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Смешивание тяжелых нейтральных лептонов и распад $\mu \to 3e$

Вероятность распада $\mu \to 3e$ вычислена в $SU_2 \times U_1$ калибровочной теории со смешиванием лептонов в предположении существования тяжелых нейтральных лептонов и правых токов. Показано, что вероятность распада $\mu \to 3e$ того же порядка, что и вероятность распада $\mu \to e\gamma$ в этой теории и при массах тяжелых лептонов порядка нескольких ГэВ может быть близкой к соответствующей экспериментальной верхней границе.

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Heavy Neutral Lepton Mixing and $\mu \to 3e$ Decay

The $\mu \to 3e$ decay rate is evaluated within the framework of $SU_2 \times U_1$ gauge theory with heavy neutral leptons, lepton mixing and right-handed currents. In the model considered the $\mu \to 3e$ branching ratio is of the same order of magnitude as the $\mu \to e\gamma$ one and may well be close to its present experimental upper bound.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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Recently, we have pointed out [1] that the $\mu \to e\gamma$ decay can occur with a branching ratio close to the present experimental upper bound if there exist heavy leptons in nature and lepton mixing takes place in the weak interaction Hamiltonian (a similar conclusion has also been drawn in ref. [2-4]). Other mechanisms for the $\mu \to e\gamma$ decay which lead to the same result are also possible [5]. The detection of the $\mu \to e\gamma$ decay would imply that the $\mu \to 3e$ decay should exist. For a given $\mu \to e\gamma$ decay rate the predicted value of the $\mu \to 3e$ decay probability is very sensitive to the mechanism assumed for the $\mu \to e\gamma$ decay. For example, in the models with mixing of doubly charged or charged heavy leptons [3], the $\mu \to 3e$ decay rate may be greater by an order of magnitude than the $\mu \to e\gamma$ one; in the case considered in ref. [5] the $\mu \to e\gamma$ decay is induced by a coupling of the muon and electron to a Higgs scalar and the $\mu \to 3e$ decay rate in this scheme is α/π times smaller than the $\mu \to e\gamma$ decay rate. So, just both the $\mu \to e\gamma$ and $\mu \to 3e$ decay rates can be used to distinguish between different mechanisms that might cause the muon number nonconservation.

In the present note we shall evaluate the $\mu \to 3e$ decay rate in $SU_2 \times U_1$ gauge theory with leptons assigned to the doublets [1,2,4]:

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L,\ \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L,\ \begin{pmatrix}N_e\\ e\end{pmatrix}_R\ \begin{pmatrix}N_\mu\\ \mu\end{pmatrix}_R . \qquad (1)$$

Here

$$\nu_e = \nu_1 \cos\theta + \nu_2 \sin\theta\ , \quad N_e = N_1 \cos\theta' + N_2 \sin\theta' ,$$
$$\nu_\mu = -\nu_1 \sin\theta + \nu_2 \cos\theta , \quad N_\mu = - N_1 \sin\theta' + N_2 \cos\theta' , \qquad (2)$$

N_1 and N_2 are heavy neutral leptons with masses M_1 and M_2 , ν_1 and ν_2 are two neutrinos with finite masses and θ and θ' are mixing angles*. In this scheme the ratio of the $\mu \to e\gamma$ decay rate to the $\mu \to e\bar{\nu}_e \nu_\mu$ one is given by the expression /1,9/:

$$B(\mu\to e\ \gamma) = \frac{\Gamma(\mu\to e\gamma)}{\Gamma(\mu\to e\bar{\nu}_e \nu_\mu)} = \frac{3}{32}\ \frac{a}{\pi}\left(\frac{M_2^2 - M_1^2}{M_w^2}\right)^2 \cos^2\theta' \sin^2\theta' , \qquad (3)$$

where M_w is the mass of charged intermediate boson ($M_w \geq 37.3$ GeV)**. For the values of the parameter $\sqrt{|M_2^2 - M_1^2|\sin\theta' \cos\theta'}$ of an order of a few GeV, $B(\mu\to e\gamma)$ is comparable with the existing experimental upper bound /7/ ($B^{exp}(\mu\to e\gamma) < 2.2\times10^{-8}$).

