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# EXCITATION OF MAGNETIC DIPOLE STATES IN 1p-SHELL NUCLEI IN RADIATIVE PION CAPTURE

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Возбуждение магнитных дипольных состояний в процессах радиационного захвата $\pi^-$-мезонов ядрами 1p-оболочки

В рамках модели оболочек с промежуточной связью рассмотрено возбуждение состояний магнитного дипольного резонанса в ядрах 1p-оболочки в процессе радиационного захвата $\pi^-$-мезонов. Проведено сравнение со спектром возбуждения M1-резонанса. Показано, что в процессе $(\pi^-,\gamma)$ преимущественно возбуждаются те же состояния, что и в случае M1-резонанса. Более сложная структура оператора перехода процесса $(\pi^-,\gamma)$ и большие значения переданного импульса приводят к усилению вероятности возбуждения состояний, которые были подавлены в случае M1-резонанса и возбуждению новых. Проанализированы основные закономерности возбуждения M1-резонанса в четных и нечетных ядрах 1p-оболочки. Приведены экспериментальные и теоретические значения вероятности B(M1) переходов.

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Dogotar G.E., Eramzhyan R.A., Kissener H.-R., Sakaev R.A.

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Excitation of Magnetic Dipole States in 1p-Shell Nuclei in Radiative Pion Capture

The excitation of the magnetic dipole resonance states in the radiative pion capture in 1p-shell nuclei is considered within the shell model with intermediate coupling.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

**Communication of the Joint Institute for Nuclear Research. Dubna 1977**

## 1. INTRODUCTION

It has been shown that in the radiative pion capture

$$\pi^- + (A,Z) \to (A,Z-1) + \gamma \qquad (1)$$

those low-lying states are strongly excited the analogs of which in the initial nucleus form M1-resonance /1-3/. This is due to the fact that in the transitions without changing parity of nuclear states the dominating component of the $(\pi,\gamma)$ amplitude in the pion capture both from s- and p-orbits is proportional to the spin isovector part of the electromagnetic M1-transition. A more complicated structure of the $(\pi,\gamma)$ amplitude and a rather large value of the transferred momentum result in some change of the relation between the intensities of lines and in the appearance of the new ones as compared with the electromagnetic case. Among the low-lying states of a daughter nucleus mostly the states forming the M1-resonance are excited in the muon capture too. In the muon capture the amplitude of the allowed transition in its structure resembles even more the isovector component of the electromagnetic transition amplitude than the $(\pi,\gamma)$ one. Therefore it is of interest to compare the excitation spectrum of atomic nuclei in all three processes in order to establish general regularities in the excitation of the M1-resonance in atomic nuclei. On the other hand, one can specify effects which are peculiar to each type of the transitions. We have partly discussed these problems in paper [3].

## 2. BASIC IDEAS

The yield of hard $\gamma$-quanta in the process (1) in 1p-shell nuclei is equal to the sum of yields $R = R_s + R_p$ due to the pion capture from s- and p-orbits. The quantity $R_\ell$ is defined by the ratio of the radiative capture rate $\lambda_{n\ell}$ to the total pion capture rate $\Lambda_{n\ell}$ from circular orbits with orbital momentum $\ell$, the principal quantum number $n = \ell + 1$ and the absorption strength $\omega_\ell$:

$$R_\ell = \frac{\lambda_{\ell+n,\ell}}{\Lambda_{\ell+1,\ell}} \omega_\ell .$$

The radiative capture rate is calculated by

$$\lambda_{n\ell} = \frac{(1+m_\pi/M_N)^2}{(1+k/AM_N)} (\frac{k}{m_\pi}) \frac{1}{(2J_i+1)(2\ell+1)} \int d\Omega_{\vec{k}} \Sigma |\langle J_f M_f | \hat{Q} | J_i M_i \rangle|^2 , \quad (2)$$

