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AN ESTIMATION OF THE PARAMETERS OF STRONG GRAVITY

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Оценка параметров сильной гравитации

В работе получены оценки параметров сильной гравитации в двух случаях:
а) когда ее протяженность порядка астрономической единицы, и
б) когда ее протяженность порядка одного анкстрема.

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An Estimation of the Parameters of Strong Gravity

The solutions of a class of generalized equations of gravitation are characterized by the fact that in newtonian approximation, besides the newtonian gravitational force, there exists a Yukawa-like additional force which is called the strong gravity.

The main result of our investigation is the estimation of the parameters of this force, the starting point being its possible perturbative action on the motion of planets and on the spectra of hadrogen atoms. Another result is the proof that strong gravity is not able to prevent the collapse of objects with masses and sizes comparable to those of quasars.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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1. INTRODUCTION

The strong gravity is derived by generalizing the equations of the gravitational field. The generalization can be achieved in various ways. The most straightforward one is to replace the usual lagrangian density

$$L_E = R = \text{scalar curvature}$$

in the integral of the gravitational field by the lagrangian density

$$L_1 = R + \alpha R^2 + \text{small terms.} \quad (1)$$

If then one goes to the weak field limit in the new equations, one obtains a generalized gravitational field the potential of which for a masspoint is /1,2,3,4/

$$U = V + W, \quad (2)$$

where

$$V = \frac{GM}{r}$$

is the newtonian potential, and

$$W = \frac{G_s M}{r} e^{-\frac{r}{\lambda}} = \frac{W_0}{r} e^{-\frac{r}{\lambda}} \quad (3)$$

is the potential of strong gravity.

In (3) and (4) M and r are the mass and the distance to the masspoint, G is the "newtonian" constant of gravitation,

G_s is the "constant of strong gravity", and λ is a parameter. Furtheron we shall use the dimensionless number

$$S = \frac{G_s}{G}.$$

The expression (3) for the strong gravity was derived also in ref.[5] However, in ref.[5] the starting point is not the generalized lagrangian (1) but a modification of the Brans-Dicke theory. This modification is due to G.Callan, S.Coleman and R.Jackiw[6] who replaced the massless scalar field in the original Brans-Dicke theory by a function of a massive scalar field.

Strong gravity was also proposed by A.Salam and his coworkers [7,8], the motivation being that there must exist a generalized theory of gravitation, built by the same principle as the photon-ρ^0-meson mixing theory [9].

There are several other theories related to strong gravity in the sense that they are operating with Lagrangian densities similar to L_I. The introduction of new Lagrangian densities has various motivations to account for the effects of the quantum gravity[10-14] to see whether the big-bang singularity can be avoided by generalizing the Lagrangian density[15] or merely to study properties of more complicated equations of gravitations[16].

It was claimed that at present to the accuracy with which astrophysical and astronomical phenomena are studied strong gravity is negligible[1,4]. The same holds for observations concerning satellite motion and measurements with gravimeters[4].

The purpose of this paper is to reinvestigate the question on a quantitative level. In this paper upper limits for C and λ were obtained for two cases: a) when λ has order of magnitude of one astronomial unit and b) when λ has the order of magnitude of the first Bohr orbit. The estimations are derived from the difference between predicted and observed perihelion shift of Mercury in the case a) (sec.§2) and on the base of predicted and observed Lamb shift of the hydrogen atom for case b) (Sec.§3).

The above estimates will be used to clarify whether repulsive forces due to strong gravity could prevent the collapse in General Relativity as conjectured by A. Salam and coworkers[8] or at least to try to explain the observed stability of quasars as suggested by the present author [17,18]. The analysis shows that strong gravity is not in the position to supply repulsive forces able to prevent the collapse of objects with masses and dimensions typical for quasars. It is also doubtfull whether strong gravity would prevent collapse of objects which are smaller than quasars.

