_2; q_1 q_2) \equiv T(s,t)$, we get the eikonal representation:

$$T(s,t) = -2is\int d^2 b\, e^{-i\vec{\Delta}\vec{b}}\left(\exp\left[\frac{ig^2}{2s}\int\frac{d\vec{k}_\perp}{(2\pi)^2}\frac{e^{-i\vec{k}_\perp\vec{b}}}{m^2+\vec{k}_\perp^2}\right]-1\right), \quad \vec{\Delta}^2 = -t. \qquad (2.4)$$

To derive the generalized eikonal representation (1.1) let us examine in the ladder approximation completely reducible (CR) diagrams, describing the process under consideration (see Fig.3).

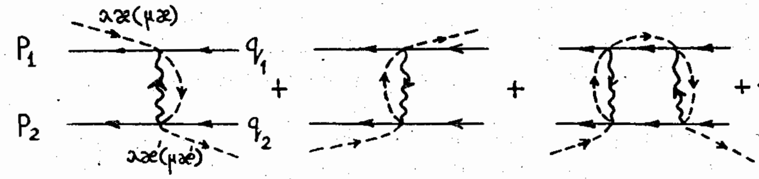

Fig.3

We shall call a diagram the CR-diagram in the ladder approximation, if it is such a reducible diagram, irreducible components of which correspond to one meson exchange (In what follows we shall omit the words "in the ladder approximation" for brevity).

For one of the CR-diagrams of the $2n^{th}$ order in g depicted in Fig.4 (the number of all CR-diagrams which differ in the way the vertices are connected by dotted line, is equal to 2^n), one gets the following expression

$$\widetilde{T}_n(s,t) = \frac{g^{2n}}{(2\pi)^{3(n-1)}}\int\prod_{j=0}^{2n-2} dk_j \overset{+}{\Delta}(k_j)\prod_{j=0}^{2n-1}\frac{d\varkappa_j}{\varkappa_j - i\varepsilon}\prod_{j=0}^{n-1}\overset{(+)}{\mathcal{D}}(\Delta'_{2j+1}+\lambda\varkappa_{2j+1}-\lambda\varkappa_{2j})\times$$
$$\times\,\delta(\Delta'_{2j+1}-\Delta'_{2j+2}+\lambda\varkappa_{2j+2}-\lambda\varkappa_{2j}), \qquad (2.5)$$

where $k_{-1}\equiv p_1$, $k_0\equiv p_2$, $k_{2n-1}\equiv q_1$, $k_{2n}\equiv q_2$, $\varkappa_0\equiv\varkappa$, $\varkappa_{2n}\equiv\varkappa'$, $\Delta'_j = k_{j-2}-k_j$.

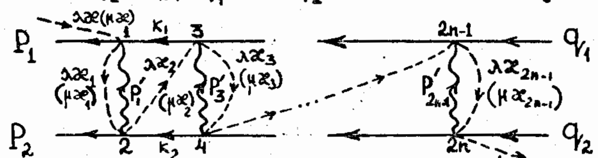

Fig.4

On integrating over k_{2j+2} $(j=0,1,\ldots n-2)$, the expression (2.5) takes the form

$$\widetilde{T}_n(s,t) = \frac{g^{2n}}{(2\pi)^{3(n-1)}}\int\prod_{j=0}^{n-2}\left[dk_{2j+1}\overset{(+)}{\Delta}(k_{2j+1})\overset{(+)}{\Delta}(\Delta_{2j+1}+\lambda\varkappa_{2j+2})\frac{d\varkappa_{2j+2}}{\varkappa_{2j+2}-i\varepsilon}\right]\times$$
$$\times\prod_{j=0}^{n-1}\left[\overset{(+)}{\mathcal{D}}(\Delta'_{2j+1}+\lambda\varkappa_{2j+1}-\lambda\varkappa_{2j})\frac{d\varkappa_{2j+1}}{\varkappa_{2j+1}-i\varepsilon}\right], \qquad \Delta_{2j+1} = P-k_{2j+1},\ P = q_1+q_2. \qquad (2.6)$$