where

$$Q = i^{-1} \sum_{j=1}^{A} f_j \exp\{-i\vec{k}\vec{r}_j\} \phi_{n\ell m}(\vec{r}_j) \quad (3)$$

and $f_j$ is the process amplitude on proton. Taking into account the linear in pion momentum q terms it has the form:

$$\sqrt{2}\ f = \tau^{(-)} i \{A(\vec{\sigma}\vec{\epsilon}_\lambda) + B(\vec{\sigma}\vec{\epsilon}_\lambda)(\vec{k}\vec{q}) + C(\vec{\sigma}\vec{k})(\vec{\epsilon}_\lambda\vec{q}) + iD\vec{\epsilon}_\lambda(\vec{q}\times\vec{k})\}, \quad (4)$$

where $\tau^{(-)}|p\rangle=\sqrt{2}|n\rangle$ ; $\vec{\epsilon}_{\lambda}$ and $\vec{k}$ are the polarization vector and the momentum of outgoing $\gamma$ -quantum, respectively. Usually the following values of the constants entering into expression (4) are used:

$$A=-0.0332\,m_{\pi}^{-1}\ ;\ B=0.0048\,m_{\pi}^{-3}\ ;\ C=-0.0329\,m_{\pi}^{-3};\ D=0.0117\,m_{\pi}^{-3}\ .$$

The pion wave function $\phi_{n\ell m}(\vec{r})$ was obtained[3] by solving the Klein-Gordon equation; the mesoatomic parameters $\omega_{\ell}$ and $\Lambda_{\ell+1,\ell}$ are given in Tables 1 and 2.

Table 1

Transition rates and yields of $\gamma$-quanta in the radiative pion capture by even 1p-shell nuclei

| Target nucleus | Final nucleus $J^{\pi}$ | E MeV | $\lambda_{1s}$ $[10^{15}sec^{-1}]$ | $\lambda_{2p}$ $[10^{11}sec^{-1}]$ | in $10^{-4}$: $R_s$ | $R_p$ | $R$ | Mesoatomic parameters |
|---|---|---|---|---|---|---|---|---|
| $^{6}Li(1^{+}0)$ | $0^{+}$ | 0 | 1.89 | 0.48 | 25.3 | 12.7 | 38.0 | $\Lambda_{1s}$ = 195 eV |
| | $2^{+}$ | 1.80 | 0.61 | 0.35 | 8.26 | 9.13 | 17.39 | $\omega_s$ = 0.4 |
| | $2^{+}$ | (4.2) | 0.22 | 0.19 | 3.01 | 4.94 | 7.95 | $\Lambda_{2p}$=0.015 eV |
| | $0^{+}$ | (6.8) | 0.41 | 0.10 | 5.53 | 2.70 | 8.23 | $\omega_p$ = 0.6 |
| $^{10}B(3^{+}0)$ | $0^{+}$ | 0 | 0.38 | 1.20 | 0.29 | 1.98 | 2.27 | |
| | $2^{+}$ | 3.37 | 1.89 | 2.56 | 1.49 | 4.21 | 5.70 | |
| | $2^{+}$ | 5.95 | 10.29 | 5.95 | 8.05 | 9.79 | 17.84 | $\Lambda_{1s}$ = 1.68 keV |
| | $2^{+}$ | 7.54 | 0.34 | 0.42 | 0.27 | 0.68 | 0.95 | $\omega_s$ = 0.2 |
| | $1^{+}$ | (8.5 ) | 0.27 | 0.74 | 0.21 | 1.22 | 1.44 | $\Lambda_{2p}$ = 0.32 eV |
| | $3^{+}$ | (9.3) | 4.23 | 2.12 | 3.31 | 3.49 | 6.80 | $\omega_p$ = 0.8 |
| | $2^{+}$ | 9.4 | 0.27 | 0.77 | 0.21 | 1.27 | 1.48 | |
| | $4^{+}$ | (11.4) | 1.21 | 1.43 | 0.95 | 2.36 | 3.31 | |
| | $3^{+}$ | (13.2) | 0.98 | 1.06 | 0.77 | 1.74 | 2.51 | |
| $^{12}C(0^{+}\ 0)$ | $1^{+}$ | 0 | 13.00 | 9.19 | 4.09 | 5.04 | 9.13 | $\Lambda_{1s}$ = 3.14 keV |
| | $2^{+}$ | 0.95 | 1.20 | 6.49 | 0.34 | 3.58 | 3.92 | $\omega_s$ = 0.15 |
| | $3^{+}$ | (5.6) | 0.61 | 4.01 | 0.22 | 2.23 | 2.42 | $\Lambda_{2p}$ = 1.02 eV |
| | | | | | | | | $\omega_p$ = 0.85 |
| $^{14}N(1^{+}\ 0)$ | $0^{+}$ | 0 | 0.32 | 2.91 | 0.04 | 0.82 | 0.86 | $\Lambda_{1s}$ = 4.48 keV |
| | $2^{+}$ | 7.01 | 17.60 | 27.10 | 2.60 | 7.63 | 10.23 | $\omega_s$ = 0.1 |
| | $2^{+}$ | 8.31 | 17.60 | 27.10 | 2.60 | 7.63 | 10.23 | $\Lambda_{2p}$ = 2.1 eV |
| | $1^{+}$ | (9.5) | 3.84 | 13.14 | 0.56 | 3.70 | 4.27 | $\omega_p$ = 0.9 |