2. SECULAR SHIFT OF MERCURY'S PERIHELION

Shapiro and his coworkers have made an attempt to determine the general relativistic effects of the combined motion of the planets of the Solar system. They have used the following method. The parameters of planetary motion with respect to their mutual disturbances have been calculated in two ways: firstly on the basis of Newton's theory and then by means of Einstein gravitational equations; afterwards the results obtained in both ways were compared with observational data. This comparison has shown that at present there are no grounds to prefer the ralativistic calculation instead of the non-relativistic one while taking into consideration the mutual influence of the Sun and the planets [19]. That is why, in order to obtain more reliable results, we shall limit ourselves to considering the effect of the parameter α in the non-linear Lagrangian $L_1 = R + \alpha R^2$ on the secular shift of Mercury's perihelion, where the success of the conventional GR is indisputable.

Hereafter we shall make an estimation of the parameters λ and W_0 in (3). For that purpose let us consider the planets motion in a centrally symmetric field of the Sun with a potential (2).

As the relativistic corrections are small we can sum them up linearly with the non-relativistic ones, and in particular with the effects of the strong gravity.

Let us pass to taking into account the effect of the strong gravity. We shall seek a solution in the form of an expansion over the small parameter W_0. The Binet equation for a central force with a potential (2) has the form [20]

$$\frac{d^2(\frac{1}{r})}{d\Theta^2} + \frac{1}{r} = \frac{GM}{K^2} + \frac{W_0}{mK^2}\left(1+\frac{r}{\lambda}\right)e^{-\frac{r}{\lambda}}. \qquad (4)$$

where $K = const$ is the surface velocity, r is the planet's radial coordinate, and Θ is the polar angle.

The polar coordinate system is situated in the plane of the non-disturbed motion of the planet, its center coincides with the center of gravity of the system Sun-planet and the straight line $\Theta = 0$ passes through the perihelion.

Let us introduce in (4) the new variable

$$u = \frac{a}{r} - \frac{GM}{K^2}. \qquad (5)$$

The equation (4) takes the form

$$\frac{d^2 u}{d\Theta^2} + u = f(u), \qquad (6)$$

where

$$f(u) = \frac{W_0}{GmM(1-e^2)}\left(1+\frac{a}{\lambda}\frac{1}{u-\frac{1}{1-e^2}}\right)\exp\left(-\frac{a}{\lambda}\frac{1}{u+\frac{1}{1-e^2}}\right). \qquad (7)$$

In formula (7) it has been taken into account that $\frac{GM}{K^2} = \frac{1}{1-e^2}$, where a and e are correspondingly the major half-axis and the eccentricity of the undisturbed motion of the planet.

Applying the method of variation of constants we can write down the solution of the differential equation (6) in the form

$$u = C_1\cos\Theta + C_2\sin\Theta + \int_0^{\Theta} f(u)\sin(\Theta-\phi)\,d\phi.$$

In (5) and (7) it has been taken into account that the undisturbed motion of the planet is along an ellipse with major half-axis a. That is why in the latter equation we substitute $C_1 = \frac{ae}{1-e^2}$ and $C_2 = 0$. If in addition we replace $f(u)$ by its expression in (7) we obtain

$$u=\frac{e}{1-e^2}\cos\Theta+\epsilon\int_0^{\Theta}\left(1+\frac{\alpha}{\lambda}\frac{1}{u+\frac{1}{1-e^2}}\right)\exp\left(-\frac{\alpha}{\lambda}\frac{1}{u+\frac{1}{1-e^2}}\right)\sin(\Theta-\phi)\,d\phi, \tag{8}$$

where $\epsilon=\frac{\zeta}{1-e^2}$.