Now integrating over $\varkappa_{2j+1}$ $(j=0,1,\ldots,n-1)$, we get:

$$\widetilde{T}_n(s,t) = \frac{g^{2n}}{(2\pi)^{3(n-1)}}\int\prod_{j=0}^{n-2}\left[dk_{2j+1}\overset{(+)}{\Delta}(k_{2j+1})\overset{(+)}{\Delta}(\Delta_{2j+1}+\lambda\varkappa_{2j+2})\frac{d\varkappa_{2j+2}}{\varkappa_{2j+2}-i\varepsilon}\right]\times$$
$$\times\prod_{j=0}^{n-1}\left[\frac{1}{2\omega'_{2j+1}}\cdot\frac{1}{\varkappa_{2j}-\lambda\Delta'_{2j+1}+\omega'_{2j+1}-i\varepsilon}\right], \qquad \omega'_{2j+1} = \sqrt{(\lambda\Delta'_{2j+1})^2-\Delta'^2_{2j+1}+m^2}. \qquad (2.7)$$

All subsequent calculations are carried out on the energy shell $(\varkappa = \varkappa' = 0)$.

Suppose that at high energies (i.e., when $s = P^2\to\infty$, $t = (p_1-q_1)^2 = \mathrm{const}$) in denominators of the type $\varkappa_{2j}-\lambda\Delta'_{2j+1}+\omega'_{2j+1}$ of the integrand (2.7) one can neglect the terms $\varkappa_{2j}$ (for the discussion of this approximation, see Appendix), i.e.,

$$\varkappa_{2j}-\lambda\Delta'_{2j+1}+\omega'_{2j+1} \longrightarrow -\lambda\Delta'_{2j+1}+\omega'_{2j+1}. \qquad (2.8)$$

In this approximation the sum of all terms, corresponding to 2^n CR-diagrams, is equal to

$$T_n(s,t) = \frac{1}{(2\pi)^{3(n-1)}}\int\prod_{j=0}^{n-2} dk_{2j+1}\, 2\overset{(+)}{\Delta}(k_{2j+1})\overset{(+)}{\Delta}(\Delta_{2j+1}+\lambda\varkappa_{2j+2})\frac{d\varkappa_{2j+2}}{\varkappa_{2j+2}-i\varepsilon}\prod_{j=0}^{n-1}V[(\vec{k}_{2j-1}(-)\vec{k}_{2j+1})^2], \qquad (2.9)$$

where $V[(\vec{k}_1(-)\vec{k}_2)^2] = \dfrac{g^2}{m^2-(k_1-k_2)^2-i\varepsilon}$, $\quad\vec{k}_1(-)\vec{k}_2 = \vec{k}_1-\dfrac{\vec{k}_2}{M}\left[k_1^0-\dfrac{\vec{k}_1\vec{k}_2}{M+k_2^0}\right]$.

We choose the 4-vector λ in the form

$$\lambda = \frac{P}{\sqrt{P^2}} = \frac{k_{2j+1}+k_{2j}}{\sqrt{(k_{2j+1}+k_{2j})^2}}. \qquad (2.10)$$

Then expression (2.9) in the c.m.s. (i.e. $\vec{q}_1 = -\vec{q}_2 = \vec{q}$, $\vec{p}_1 = -\vec{p}_2 = \vec{p}$) can be written as follows

$$T_n(s,t) = \frac{1}{(2\pi)^{3(n-1)}}\int\prod_{j=0}^{n-2}\left[\frac{d\Omega_{2j+1}}{8E_{2j+1}(E_{2j+1}-E_q-i\varepsilon)}\right]\prod_{j=0}^{n-1}V[(\vec{k}_{2j-1}(-)\vec{k}_{2j+1})^2], \qquad (2.11)$$

where $E_{2j+1} = \sqrt{M^2+\vec{k}^2_{2j+1}}$, $E_p = E_q = \sqrt{M^2+\vec{q}^2}$, $d\Omega_{2j+1} = \dfrac{d\vec{k}_{2j+1}}{E_{2j+1}}$.