The diagrams giving the main contribution to the $\mu \to 3e$ decay amplitude in the 't Hooft-Feynman gauge are shown in Fig. 1

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*Some features of this model (including its quark sector constructed on the basis of quark-lepton symmetry) are considered in ref. /6/

** Expression (3) is obtained under the assumption $M_i^2 \ll M_w^2$ (i=1,2) which will be used also in our further calculations.

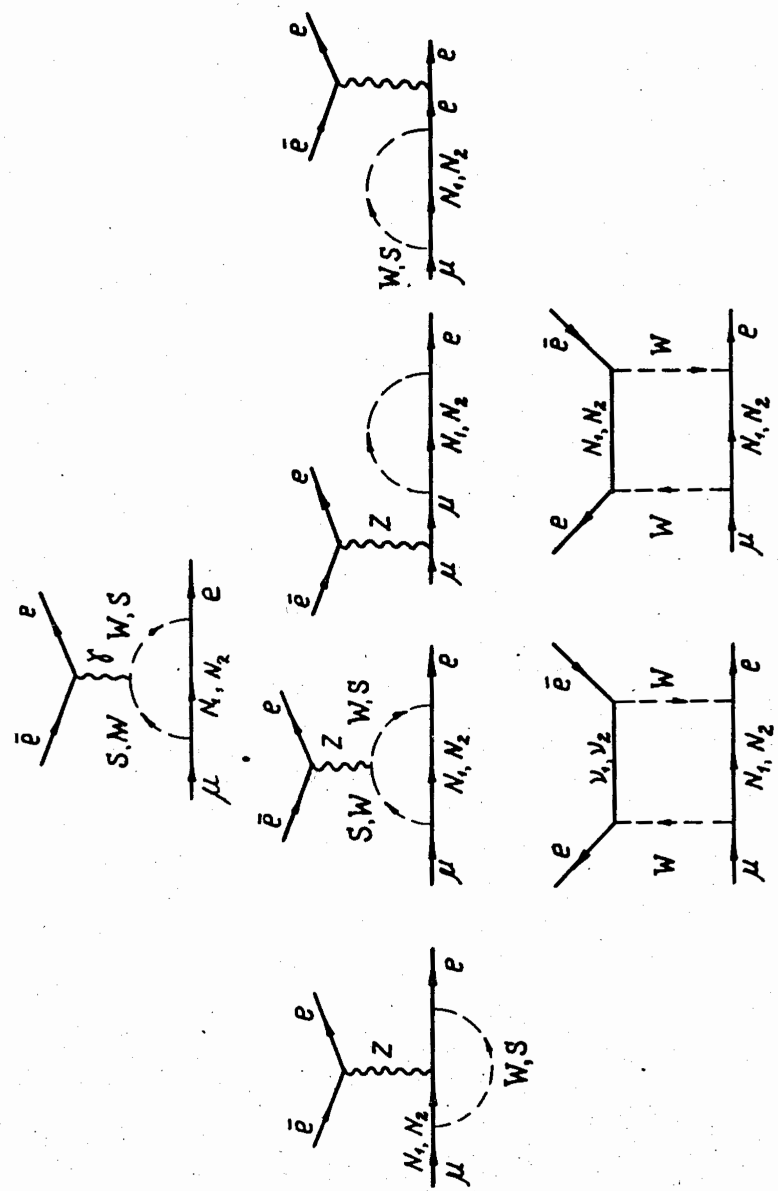

Fig. 1. Leading contribution diagrams for the $\mu \to 3e$ decay.

($s^{\pm}$ is an unphysical Higgs scalar [8]). Let $M^{(\gamma)}$ represent the contribution of the diagrams with virtual photon, $M^{(z)}$, the contribution of the diagrams with virtual Z° -boson and $M^{(w)}$, that of the box diagrams with two virtual W -bosons. Using the results of ref. [9] we get:

$$M^{(\gamma)} = i\frac{G\alpha}{2\pi\sqrt{2}}\sin\theta'\cos\theta'\frac{M_2^2-M_1^2}{M_w^2}\;\bar{u}(k_1)\Gamma_\rho(p-k_1)\times$$

$$\times u(p)\bar{u}(k_2)\gamma_\rho u(-k_3)\frac{1}{(p-k_1)^2}, \qquad (4)$$

where p , k_1 , k_2 , k_3 are the momenta of the muon, two electrons and positron, respectively, and

$$\Gamma_\rho(q)=(1+\gamma_5)(\frac{m_\mu}{4}\sigma_{\rho\alpha}q_\alpha+\gamma_\rho q^2-\hat{q}q_\rho). \qquad (5)$$

(m_μ is the muon mass).