Eq. (8) is the nonlinear integral equation for the unknown function $u(\Theta)$. As $\epsilon\ll 1$ and $e<1$ the right-hand side of (8) is a contraction operator. Therefore (8) can be solved through the method of successive approximations of Kachopoli-Banach [21]. It is sufficient to retain the first iteration and in it only terms linear in ϵ. Then we obtain the solution in the form

$$u=\frac{e}{1-e^2}\cos\Theta+\epsilon\int_0^{\Theta}\left(1+\frac{\alpha}{\lambda}\frac{1-e^2}{1+e\cos\phi}\right)\exp\left(-\frac{\alpha}{\lambda}\frac{1-e^2}{1+e\cos\phi}\right)\sin(\Theta-\phi)\,d\phi. \tag{9}$$

The law of planetary motion, with an accuracy up to terms which are linear in ϵ, is given by formulas (5) and (9). Let the shift of the perihelion for one revolution of the planet be θ. This means that if we substitute $2\pi+\theta$ for θ in (9) then $\left.\frac{dr}{d\Theta}\right|_{\Theta=2\pi+\theta}$ will be equal to zero. From (5) it follows that for this to be fulfilled $\frac{du}{d\Theta}$ should be equal to zero for $\Theta=2\pi+\theta$. Therefore, in order to determine θ it is necessary to differentiate (9) over Θ and then to substitute in the left-hand side of (9) $\left.\frac{du}{d\Theta}\right|_{\Theta=2\pi+\theta}=0$ and $\Theta=2\pi+\theta$ in the right-hand side.

In this way we obtain with an accuracy up to the first degree in θ

$$\theta=\epsilon\frac{1-e^2}{e}\int_0^{2\pi}\left(1+\frac{\alpha}{\lambda}\frac{1-e^2}{1+e\cos\Theta}\right)\left[\exp\left(-\frac{\alpha}{\lambda}\frac{1-e^2}{1+e\cos\Theta}\right)\right]\cos\Theta\,d\Theta. \tag{10}$$

In obtaining (10) it has been taken into account that $\epsilon\ll 1$. Let us investigate (10) supposing that $\alpha/\lambda\gg 1$. In order to take into account the speed of decreasing of the exponential multiplier in the vicinity of the point $\phi=0$ we rewrite (10) in the following form:

$$\theta=\epsilon\frac{1-e^2}{e}\left[\int_{-\pi}^{+\pi}\left(1+\frac{\alpha}{\lambda}\frac{1-e^2}{1+e\cos\theta}\right)\exp\left[-\frac{\alpha}{\lambda}(1-e^2)\left(\frac{1}{1+e\cos\phi}-\frac{1}{1+e}+\right.\right.\right.$$

$$\left.\left.\left.+\frac{1}{1+e}\right)\right]\cos\Theta\,d\Theta=\epsilon\frac{1-e^2}{e}\int_{-\pi}^{+\pi}\left(1+\frac{\alpha}{\lambda}\frac{1-e^2}{1+e\cos\theta}\right)\left\{\exp\left[-\frac{\alpha(1-e^2)}{\lambda}\right]\times\right.$$

$$\times\exp\left[-\frac{2\alpha}{\lambda}(1-e)e\,\frac{\sin^2\Theta/2}{1+e\cos\Theta}\right]\cos\Theta\,d\Theta.$$

Further, considering that $\alpha/\lambda\gg 1$, we substitute in the latter integral 1 for $\cos\Theta$ and $\frac{\Theta}{2}$ for $\sin\frac{\Theta}{2}$. If apart from this we substitute the limits $-\infty$, $+\infty$ for $-\pi,+\pi$, respectively we obtain

$$\theta=J\frac{\zeta}{e}\left[1+\frac{\alpha}{\lambda}(1-e)\right]\sqrt{2\frac{\lambda}{\alpha}\frac{1+e}{1-e}}\,\frac{1}{e}\exp\left[-\frac{\alpha}{\lambda}(1-e)\right], \tag{11}$$

where $J=\int_{-\infty}^{+\infty}e^{-t^2}dt=\sqrt{\pi}$.