Table 2

Transition rates and yields of $\gamma$-quanta in the radiative pion capture by odd-even 1p -shell nuclei

| Target nucleus | Final nucleus $J^\pi$ | $E$ MeV | $\lambda_{1s}$ $[10^{15} sec^{-1}]$ | $\lambda_{2p}$ $[10^{11} sec^{-1}]$ | in $10^{-4}$ $R_s$ | $R_p$ | $R$ | Mesoatomic parameters |
|---|---|---|---|---|---|---|---|---|
| $^{7}Li(\frac{3}{2}^- \frac{1}{2})$ | $3/2^-$ | 0 | 0.107 | 0.120 | 1.45 | 2.97 | 4.43 | $\Lambda_{1s}$ = 195 eV |
| | $1/2^-$ | (1.2) | 0.200 | 0.076 | 2.69 | 1.87 | 4.56 | $\omega_s$ = 0.4 |
| | $3/2^-$ | (3.3) | 0.013 | 0.016 | 0.18 | 0.38 | 0.56 | $\Lambda_{2p}$ = 0.016 eV |
| | $5/2^-$ | (3.4) | 0.036 | 0.022 | 0.48 | 0.56 | 1.03 | $\omega_p$ = 0.6 |
| $^{9}Be(\frac{3}{2}^- \frac{1}{2})$ | $3/2^-$ | 0 | 0.414 | 0.344 | 1.38 | 0.99 | 2.36 | $\Lambda_{1s}$ = 0.591 keV |
| | $1/2^-$ | 2.7 | 0.020 | 0.052 | 0.07 | 0.16 | 0.22 | $\omega_s$ = 0.3 |
| | $5/2^-$ | (3.7) | 0.324 | 0.222 | 1.08 | 0.63 | 1.72 | $\Lambda_{2p}$ = 0.16 eV |
| | $3/2^-$ | (4.9) | 0.416 | 0.113 | 1.39 | 0.32 | 1.71 | $\omega_p$ = 0.7 |
| | $7/2^-$ | (6.2) | 0.037 | 0.093 | 0.12 | 0.27 | 0.39 | |
| | $3/2^-$ | (7.2) | 0.042 | 0.102 | 0.14 | 0.29 | 0.43 | |
| | $5/2^-$ | (7.8) | 0.079 | 0.133 | 0.27 | 0.39 | 0.64 | |
| | $7/2^-$ | (9.5) | 0.062 | 0.158 | 0.21 | 0.45 | 0.66 | |
| $^{11}B(\frac{3}{2}^- \frac{1}{2})$ | $1/2^-$ | 0.3 | 1.80 | 1.328 | 1.41 | 2.19 | 3.60 | $\Lambda_{1s}$ = 1.68 keV |
| | $3/2^-$ | (2.1) | 2.37 | 1.741 | 1.86 | 2.86 | 4.72 | $\omega_s$ = 0.2 |
| | $5/2^-$ | (4.2) | 0.599 | 1.420 | 0.47 | 2.33 | 2.80 | $\Lambda_{2p}$ = 0.32 eV |
| | $3/2^-$ | (4.8) | 0.826 | 0.550 | 0.65 | 0.90 | 1.55 | $\omega_p$ = 0.8 |
| | $7/2^-$ | (7.6) | 0.086 | 0.427 | 0.07 | 0.70 | 0.77 | |
| | $5/2^-$ | (7.6) | 0.200 | 0.380 | 0.16 | 0.62 | 0.78 | |
| $^{13}C(\frac{3}{2}^- \frac{1}{2})$ | $3/2^-$ | 0 | 11.84 | 16.31 | 3.72 | 8.94 | 12.66 | $\Lambda_{1s}$ = 3.14 keV |
| | | | | | | | | $\omega_s$ = 0.15 |
| | | | | | | | | $\Lambda_{2p}$ = 1.02 eV |
| | | | | | | | | $\omega_p$ = 0.85 |