The evaluation of the effective $\mu e Z$ vertex is analogous to the evaluation of the effective $\lambda n Z$ vertex in the Weinberg-Salam model with GIM mechanism carried out in detail in ref. [10]. It leads to the following expression for the $M^{(z)}$ term:

$$M^{(z)} = -i\frac{G^2}{4\pi^2}\sin\theta'\cos\theta'(M_2^2\ln\frac{M_w^2}{M_2^2}-M_1^2\ln\frac{M_w^2}{M_1^2}-3(M_2^2-M_1^2))\times$$

$$\times\bar{u}(k_1)\gamma_\rho(1-\gamma_5)u(p)\bar{u}(k_2)\gamma_\rho(1-2\sin^2\theta_w)u(-k_3), \qquad (8)$$

θ_w being the Weinberg angle.

The terms proportional to the masses and momenta of the real particles can be neglected when evaluating the $M^{(w)}$ term. Our result for $M^{(w)}$ is:

$$M^{(w)} = i\frac{G^2}{8\pi^2}\sin\theta'\cos\theta'\bar{u}(k_1)\gamma_\rho(1-\gamma_5)u(p)\bar{u}(k_2)\gamma_\rho((5+3\gamma_5)\times$$

$$\times(M_2^2\ln\frac{M_w^2}{M_2^2}-M_1^2\ln\frac{M_w^2}{M_1^2}-M_2^2+M_1^2)+(1-\gamma_5)\times$$

$$\times(M_1^2\cos^2\theta'-M_2^2\sin^2\theta'+\cos 2\theta'\frac{M_1^2M_2^2}{M_2^2-M_1^2}\ln\frac{M_1^2}{M_2^2}))u(-k_3). \qquad (7)$$

Taking into account the relation $\frac{G}{\sqrt{2}} = \frac{\alpha\pi}{2M_w^2}\frac{1}{\sin^2\theta_w}$ we get for the $\mu\to 3e$ decay amplitude:

$$M(\mu\to 3e) = i\frac{G\alpha}{2\pi\sqrt{2}}\frac{M_2^2-M_1^2}{M_w^2}\sin\theta'\cos\theta'\{(\bar{u}(k_1)\frac{m_\mu}{4}\times$$

$$\times\frac{\sigma_{\rho\alpha}(p-k_1)_\alpha(1+\gamma_5)}{(p-k_1)^2}u(p)\bar{u}(k_2)\gamma_\rho u(-k_3)-(k_1\leftrightarrow k_2))+$$

$$+[\bar{u}(k_1)\gamma_\rho(1-\gamma_5)u(p)\bar{u}(k_2)\gamma_\rho(a+b\gamma_5)u(-k_3)-$$

$$-(k_1\leftrightarrow k_2)]\}, \qquad (8)$$

where

$$a = (1+\frac{3}{4\sin^2\theta_w})\frac{M_2^2\ln\frac{M_w^2}{M_2^2}-M_1^2\ln\frac{M_w^2}{M_1^2}}{M_2^2-M_1^2}-2+\frac{1}{4\sin^2\theta_w}(1+$$

$$+\frac{M_1^2\cos^2\theta' - M_2^2\sin^2\theta'}{M_2^2-M_1^2} + \frac{M_1^2M_2^2}{(M_2^2-M_1^2)^2}\ln\frac{M_1^2}{M_2^2}\cos 2\theta') \qquad (9)$$

and

$$b = \frac{3}{4\sin^2\theta_w}\left(\frac{M_2^2\ln\frac{M_w^2}{M_2^2} - M_1^2\ln\frac{M_w^2}{M_1^2}}{M_2^2-M_1^2} - 1\right) - \qquad (10)$$

$$-\frac{1}{4\sin^2\theta_w}\left(\frac{M_1^2\cos^2\theta' - M_2^2\sin^2\theta'}{M_2^2-M_1^2} + \frac{M_1^2M_2^2}{(M_2^2-M_1^2)^2}\ln\frac{M_1^2}{M_2^2}\cos 2\theta'\right).$$