For $e\ll 1$, and taking into account that $\alpha/\lambda\gg 1$, we obtain

$$\theta=\zeta\sqrt{2\pi}\sqrt{\frac{\alpha}{\lambda}}\,\frac{1}{e\sqrt{e}}\exp\left[-\frac{\alpha}{\lambda}(1-e)\right]. \tag{12}$$

We must point out the circumstances that the above calculations can be given a strict formulation. For that purpose it is necessary to introduce a new variable $\tau=\phi^2$ to give the integral the form of a Laplace transformation and to proceed to Abel asymptotic as $\alpha/\lambda\to\infty$ [22].

Let us consider also the opposite case $\alpha/\lambda\ll 1$, where the exponential function in (10) can be replaced approximately by the expression

$$1-\frac{\alpha}{\lambda}\frac{1-e^2}{1+e\cos\Theta}+\frac{1}{2}\frac{\alpha^2}{\lambda^2}\frac{(1-e^2)^2}{(1-e\cos\Theta)^2}.$$

Let us put the latter expression in (10) and substitute with the approximation $1-e\cos\Theta+$ $+e^2\cos^2\Theta$ for $\frac{1}{1+e\cos\Theta}$. If after the substitution we retain in the expression under the integral sign the values of the eccentricity e not higher than second degree we shall obtain

$$\theta=\zeta\pi\frac{1-3e^2}{1-e^2}\frac{\alpha^2}{\lambda^2}. \qquad (13)$$

Let us pass on to the evaluation of ζ. It can be seen from (12) and (13) that the effect of the additional force is maximal at $\lambda\approx\alpha$ and away from this value it decays rapidly as $\alpha\to 0$ or $\alpha\to\infty$. That is why we shall use the obtained expressions for the evaluation of the parameters of the additional force supposing that $\lambda\approx\alpha$ and $e^2\ll 1$. Then from (13) we obtain

$$\zeta\approx 0{,}3\,\theta. \qquad (14)$$

The latter formula offers the possibility for evaluation of ζ. For that purpose we use the data for the shift of Mercury's perihelion because it is at least half an order greater than the perihelion motion of other planets.

Other methods for evaluating the accuracy of predicting the GR can also be suggested but the method connected with the shift of Mercury's perihelion is the most reliable[19]. This can be explained by the fact that the shift of the perihelion is a cumulative quantity that is calculated

on the basis of observations for two centuries. In such circumstances the effect of processes having accidental or periodical character (e.g., solar activity) is considerably reduced. Following this argument we could possibly understand why the attempts of Shapiro, already mentioned above, to verify GR by analysing the simultaneous motion of several planets for several months were not successful. It is true that until recently it was thought that the shift of Mercury's perihelion could be strongly influences because of the relatively high degree of flattening of the Sun[23,24]. The latest measurements however have shown that the difference between the polar and the equatorial diameters of the Sun is equal to 18.4 ± 1.25 mili-arcsec. This value is not sufficiently great in order to exert substantial influence on the shift of the perihelion. Apart from this it is comparable with the value of 16 mili-arcsec. for a uniformly revolving Sun[25,26].

The theory of Einstein predicts for the shift of Mercury perihelion the value of $\theta_E=43{,}03''$ for a century or $5.05\cdot10^{-7}$ rad./rev. The observed value amounts to $\theta_0=43.11\pm0.45''$ for a century. The value predicted by the theory lies in the range of observation of errors. Let us take the most unfavourable assumption namely that the half-interval $0.45''$ of the error of θ_0 is conditioned by the strong gravity, or in other words let us suggest that $\theta\approx0{,}01\,\theta_E$. Then from (14) we obtain for Mercury ($e=0.2$)

$$\zeta\approx1{,}4\cdot10^{-9} \quad \text{if} \quad \lambda\approx\alpha \qquad (15)$$

where $a = 5.5\ 10^{12}$ cm in the major half-axis of Mercury's orbit.

