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Электромагнитный формфактор протона в модели векторной доминантности, модифицированной на малых расстояниях

Произведено сравнение предсказаний модифицированной на малых расстояниях модели векторной доминантности с имеющимися экспериментальными данными по упругому ep-рассеянию. Показано, что учет вклада центральной части протона с радиусом равным его комптоновской длине волны улучшает качество описания.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Bilenkaya S.I., Skachkov N.B., Solovtsov I.L. **E2 - 10404**

Proton Electromagnetic Form Factor in the Vector Dominance Model Modified at Short Distances

Predictions of a vector dominance model modified at short distances are compared with the experimental data available on elastic ep-scattering. It is shown that the consideration of the proton central part with the radius equal to its Compton wave length gives a better description .

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

The vector dominance model (VDM) is successfully used in studying many problems of physics of the hadron electromagnetic interactions. However, the VDM fails to describe the experimentally observed rapid $1/t^2$ decrease of electromagnetic nucleon form factors at large $|t|$. Many attempts have been made to improve the situation. For example, the authors of [1] considered the contributions from unobserved hypothetical vector mesons ω', ϕ', ρ', etc., in addition to the contribution from experimentally established ρ, ω, ϕ and $\rho''(1550)$. In ref. [2] the dipole behaviour of the nucleon form factors was achieved by introducing the phenomenological one parameter dependence of the meson-nucleon interaction constant $g(t)$ on momentum transfer.

In this paper we analyse all the existing experimental data on elastic ep-scattering using the model of proton form factor proposed in ref. [3]. The model is based on the invariant description of the particle structure in the relativistic coordinate space [4] in which the particle Compton wave length plays the role of a natural scale. In [3] it is shown that all the discovered vector mesons ρ, ω, ϕ and $\rho''(1550)$ contribute to the proton structure only at distances larger than its Compton wave length. The main point of this model [3] is to allow for vector mesons and the proton central part (with the radius equal to its Compton wave length) contribution to the form factor. Note that the contribution from the central part can be calculated within the formalism and has clear physical meaning.

From a preliminary analysis made in [5] for the experimental data on the proton magnetic form factor it follows that to achieve a good description it is enough to take into account the contribution from the experimentally observed ρ, ω, ϕ and $\rho''(1550)$ mesons only. It is natural that to conclude about the agreement of the model with experiment, it is necessary to make the comparison with all the existing experimental data on elastic ep-scattering.

2. *PARTICLE STRUCTURE IN THE RELATIVISTIC COORDINATE SPACE*

In ref. [3] a new method was proposed for description of the spatial structure of particles in terms of a new relativistic relative coordinate introduced in [4]. The expansion over unitary irreducible representations of the Lorentz group [6] is ised instead of the Fourier transform in passing to a configurational representation. For the proton form factor F(t) due to its spherical symmetry this transformation is as follows [4,6]:

$$F(t) = 4\pi \int \frac{\sin rMy}{rM \operatorname{sh} y} F(r) r^2 dr, \tag{1}$$

where M - is the proton mass, and the hyperbolic angle y is the rapidity corresponding to the momentum transfer $t = (p-k)^2$: $y = \operatorname{Arch} \frac{2M^2 - t}{2M^2}$.

The important property of the relativistic coordinate $\vec{r}$, introduced by (1) is that its modulus r is a relativistic invariant (since it parametrizes the eigenvalues of the invariant operator Ĉ, the Casimir operator of the Lorentz group [3]). Therefore the function F(r) like F(t) is also the relativistic invariant and describes the particle structure in an arbitrary reference frame. In terms of F(r) the invariant mean-square radius

$$\langle r_0^2 \rangle = 6 \frac{\partial F(t)}{\partial t}\Big|_{t=0} \Big/ F(0)$$

is written as follows [3]

$$\langle r_0^2 \rangle = \frac{\langle \hat{C} F(t) \rangle_{t=0}}{F(0)} = \frac{\hbar^2}{M^2 c^2} + \frac{\int r^2 F(r)\, d\vec{r}}{\int F(r)\, d\vec{r}} = \frac{\hbar^2}{M^2 c^2} + \langle r^2 \rangle. \quad (2)$$