The reduced probability of the magnetic dipole transition B(M1) is determined in a standard way.

The states of normal parity of atomic nuclei (except for $^{6}$Li ) have been described by the wave functions calculated with the Cohen-Kurath parameters [4] in the version (8-16) 2BME. The wave functions of the nuclear system with atomic number $A=6$ have been used from the paper by Barker [5]. There is usually the dominating component $|1p^n[\lambda]^{2T+1,2S+1}L_J>$ in the wave functions of the low-lying states or 1p-shell nuclei if they are written in the scheme of L-S coupling. If this component is known one can understand qualitatively the regularities of the nuclear excitation. Therefore when analyzing the obtained results we shall refer to paper [6] which contains the structure of the wave functions of 1p-shell nuclei within the scheme of L-S coupling. The symbol $[\lambda]$ denotes the Young spatial scheme which describes the symmetry of the wave function, T is the isospin, S - spin, L - orbital and J- total moments of the nuclear system; n is the number of particles in 1p-shell.

## 3. THE RESULTS

The quantities B(M1) and R are given in Figs. 1-4. Tables 1 and 2 represent the same quantities and also $\lambda_{\rho+1,\rho}$ and $R_\rho$. The calculated values of the level energies somewhat differ from the experimental ones. One can uniquely determine the correspondence between the theoretical and experimental positions of levels, given in the figures, by the quantum numbers of levels.

In Figures 1-4 the excitation energy of nuclear states is reckoned from the ground state of the target nucleus. In the tables when the experimental positions of levels are unknown, the theoretical values are given (in brackets).