3. ESTIMATION OF G_s AND λ FOR ATOMIC DIMENSIONS

Let us discuss briefly some of the possible effects of the strong gravity on the spectrum of hydrogen atom.

Let us assume that in Schrodinger's equation

$$\Delta\Psi + \frac{2m}{h^2}[E-U]\Psi = 0$$

the potential has the form

$$U = V + W, \qquad (16)$$

where

$$V = \frac{-e^2}{r}$$

is the electrostatic potential and

$$W = \frac{G_s m M}{r} e^{-\frac{r}{\lambda}} = \frac{W}{r} e^{-\frac{r}{\lambda}}$$

is the potential of strong gravity.

In the last two formulae e is the charge of the electron and proton and m and M are their masses.

Taking into account the additional shift of the transition energy of the jump $2s_{1/2} - 2p_{1/2}$ under the influence of W which is considered as a small correction we obtain

$$\Delta E_W = -\frac{W_0}{\alpha_0} \frac{\left(\frac{\alpha_0}{\lambda}\right)^2}{\left(1 + \frac{\alpha_0}{\lambda}\right)^4}, \qquad (17)$$

where $\alpha_0 = \frac{h^2}{me^2}$ is the radius of the first Bohr orbit. ΔE_W has a maximum value

$$|\Delta E_{W_{max}}| = \frac{9}{512}\frac{W_0}{\alpha_0}, \qquad (18)$$

which is reached at $\lambda = 3\alpha_0$.

ΔE_W is superimposed on Lamb's shift ΔE_L of the hydrogen line with an obtained experimental value of $\Delta E_L^{exp} = 1057.893(20)$ MHz /27/.

The theoretical value ΔE_W^{th} of this quantity is equal to 1057,864(14)MHz /28/. Proceeding from these data let us evaluate the parameter W of the strong gravitation. For this purpose let us assume that it is admissible to identify ΔE_W with $E_L^{exp} - E_L^{th} = 0.029$ MHz.

Under this assumption we obtain

$$W_0 /_{\lambda = 3a_o} \approx 0{,}8 \cdot 10^{-27} \text{ erg.cm} \qquad (19)$$

and replacing W_0 by $G_s\, mM$

$$G_s \lesssim 0{,}65 \cdot 10^{25} \text{ erg.cm/g}.$$

For the number ζ we have

$$\zeta /_{\lambda \approx 3a_o} \approx 10^{30}. \qquad (20)$$

4. THE EFFECT OF STRONG GRAVITY ON THE STABILITY OF QUASARS

We shall estimate the forces acting in the quasar on the base of a rough model. In this model the quasar will be considered as a system of two attracting each other mass points in those cases when we are interested in the force and the energy of attraction. Each of the mass points has mass $M/2$, where M is the whole mass of the plasma. The mass of the photon gas is disregarded and does not enter in M. If OXYZ is a Cartesian coordinate system with a center coinciding with the centrum of the quasar, then one of the masses is located in a point $X=R_q/2$, $Y=Z=0$, and the other one - in a point $X=-R_q/2$, $Y=Z=0$, where R_q is the radius of the quasar.

As far as the force and the energy of pressure are concerned, the quasar is considered as a sphere with radius R_q where the pressure is the same over the whole sphere. The pressure will be equalized with the photon gas pressure with the plasma pressure being neglected.

The validity of the model is motivated in the following way. The mass M_{ph} of the photon gas could be ignored in comparison with the mass M_{pl} of the plasma because $\frac{M_{ph}}{M_{pl}} \approx \frac{R_g}{R_q}$ where $R_g = \frac{2GM}{c^2}$ is the gravitational radius. As by quasars we usually have $\frac{R_g}{R_q} \ll 1$, then $M \approx M_{pl}$. On the other hand, because[29] $\frac{P_{pl}}{P_{ph}} \approx 9\sqrt{\frac{M_\cdot}{M}}$ and $M \geq 10^6\ M$. we can substitute $p \approx p_{pl}$. In the framework of the accepted model the Newtonian force N_q with which are attracted the two parts of the quasar is given by the expression