Since high energies are carried by nucleon lines (i.e., the essential contribution in (2.9) comes from the regions $E_{2j+1}\sim E_q$), we can substitute $E^{-1}_{2j+1}(E_{2j+1}-E_q-i\varepsilon)^{-1}$ by $E_q^{-1}(E_{2j+1}-E_q-i\varepsilon)^{-1}$ and

$$T_n(s,t) = (2\pi)^{-3(n-1)}\left(\frac{1}{8E_q}\right)^{n-1}\int\prod_{j=0}^{n-2}\frac{d\Omega_{2j+1}}{E_{2j+1}-E_q-i\varepsilon}\prod_{j=0}^{n-1}V[(\vec{k}_{2j-1}(-)\vec{k}_{2j+1})^2]. \qquad (2.12)$$

Now for finding the asymptotics of (2.12), we apply the technique, developed in ref./4/. To this end, putting

$$\vec{p}(-)\vec{q}=\vec{\Delta},\quad \vec{k}_{2j+1}(-)\vec{q}=\vec{\lambda}_j\left(\vec{p}=\vec{\Delta}(+)\vec{q},\ \vec{k}_{2j+1}=\vec{\lambda}_j(+)\vec{q}\right) \tag{2.13}$$

and taking into account relations $d\Omega_{2j+1}=d\Omega_{\vec{\lambda}_j}$ in (2.12), we write it in the following form:

$$T_n(s,t)=(2\pi)^{-3(n-1)}(8E_q)^{-(n-1)}\int\prod_{j=1}^{n-1}\left[\frac{d\Omega_{\vec{\lambda}_j}}{E_{\vec{\lambda}_j(+)\vec{q}}-E_q-i\varepsilon}\right]V[(\vec{\Delta}(-)\vec{\lambda}_1)^2]V[(\vec{\lambda}_1(-)\vec{\lambda}_2)^2]\cdots V(\vec{\lambda}^2_{n-1}) \tag{2.14}$$

Using relations $(\vec{\lambda}_i(-)\vec{\lambda}_j)^2=(\vec{\lambda}_i\oplus\vec{\lambda}_j^{-1})^2$ (see /4/), we pass to horospherical coordinates $\vec{\Delta}=(a,\tilde{\gamma}),\ \vec{\lambda}_j=(a_j,\tilde{\gamma}_j)$ in (2.14) by means of formulae $E_{\lambda_j}+\lambda_{j3}=Me^{a_j}$, $E_{\lambda_j}-\lambda_{j3}=Me^{-a_j}+\frac{1}{M}\tilde{\gamma}_j^2e^{a_j}$, $\tilde{\lambda}_j=(\lambda_{j1},\lambda_{j2})$, $d\Omega_{\lambda_j}=e^{2a_j}da_jd^2\tilde{\gamma}_j$.

Then we get:

$$T_n(s,t)=(2\pi)^{-3(n-1)}(8E_q^2)^{-(n-1)}\int\prod_{j=1}^{n-1}\left[\frac{e^{2a_j}da_jd^2\tilde{\gamma}_j}{e^{a_j}-1-i\varepsilon}\right]\prod_{j=1}^{n}V(\vec{\Delta}_j'^2), \tag{2.15}$$

where $\vec{\Delta}'=\vec{\Delta}\oplus\vec{\lambda}_1^{-1},\ \vec{\Delta}_2'=\vec{\lambda}_1\oplus\vec{\lambda}_2^{-1},\dots,\ \vec{\Delta}_{n-1}'=\vec{\lambda}_{n-2}\oplus\vec{\lambda}_{n-1}^{-1},\ \vec{\Delta}_n'=\vec{\lambda}_{n-1}$.

The definition of operations (+) and $\oplus$ are given, for example, in /7/. We have chosen $\vec{q}$ in the form $\vec{q}=(0,0,q)$ and have taken into account the fact, that when $s\gg M^2,|t|$ the following approximation holds

$$E_{\vec{\lambda}_j(+)\vec{q}}-E_q=(E_{\lambda_j}E_q+\lambda_{j3}q)/M-E_q\simeq E_q\left(\frac{E_{\lambda_j}+\lambda_{j3}}{M}-1\right)=E_q(e^{a_j}-1).$$