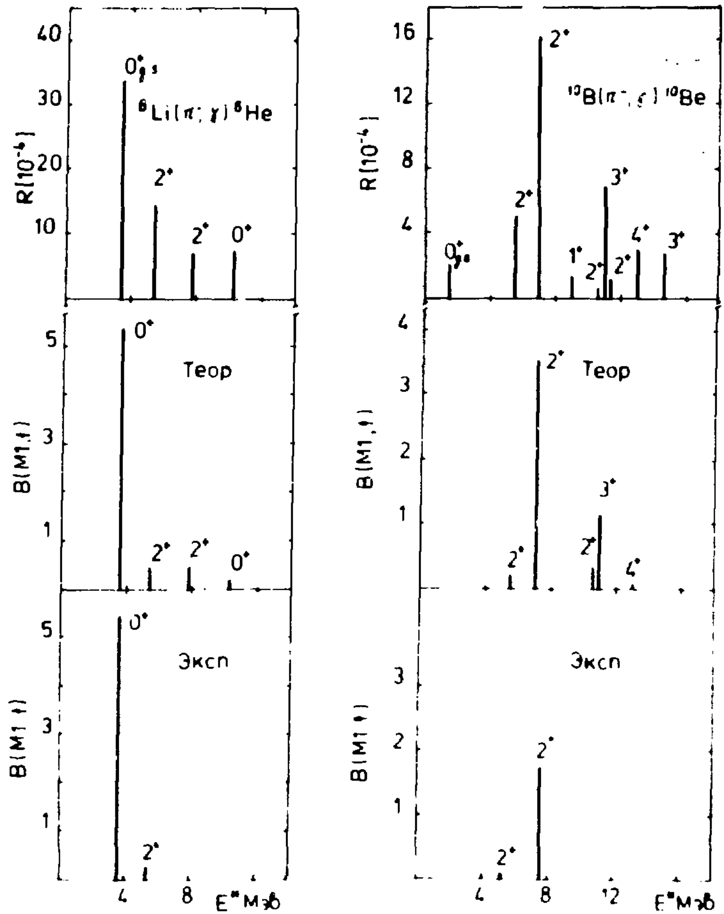


Fig. 1. Excitation of M1-resonance in radiative pion capture and electromagnetic transitions at long-range limit on $^6$Li and $^{10}$B.

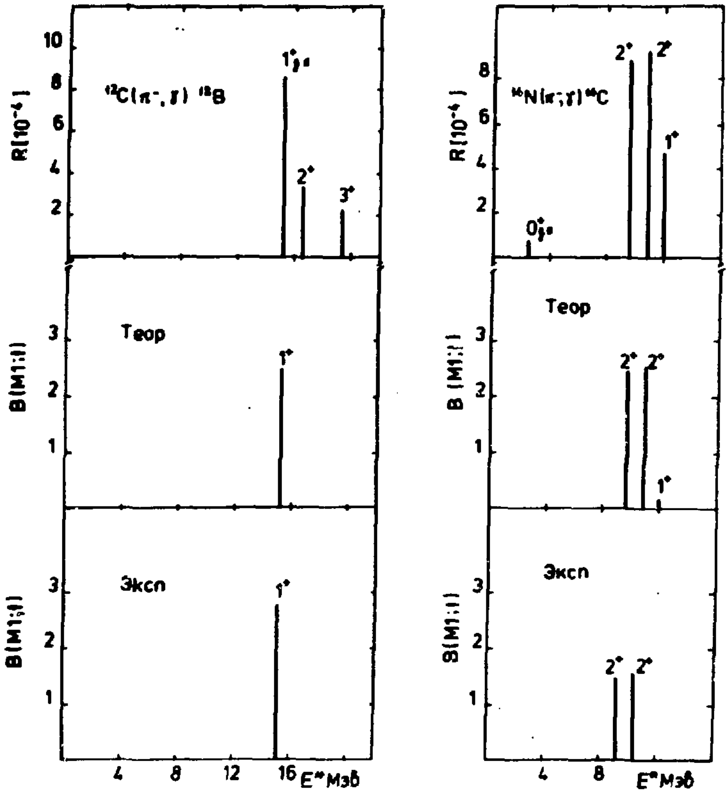


Fig. 2. Excitation of M1-resonance in radiative pion capture and electromagnetic transitions at long-range limit on $^{12}C$ and $^{14}N$.

The figures and tables present the strongest transitions. The contribution of the rest ones is small and can be neglected.