The ratio of the $\mu\to 3e$ decay rate to the $\mu\to e\,\bar{\nu}_e\,\nu_\mu$ one is given by the following expression:

$$B(\mu\to 3e) = \frac{\Gamma(\mu\to 3e)}{\Gamma(\mu\to e\bar{\nu}_e\nu_\mu)} = \frac{\alpha^2}{16\pi^2}\left(\frac{M_2^2-M_1^2}{M_w^2}\right)^2 \sin^2\theta'\cos^2\theta' \times$$

$$\times\left[\left(\ln\frac{m_\mu}{m_e} - \frac{11}{8}\right) - (3a-b) + (a+b)^2 + 2(a-b)^2\right], \qquad (11)$$

where the term with factor $(\ln\frac{m_\mu}{m_e} - \frac{11}{8})$ is due to the contribution of the expression in the round brackets in (8), the term with factor $((a+b)^2+2(a-b)^2)$ is due to the contribution of the expression in the square brackets in (8) and that with factor $-(3a-b)$, respectively, to the interference of these two expressions (m_e is the electron mass). Leaving only the leading logarithmic terms in the case $M_2^2 \gg M_1^2$ we have:

$$B(\mu\to 3e) = \frac{3}{64}\,\frac{\alpha^2}{\pi^2}\sin^2\theta'\cos^2\theta'\left(\frac{M_2^2}{M_w^2}\ln\frac{M_w^2}{M_2^2}\right)^2 \times$$

$$\times\ \frac{4\sin^4\theta_w + 4\sin^2\theta_w + 3}{\sin^4\theta_w}\ .$$

Some values of the quantities $B(\mu\to 3e)$ and $\Gamma(\mu\to 3e)/\Gamma(\mu\to e\gamma)$ for various values of the mass M_2 in the case $M_2^2 \gg M_1^2$ and for $M_w = 60$ GeV, $\sin^2\theta_w = 3/8$ and $\theta' = \pi/4$ (maximal mixing) are presented in the table:

Table

Values of the quantities $B(\mu\to 3e)$ and $\Gamma(\mu\to 3e)/\Gamma(\mu\to e\gamma)$ for different values of the mass M_2 in the case $M_2^2 \gg M_1^2$ and for $\theta' = \frac{\pi}{4}$ ($\sin^2\theta_w = \frac{3}{8}$, $M_w = 60$ GeV).

M_2 (GeV)	$B(\mu\to 3e)$	$\Gamma(\mu\to 3e)/\Gamma(\mu\to e\gamma)$
2	1.5×10^{-10}	2.2
3	5.8×10^{-10}	1.7
4	1.2×10^{-9}	1.1

The existing experimental upper bound for the $\mu\to 3e$ branching ratio is [11]

$$B^{exp}(\mu\to 3e) < 1.9\cdot 10^{-9}\ . \qquad (12)$$

So we see that for the heavy neutral lepton masses of an order of a few GeV the $\mu\to 3e$ decay rate may well be close to its experimental upper limit. The $\mu\to e\gamma$ and $\mu\to 3e$ decay rates are of the same order of magnitude in the model we consider. Our results indicate that the $\mu \to 3e$ decay should be investigated even with an accuracy several times better than the existing one.

We would like to conclude with the following remark. We assume that the fermions of the theory acquire mass by gauge-invariant bare mass terms and couplings to Higgs triplet and doublet whose neutral members have nonzero vacuum expectation values. Thus terms like $\bar{\nu}_{eL} N_{eR}$ and $\bar{\nu}_{\mu L} N_{\mu R}$ are absent in the interaction Hamiltonian. In ref. [12] it was suggested that only Higgs doublets are present in the theory. In that case the abovementioned terms appear inevitably leading to a mixing between neutrinos and heavy leptons in the left-handed doublets of eq. (1). The $\mu \to e\gamma$ decay rate in this theory is 25 times larger than the one of eq. (3) [12]. We estimate the $\mu \to 3e$ decay rate to be in this scheme by an order of magnitude smaller than the $\mu \to e\gamma$ decay rate for a reasonable range of heavy neutral lepton masses ($M_i \ll M_W$).

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