Since $\vec{\Delta}'=(a_k',\tilde{\gamma}_k')$ $(k=1,2,\dots,n)$, then $a_k'=a_{k-1}-a_k,\ \tilde{\gamma}_k'=e^{a_k}(\tilde{\gamma}_{k-1}-\tilde{\gamma}_k)$ when $1\le k\le n-1$ and $a_k'=a_{n-1},\ \tilde{\gamma}_k'=\tilde{\gamma}_{n-1}$ when k = n, where, by definition, $a_0\equiv a,\ \tilde{\gamma}_0\equiv\tilde{\gamma}$. Using now in (2.15) the operator Fourier-transformation on the group $T(3)$

$$V(\vec{\Delta}_j')\equiv V(a_j',\tilde{\gamma}_j')=\int dz_jd^2\tilde{\varrho}_1^{(j)}d^2\tilde{\varrho}^{(j)}\langle\tilde{\varrho}_1^{(j)}|\hat{V}(z_j)|\tilde{\varrho}^{(j)}\rangle e^{-ia_j'z_j-i\tilde{\gamma}_j\tilde{\varrho}^{(j)}-a_j'}$$

and performing the integration over all $\tilde{\gamma}_j$, we arrive at the expression

$$T_n(s,t)=(16\pi E_q^2)^{-(n-1)}e^{-a}\int e^{-i\tilde{\gamma}\tilde{\varrho}}d^2\varrho\int\prod_{j=1}^{n-1}\left[\frac{e^{i\sum_{j=1}^{n-1}a_j(z_j-z_{j+1})-iaz_1}da_j}{e^{a_j}-1-i\varepsilon}\right]\times$$
$$\times\int\prod_{j=1}^{n}dz_jd^2\tilde{\varrho}_1^{(j)}\langle\tilde{\varrho}_1^{(j)}|\hat{V}(z_j)|\tilde{\varrho}e^{-a_j}\rangle. \tag{2.16}$$

Taking into account, that in high energy regime the relations $a\approx 0,\ \tilde{\gamma}\approx\tilde{\Delta},\ |t|\approx\tilde{\gamma}^2$ hold, after the integration over a_j $(j=1,2,\dots,n-1)$, one gets the following representation

$$T_n(s,t)=-2is\int e^{-i\tilde{\Delta}\tilde{\varrho}}d^2\tilde{\varrho}d^2\tilde{\varrho}_1\left(\frac{i}{2s}\right)^n\int\hat{\theta}(z_1-z_2)\cdots\hat{\theta}(z_{n-1}-z_n)\langle\tilde{\varrho}_1|\hat{V}(z_1)\cdots\hat{V}(z_n)|\tilde{\varrho}\rangle dz_1\cdots dz_n \tag{2.17}$$

Thus, we have demonstrated that the summation of the chain of diagrams (Fig.3) at high energies leads to the representation (1.1) for the amplitude $T(s,t)=\sum_{n=1}^{\infty}T_n(s,t)$, which was derived in /4/ from the QPE.

3. A GENERALIZED EIKONAL REPRESENTATION IN THE THREE-DIMENSIONAL FORMULATION OF QFT ON THE LIGHT CONE

According to the diagram technique of the three-dimensional formulation of QFT on the light cone /11/, to dotted line there corresponds 4-momentum $\mu\varkappa$, where 4-vector μ, in contrast to 4-vector λ, is light-like: $\mu^2=\mu_0^2-\vec{\mu}^2=0,\ \mu_0>0$. In this case to the CR-diagram of the 2nth order in g, which is depicted in Fig.4, there corresponds the following expression

$$\tilde{T}_n(s,t)=\frac{g^{2n}}{(2\pi)^{3(n-1)}}\int\prod_{j=0}^{n-2}dk_{2j+1}\overset{(+)}{\Delta}(k_{2j+1})\overset{(+)}{\Delta}(\Delta_{2j+1}+\mu\varkappa_{2j+2})\frac{d\varkappa_{2j+2}}{\varkappa_{2j+2}-i\varepsilon}\prod_{j=0}^{n-1}\overset{(+)}{\mathcal{D}}(\Delta_{2j+1}'+\mu\varkappa_{2j+1}-\mu\varkappa_{2j+2})\frac{d\varkappa_{2j+1}}{\varkappa_{2j+1}-i\varepsilon} \tag{3.1}$$