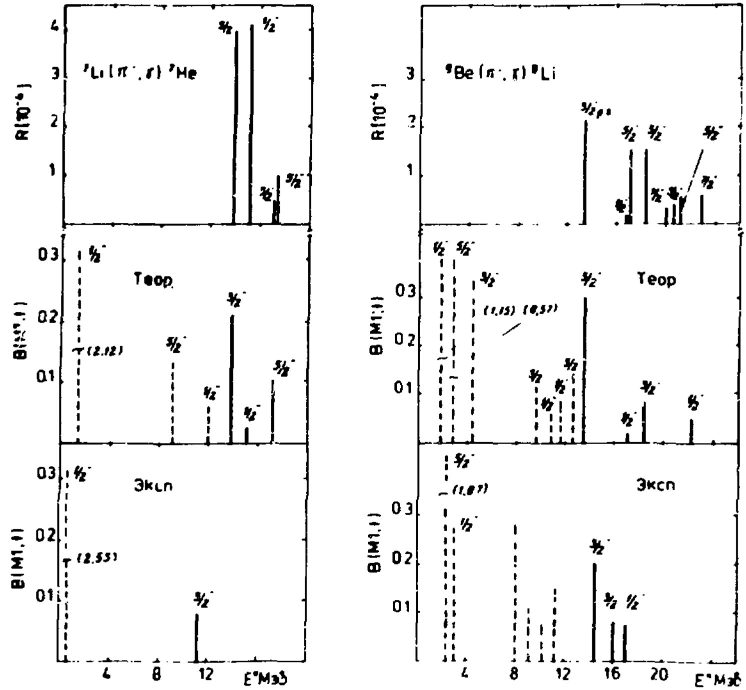


Fig. 3. Excitation of M1-resonance in radiative pion capture and electromagnetic transitions at long-range limit on $^7$Li and $^9$Be.

## 3.1. Odd-Odd Nuclei

$^6Li(1^+, 0)$. In $^6$Li there occurs a strong strength concentration of the M1-transition on the level with quantum numbers $J^\pi T$; $E = 0^+ 1$; 3.56 MeV (Fig. 1a). The spatial

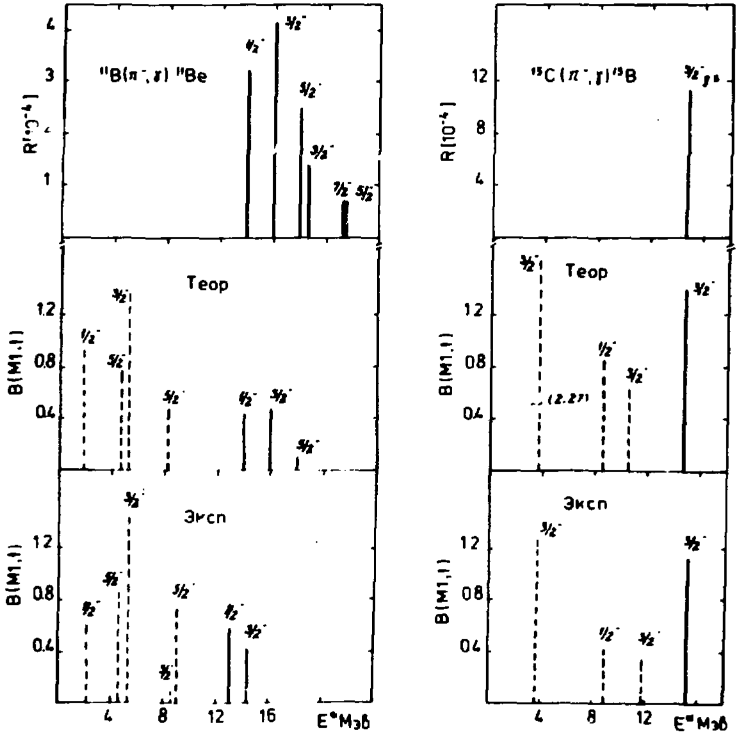


Fig. 4. Excitation of M1-resonance in radiative pion capture and electromagnetic transitions at long-range limit on $^{11}B$ and $^{13}C$.

parts of the wave functions of the ground state and of the resonance are almost completely overlapping:

$$\psi(1^+0) \sim 0{,}97\,|1p^2[2]\,^{13}S_1> ;\ \psi(\,^{-+}1) \sim 0{,}97\,|1p^2[2]^{31}S_0> .$$