$$N_q = -\frac{GM^2}{4R_q^2}. \qquad (21)$$

The gas force is calculated according to the formula $G_q = 12R^2p$. (Here and further on it is accepted that $\pi \approx 3$). The pressure and the entropy S of the photon gas are given by the expressions $p = \frac{aT^4}{3}$ and $S = 4/3.\ VT^3$ respectively, where $V = 4R_q^3$ is the volume of the quasar and $a = 7.7\times10^{-15}$ erg.cm^{-3} gr^{-4} is a constant. Following the elimination

of T from the formulas for p and S we obtain

$$p = \left(\frac{3}{a}\right)^{1/3} S^{4/3} R_q^{-4}$$

and consequently

$$G_q = 12\left(\frac{3}{a}\right)^{1/3} S^{4/3} \frac{1}{R_q^2}. \qquad (22)$$

Let us pass on the evaluation of the post-newtonian force of the quasar P_q.

The energy E_p, corresponding to the post-newtonian force P_q, or, in other words, the energy, which is obtained with an accuracy of the order R_g/R_q on the basis of the presence of GR effects, is given by the expression[29]

$$E_p = -\frac{GM^2}{R_q}\,\frac{R_g}{R_q}.$$

Then, according to the accepted model, we have the relation

$$E_p(R_q) = 2\int_{R_q/2}^{\infty} P_q(\chi)\,d\chi = \frac{-GM^2 R_g}{R_q^2}$$

from where we obtain

$$P_q = -\frac{GM^2 R_g}{4R_q^3}. \qquad (23)$$

Let us consider now the equilibrium conditions of the quasar on the basis of the post-newtonian approximation of the classical equation of Einstein of the gravitation field. Making use of our rough estimations we can write down the equilibrium condition as follows:

$$G_q + N_q + P_q = 0. \qquad (24)$$

If in (24) we substitute (21), (22) and (23), we obtain

$$\frac{A}{R_q^2} - \frac{GM^2 R_g}{4R_q^3} = 0 \qquad (25)$$

where $A = 12\left(\frac{3}{a}\right)^{1/3} S^{4/3} - \frac{GM^2}{4}$. At $S = S^* = \frac{a^{1/4}(GM)^{3/4}}{24}$ the minued in (25) disappears and that corresponds to the equilibrium condition in newtonian approximation. As $S^* = \frac{16}{3} a R_q^3 T^3$, then according to the equilibrium condition in newtonian approximation the temperature of the quasar should be

$$T_q = \sqrt[4]{\frac{G}{a}}\, \frac{1}{4\sqrt[3]{2}} \sqrt{M}\,. \qquad (26)$$

The presence of the subtrahend in (25) however shows that by more strict consideration there will be no equilibrium. There remains the post-newtonian force which could be balanced by the repulsive force Y_q due to the strong gravity. In this case instead of the equilibrium condition (25) we should have the following equilibrium condition:

$$G_q + N_q + P_q + Y_q = 0, \qquad (27)$$

where Y_q is the interaction force between the two parts of the quasar, generated by the Yukawa force.

The Yukawa force is essentially depending on λ.

Let us consider this question in more detail.

A. Let us suppose that $\lambda \approx r_H$, where r_H is the radius of the nucleon. Then, it is natural to suggest[3] that the potential of strong gravity is identical to the ordinary Yukawa potential from the theory

of strong interactions, i.e., to the potential of the nuclear forces. If this is the case, then the contribution of the force could be described by means of the right-hand part of the Einstein gravitational equations. In other words, as in the studies of Openheimer and his assistants, it is necessary by the calculation of T_{ik} to take into account the nuclear forces. By that approach however it is known[29] that the object loses stability already at $M \approx 2M$.