After performing integration over $\varkappa_j$ $(j=1,2,\dots 2n-1)$, instead of (3.1) we get *)

$$\tilde{T}_n(s,t)=\frac{g^{2n}}{(2\pi)^{3(n-1)}}\int\prod_{j=0}^{n-2}dk_{2j+1}\overset{(+)}{\Delta}(k_{2j+1})\frac{\Theta(\mu\Delta_{2j+1})}{M^2-\Delta^2_{2j+1}-i\varepsilon}\prod_{j=0}^{n-1}\frac{\Theta(\mu\Delta_{2j+1}')}{\varkappa_{2j}+\frac{M^2-\Delta'^2_{2j+1}}{2\mu\Delta'_{2j+1}}-i\varepsilon}\frac{1}{|2\mu\Delta'_{2j+1}|}, \tag{3.2}$$

where $\varkappa_{2j}=(M^2-\Delta^2_{2j+1})/|2\mu\Delta_{2j+1}|$, $j=1,2,\dots n-1$.

Suppose that at high energies in the denominators of the form $\varkappa_{2j}+(M^2-\Delta'^2_{2j+1})/|2\mu\Delta'_{2j+1}|$, one can neglect the terms $\varkappa_{2j}$ (cf. with /2.8/). Then the sum of all terms, which correspond to 2^n CR-diagrams, is equal to (in the following $\varkappa=\varkappa'=0$)

$$T_n(s,t)=(2\pi)^{-3(n-1)}\int\prod_{j=0}^{n-2}dk_{2j+1}\overset{(+)}{\Delta}(k_{2j+1})\frac{\Theta(\mu\Delta_{2j+1})}{M^2-\Delta^2_{2j+1}-i\varepsilon}\prod_{j=0}^{n-1}V[(\vec{k}_{2j-1}(-)\vec{k}_{2j+1})^2]. \tag{3.3}$$

In (3.3) it is convenient to pass to the c.m.s. ($\vec{P}=0$), where

$$T_n(s,t)=(2\pi)^{-3(n-1)}(8E_q)^{-(n-1)}\int\prod_{j=0}^{n-2}\frac{d\Omega_{2j+1}}{E_{2j+1}-E_q-i\varepsilon}\Theta(2E_q-E_{2j+1}+\vec{n}\vec{k}_{2j+1})\prod_{j=0}^{n-1}V[(\vec{k}_{2j-1}(-)\vec{k}_{2j+1})^2],\ \vec{n}=\frac{\vec{\mu}}{\mu_0}. \tag{3.4}$$

In the high energy limit, i.e. when $2E_q=\sqrt{s}\mapsto\infty$, the Θ-functions in (3.4) can be replaced by unity. As a result the expression (3.4) for T_n takes the form

$$T_n(s,t)=(2\pi)^{-3(n-1)}(8E_q)^{-(n-1)}\int\prod_{j=0}^{n-2}\frac{d\Omega_{2j+1}}{E_{2j+1}-E_q-i\varepsilon}\prod_{j=0}^{n-1}V[(\vec{k}_{2j-1}(-)k_{2j+1})^2], \tag{3.5}$$

*) As it is known, the points $\mu\Delta_{2j+1}=\mu\Delta'_{2j+1}=0$ do not contribute in (3.1) (see /11/).

which exactly coincides with the expression (2.12) of the preceding section.

Thus, we again obtain the representation (1.1) for the scattering amplitude, which differs from the usual eikonal representation (2.4) by a more complicated dependence of phase function on the energy and potential. For example, in the case when the matrix $\langle \tilde{\varrho}_1 | \hat{V}(z) | \tilde{\varrho} \rangle = \delta(\tilde{\varrho}_1 - \tilde{\varrho}) V_s(z, \tilde{\varrho})$ is diagonal, the logarithmic dependence arises /4,7/:

$$T(s,t) = -4\pi i s \int_0^\infty \varrho \, d\varrho \, J_0(\sqrt{-t}\,\varrho) \left\{ e^{i \int_{-\infty}^{\infty} \ln\left(1 + \frac{V_s(z,\tilde{\varrho})}{2s}\right) dz} - 1 \right\} \qquad (3.6)$$