This provides the concentration of the M1-transition. In $(\pi,\gamma)$ and $(\mu,\nu)$ processes among the transitions to the low-lying levels of $^6He$ the strongest is the ground state one ($J^\pi T = 0^+1$). The dominating component in the wave function of the $J^\pi T$; $E = 2^+1$; 1.8 MeV state in $^6He$ has the orbital moment $L = 2(^{31}D_2)$. The M1-transition to an analog of this level in $^6Li$ proceeds only due to small admixtures to the wave function and therefore its intensity is very small. In $(\pi,\gamma)$ and $(\mu,\nu)$ processes the level $J^\pi T = 2^+1$ is excited much stronger due to a more complicated structure of the transition amplitude and large values of the transferred momentum. In particular, in the $(\pi,\gamma)$ process the ratio $R(2^+)/R(0^+)$ is very close to 0.5.

In the excitation spectra there is no resonance with the quantum number $J^\pi T = 1^+1$. This resonance is specified by the wave function $|1p^2[11]\,^{33}P_1>$ with the antisymmetric spatial component whereas the $^6Li$ ground state is mainly specified by the symmetric component. The transition between these states due to the operator $\tau\vec{\sigma}$ is forbidden. The suppression of intensity occurs also for the $(\pi,\gamma)$ process.

$^{14}N(1^+0)$. In the wave function of the ground state of this nucleus the dominating is the component $|1p^{10}[442]\,^{13}D_1>$. The M1-resonance is connected with the $J^\pi T = 2^+1$ state excitation. In the wave function of this resonance the dominating is the component $|1p^{10}[442]\,^{31}D_2>$ which provides almost a complete overlapping of spatial parts. Due to the admixtures of higher $2\hbar\omega$ configurations this level is splitted into two levels so that each of them contains the component $|1p^{10}[442]^{31}D_2\rangle$ almost with equal weight. In Fig. 2b this specific property of the excitation of this level is taken into account. Like in $^6$Li the level $J^\pi T = 1^+1$ in the M1-transitions is weakly excited due to a poor overlapping of the nuclear wave functions specified by the selection rules according to the Young scheme. This level is excited somewhat stronger in the $(\pi,\gamma)$ process.

$^{10}B(3^+0)$. In the ground state wave function of $^{10}$B the main contribution (about 60%) comes from the component $|1p^6[42]^{13}D_3\rangle$. The M1-resonance in $^{10}$B is connected with the excitation of the levels $J^\pi T = 2^+1$ through the component $|1p^6[42]^{31}D_2\rangle$ (Fig. 1b). In comparison with $^6$Li and $^{14}$N the M1-resonance is noticeably spread since the leading components are also spread. The excitation of states with the spins $J^\pi = 3^+$ and $4^+$ in M1-transitions proceeds with lower intensity, since in the wave functions of these resonances the component with the Young scheme [42] is specified by a state with an orbital moment different from $L = 2$. Therefore, the excitation of resonances with the quantum numbers $J^\pi = 3^+$ and $4^+$ is due to the admixtures to the leading component of the ground state.

The spectrum of nuclear state excitations in the $(\pi,\gamma)$ process is much richer than in the electromagnetic one. Nevertheless, the main maximum is connected with the excitation of the same level $J^\pi T$; $E = 2^+1$; 5.96 MeV as in purely electromagnetic and muon capture processes.