B. Let us consider now the case $R_q > \lambda > r_H$. The strong gravity is supposed to be repulsive. In this case because of the counteraction of the strong gravity, the macroscopic effect will consist in the formation of surface tension on the surface of the quasar, as well as state of strain inside the quasar with isotropic tension $\sigma < 0$.

C. Finally, let us suppose that $\lambda > R_q$. In this case the potential of strong gravity would be equal to the newtonian potential. And that means that (27) will coincide with (25). Therefore, in that case there will be no equilibrium.

And so, from our analysis it follows that the strong gravity could generate a repulsive force with the necessary properties if we have case B, i.e., if the specific Yukawa forces were acting at longer distance than the nuclear forces under the condition that their radius of action was smaller than the radius of the quasar. It is interesting to note that short-range action of a similar type has been proposed on other grounds[29,30].

For the sake of simplicity let us suppose that the plasma consists only of nucleons and electrons and let ℓ be the average distance between two nucleons. In principle, however, this suggestion could not be true, but in such case it is easy to adjust the conclusions we are to make further.

The force F_s of the Yukawa type gravitational interaction between two nucleons with a distance between them equal to r can be written down approximately as follows:

$$F_s = 0, \quad r > \lambda,$$

$$F_s \approx \frac{G_s M^2}{r^2}\left(1 + \frac{r}{\lambda}\right), \quad r < \lambda .$$

Let us make an estimation of the repulsive force Y_q acting between the two halfs of the quasar, which would be necessary to maintain its stability. It is sufficient to investigate the case $\lambda \approx \ell$. For the stability to exist it is necessary that the post-newtonian force P_q be equal to the repulsive force Y_q. As the latter is due to short-range forces we can put $Y_q = \pi R_q^2 \sigma$.

For the special case $\lambda \approx \ell$

$$\sigma \approx \frac{F_s}{\ell^2}$$

so that in a rough approximation we have

$$Y_q \approx \pi \left(\frac{R_q}{\ell}\right)^2 F_s .$$

Now, equating the expressions for P_p and Y_q for a quasar with a mass $M_q \approx 10^9$ M and radius $R_q \approx 10^{16}$ cm, we get

$$\ell \approx 1{,}5 \cdot 10^{-6} \text{ cm}$$

$$F_s \approx 1{,}5 \cdot 10^{-1} \text{ dyn.}$$

As

$$F_s = \frac{d}{dr}\left(\frac{W_0}{r} e^{-\frac{r}{\lambda}}\right)$$

for $r \approx \ell \approx \lambda$ we have

$$F_5 \approx \frac{2W_0}{\ell^2} .$$

From the latter formula we derive

$$W_0 = W_0^q \approx 1{,}7 \cdot 10^{-13} \text{erg·cm} .$$

So, we obtained an estimation for the value of which would ensure the stability of the quasar.

Let us now look for the possible value of W_0^q as derived from the behaviour of the H atom.

From (17) we have

$$\Delta E_W \approx \frac{10^4}{4{,}5} \cdot W_0 .$$

Taking for ΔE_W as above the value $0.8 \cdot 10^{-27}$ ergcm we have

$$W_0 = W_0^H \approx 1{,}4 \cdot 10^{-26} \text{ erg.cm.}$$

For the comparison we must take $W_0^{H}* = \frac{m_p}{m_e} W_0^H = 1840\, W_0^H = 2{,}5 \cdot 10^{-23}$ erg.cm instead of W_0^H.

The comparison between W_0^q and $W_0^{H}*$ shows that even if shortrange forces would be 10^{10} times weaker than is necessary to maintain the equilibrium of quasars, they would be discovered from the behaviour of atomic systems. As is easily seen, this conclusion is true not only for $r \approx \ell \approx \lambda$ but also for $\ell \gtrsim \lambda \geq 0{,}01\ell$.

So, one is led to the conclusion that the strong gravity does not explain the observed stability of quasars and probably of objects much smaller than quasars.

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