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A P P E N D I X

When deriving the expression (2.9) we have used the approximation (2.8), which is analogous to the approximation $\sum k_i k_j = 0$ /2,3/. It can be shown that this approximation preserves the asymptotics of the amplitude. By the conservation of asymptotics we mean the following /12/. Let $\tilde{T}_n(s,t)$ be the exact contribution of CR-diagrams of 2nth order in g to the amplitude, and $T_n(s,t)$ in the approximation (2.8). Then, as $s \to \infty$, $t = \mathrm{const}$,

$$\tilde{T}_n(s,t) \longrightarrow \tilde{\alpha}(t)\tilde{\beta}(s),$$
$$T_n(s,t) \longrightarrow \alpha(t)\beta(s). \qquad (A.1)$$

If $\beta(s) = \tilde{\beta}(s)$, then asymptotics does not change, though $\alpha(t)$ and $\tilde{\alpha}(t)$ are always different.

For example, let us consider the CR-diagrams of fourth order (Fig.5), which give the following contribution to the amplitude:

$$T_2(s,t) = \mathrm{const} \int \overset{(+)}{\Delta}(k) \overset{(+)}{\Delta}(k') \overset{(+)}{\mathcal{D}}(p') \overset{(+)}{\mathcal{D}}(q') \prod_{j=1}^{3} \frac{d\varkappa_j}{\varkappa_j - i\epsilon} \left[\delta(q_1 - k + q' + \lambda\varkappa_2 - \lambda\varkappa_3) \times \right. \qquad (A.2)$$
$$\times \delta(k - p_1 + p' - \lambda\varkappa_1)\delta(q_2 - k' - q' + \lambda\varkappa_3) + \delta(q_1 - k - q' + \lambda\varkappa_3)\delta(k - p_1 - p' + \lambda\varkappa_1 - \lambda\varkappa_2) \times$$
$$\times \delta(q_2 - k' + q' + \lambda\varkappa_2 - \lambda\varkappa_3) + \delta(q_1 - k + q' + \lambda\varkappa_2 - \lambda\varkappa_3)\delta(k - p_1 - p' + \lambda\varkappa_1 - \lambda\varkappa_2) \times$$
$$\left. \times \delta(q_2 - k' - q' + \lambda\varkappa_3) + \delta(q_1 - k - q' + \lambda\varkappa_3)\delta(k - p_1 + p' - \lambda\varkappa_1)\delta(q_2 - k' + q' + \lambda\varkappa_2 - \lambda\varkappa_3) \right] \times$$
$$\times dk \, dk' \, dp' \, dq'.$$

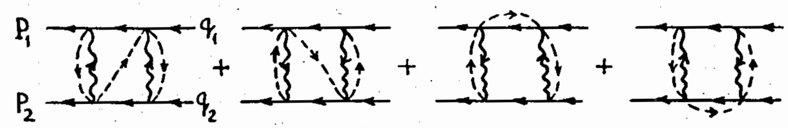

Fig.5

We write the relation (A.2) in the form

$$\tilde{T}_2(s,t) = T_2(s,t) + A_2(s,t), \qquad (A.3)$$

where $\tilde{T}_2$ is defined by formula (2.9) with n=2, and for the case when λ is taken in the form (2.10), A_2 in the c.m.s. has the form $\left(E'_k = \sqrt{m^2 + \vec{k}^2} \right)$

$$A_2(s,t) = -\mathrm{const} \int \frac{d\Omega_k}{E_k} \left\{ \frac{1}{E'_{p-k}[m^2 - (k-q_1)^2 - i\varepsilon][E'^2_{p-k} - (E_q - E_k)^2 - i\varepsilon]} + \right.$$
$$\left. + \frac{1}{E'_{q-k}[m^2 - (k-p_1)^2 - i\varepsilon][E'^2_{q-k} - (E_q - E_k)^2 - i\varepsilon]} + \frac{1}{E'_{p-k}E'_{q-k}[E'^2_{q-k} - (E_q - E_k)^2 - i\varepsilon][E'^2_{p-k} - (E_q - E_k)^2 - i\varepsilon]} \right\}$$