### 3.2. Even-Even Nucleus $^{12}C$

The transition to the level $J^{\pi}T;E = 1^{+}1;$ 15.1 MeV takes almost the whole strength of magnetic dipole transitions (Fig. 2a). The M1-transitions with $\Delta T = 1$ in an even-even nucleus is characterized by rather a poor overlapping of the spatial components of the ground state and resonance wave function. The wave function of the $^{12}C$ ground state contains mainly a component with the Young scheme [44]:

$$\psi(0^{+},0) \approx 0{,}85[44]^{11}S + 0{,}50[431]^{13}P.$$

The component with this Young scheme is not realized in the wave functions of resonances with $T = 1$. Therefore it does not contribute to the formation of the M1-resonance. Thus, higher multipoles in $(\pi,\gamma)$ amplitude should be exhibited in transitions in even-even nucleus. The $J^{\pi}T = 2^{+}1$ state is strongly excited. Nevertheless, the main maximum is connected with the excitation of that level, which forms the M1-resonance.

### 3.3. Odd-A Nuclei

In odd-A 1p-shell nuclei the M1-resonance with $\Delta T = 1$ is strongly suppressed due to a poor overlapping of the nuclear wave functions. In fact, in the ground state of odd-A nuclei the leading are the following components of the wave functions:

$$^{7}Li: \psi(3/2^{-}\ 1/2) \approx 0{,}99|1p^{3}[3]^{22}P_{3/2}>$$

$$^{9}Be: \psi(3/2^{-}\ 1/2) \approx 0{,}90|1p^{5}[41]^{22}P_{3/2}> + 0{,}40|1p^{5}[41]^{22}D_{3/2}>,$$

$$^{11}B:\psi(3/2^-\ 1/2) \approx 0{,}64|1p^7[43]^{22}P_{3/2}> + 0{,}57|1p^7[43]^{22}D_{3/2}>$$

$$^{13}C:\psi(1/2^-\ 1/2) \approx 0{,}89|1p^9[441]^{22}P_{1/2}> .$$

Such Young schemes are not realized in the wave functions of daughter nuclei $(A,Z-1)$ if they have the isospin equal to $T = T_i + 1$. The same holds for their analog states in the target nuclei. Owing to this the suppression of the M1-resonance with $\Delta T = 1$ is so strong that the resonances with $\Delta T = 0$ become dominating. The latter are represented in Figs. 3 and 4 by the dashed lines. In the $(\pi,\gamma)$ process only the resonances with $\Delta T = 1$ are excited. Thus, using this process, one can obtain a more detailed information on the structure of the M1-resonance with $\Delta T = 1$ in odd-A nuclei.

## 4. CONCLUSION

As it follows from the nuclear excitation spectra mostly the states forming the M1-resonance in the electromagnetic processes are excited in the $(\pi,\gamma)$ process at low excitation energies. A more complicated structure of the $(\pi,\gamma)$ amplitude and a larger value of the transferred momentum results in the change of the relation between the excitation intensities of different levels and in the appearance of the new ones as compared with electromagnetic transitions. Thus, the study of the nuclear excitation spectrum

at low excitation energies in the $(\pi,\gamma)$ process allows one to obtain further information on the M1-resonance structure.
The theory describes rather well the gross-structure of the M1-resonance. However, in a number of cases the value of B(M1) is somewhat overestimated in comparison with the measured one ($^{10}B$, $^{14}N$). An analogous situation is in the $(\pi,\gamma)$ process. To obtain a better agreement of the calculated values with the experiment, one should somewhat change the residual interaction parameters used in the calculation. The theory encounters some difficulties in describing the M1 -resonance with $\Delta T=1$ in odd-A nuclei. Namely. the main components of the ground state wave function of the nucleus-target do not contribute to the transition and their admixtures are affected strongly by the residual interaction of nucleons in a nucleus. In the $(\pi,\gamma)$ process the leading components of the target and the final nucleus wave functions are connected through the higher multipole components of the transition operator. Therefore, in this case the suppression of the transition is not so large and the calculated values of the capture rates in $(\pi,\gamma)$ process are less sensitive to the parameters of the residual interaction of nucleons in a nucleus.

In conclusion we should like to note that in some nuclei the resonances with opposite parity may be located in the low-energy excitation region, too /3/.This fact should be taken into account when the experimental data on $(\pi,\gamma)$ reaction are analysed.

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