The approximation (2.8) implies that the terms $A_n(s,t) = \tilde{T}_n(s,t) - T_n(s,t), n \geq 2$ (see /A.1/) are neglected. Here we will demonstrate that the asymptotics of $A_2(s,t)$ is of the form $\frac{1}{s}$. Because of (2.13) and the relation

$$E_{\vec{p}(\pm)\vec{k}} \equiv \sqrt{M^2 + (\vec{p}(\pm)\vec{k})^2} = \frac{E_p E_k \pm \vec{p}\vec{k}}{M}, \quad (p^2 = k^2 = M^2)$$

we have

$$m^2 - (k - q_1)^2 = E'^2_{q-k} - (E_q - E_k)^2 = f(\vec{\lambda}),$$
$$m^2 - (k - p_1)^2 = E'^2_{p-k} - (E_q - E_k)^2 = f(\vec{\Delta}(-)\vec{\lambda}),$$

where $f(\vec{k}) = 2ME_k + m^2 - 2M^2$ and when $s \gg m^2, |t|, M^2$

$$E'^2_{p-k} \simeq E'^2_{q-k} \simeq E_q \left| 1 - \frac{E_\lambda + \lambda_3}{M} \right|.$$

Consequently,

$$A_2(s,t) = -\frac{\mathrm{const}}{s} \int d\Omega_{\vec{\lambda}} \frac{\left(1 - \frac{E_\lambda + \lambda_3}{M}\right)^{-2}}{f(\vec{\lambda}) f(\vec{\Delta}(-)\vec{\lambda})} \left[2 + \mathrm{sign}\left(1 - \frac{E_\lambda + \lambda_3}{M}\right) \right] \qquad (A.4)$$

what was to be proven.

In the same way one can prove that after omitting $\varkappa_{2j}$ in denominators of the integrand in the expression (3.2), its asymptotics does not change. Indeed, confining ourselves to the CR-diagram of the fourth perturbation expansion order (see Fig.5), we get the following expression

$$A_2(s,t) = -\mathrm{const}\int \frac{\theta(\mu P-\mu\kappa)}{2\mu(P-\kappa)} \frac{d\Omega_{\vec{\kappa}}}{[m^2-(\kappa-q_1)^2-i\varepsilon][m^2-(\kappa-p_1)^2-i\varepsilon]} \left\{ \frac{\theta(\mu p_1-\mu\kappa)\theta(\mu\kappa-\mu q_1)}{\varkappa_1+\varkappa_2-i\varepsilon} + \right.$$

$$\left. + \frac{\theta(\mu q_1-\mu\kappa)\theta(\mu\kappa-\mu p_1)}{\varkappa_2+\varkappa_3-i\varepsilon} + \theta(\mu\kappa-\mu q_1)\theta(\mu\kappa-\mu p_1)\left[\frac{1}{\varkappa_1+\varkappa_2-i\varepsilon} + \frac{1}{\varkappa_2+\varkappa_3-i\varepsilon} - \frac{\varkappa_2}{(\varkappa_1+\varkappa_2-i\varepsilon)(\varkappa_2+\varkappa_3-i\varepsilon)}\right]\right\} \quad (A.5)$$

where $\varkappa_1 = \frac{m^2-(\kappa-q_1)^2}{2\mu(\kappa-q_1)}$, $\varkappa_2 = \frac{m^2-(P-\kappa)^2}{2\mu(P-\kappa)}$, $\varkappa_3 = \frac{m^2-(\kappa-p_1)^2}{2\mu(\kappa-p_1)}$.

We orient, as above, the vector $\vec{q}$ along, and the vector $\vec{n}$ against z axis. Then at high energies in c.m.s. one has relations $\mu q_1 \simeq \mu p_1 \simeq 2E_q = \mu P$ and the integrand in (A.5) vanishes. It means that as $E_q \to \infty$ the asymptotics of $A_2(s,t)$ is smaller than that of $T_2(s,t)$.

In conclusion it is to be noted, that though when obtaining the representation (1.1) we have not taken into account all generalized ladder diagrams (which are used in deriving the usual eikonal representation), the asymptotics of the amplitude is not changed, since the sum of all omitted diagrams in each 2nth order tends to zero like $1/s^{n-1}$.

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