Consequently, for $\langle r^2 \rangle$ positive* both the new coordinate and $F(r)$ describe not the total particle structure but only the region at distances larger than its Compton wave length. To the value of the mean square radius $\langle r_0^2 \rangle = \hbar^2 / M^2 c^2$, i.e., to the central part there corresponds the spatial distribution $F(r) = \delta(r) / 4\pi r^2$, which produces (cf. eq. (1)) the following contribution to the form factor from that sphere

$$F_{r_0 = \frac{\hbar}{Mc}}(t) = \frac{\sin rMy}{rM \operatorname{sh} y}\Big|_{r=0} = \frac{y}{\operatorname{sh} y} = \frac{2M^2 \operatorname{Arch} \frac{2M^2 - t}{t}}{\sqrt{t(t - 4M^2)}}. \quad (3)$$

As is shown in [3], the standard form factor $F(t)$ for $\langle r^2 \rangle > 0$ can be represented in the form $F(t) = \frac{y}{\operatorname{sh} y} \Phi(y)$

where the "external" form factor $\Phi(y)$ corresponds to the particle structure outside the sphere with $r_0 = \hbar / Mc$. Note that the factor $y/\operatorname{sh} y$ and the corresponding region with $r_0 = \hbar / Mc$ have no nonrelativistic analogs since $\hbar/Mc \xrightarrow[c\to\infty]{} 0$ and $y/\operatorname{sh} y \xrightarrow[c\to\infty]{} 1$.

Now let us find, in the new coordinate space, what proton region is described by the vector meson contribution. Recall that the VDM allowing for ρ, ω, ϕ mesons describes satisfactorily the experimental data on proton form factor only at small momenta transfer $0 \le -t \le 1\,(\mathrm{GeV}/c)^2$. At large $|t|$ the propagator set of the type $1/(\mu_V^2 - t)$ did not give the experimentally observed rapid "dipole" $1/t^2$ decrease of the proton form factor. However, the transform of a propagator of this type in the r-space depends

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* For this purpose it is sufficient that $F(r)$ is of constant sign.

essentially on the mass ratio of a particle itself M and intermediate particle μ:

$$F(r)=\begin{cases}\dfrac{1}{4\pi r}\,\dfrac{\operatorname{ch} rMa}{\operatorname{sh} rM_\pi} & \mu^2<4M^2,\quad a=\arccos\dfrac{\mu^2-2M^2}{2M^2} \qquad (4)\\[2ex] \dfrac{1}{4\pi r}\,\dfrac{\cos rMb}{\operatorname{sh} rM_\pi} & \mu^2>4M^2,\quad b=\operatorname{Arch}\dfrac{\mu^2-2M^2}{2M^2}. \qquad (5)\end{cases}$$

From formulae (2) and (4–5) we find

$$\langle r^2\rangle=\frac{6M^2-\mu^2}{\mu^2M^2}. \qquad (6)$$

Since the masses of the discovered ρ , ω , ϕ and $\rho''(1550)$ mesons obey the inequality $\mu_V^2<4M^2$, the function $F_p(t)$ characterizing their contribution to the proton structure in terms of a new coordinate space has the form (4), i.e., it is of constant sign, and $\langle r_p^2\rangle$ (6) is positive. Consequently, these vector mesons produce the proton structure at distances exceeding its Compton wave length and contribute to its "external" form factor $\Phi(y)$.*

Therefore to describe the total proton structure in the momentum space, it is necessary to add the contri-

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*Note that for a pion due to inequality $\mu_V^2>4M_\pi^2$ to the same vector mesons there correspond the oscillating functions $F_\pi(r)$ and negative $\langle r^2\rangle$. By (2) the contribution of negative $\langle r^2\rangle$ leads to the value of pion $\langle r_0^2\rangle$ that is consistent with experimental data.

bution from its central part. As a result, the proton electromagnetic form factor takes the form

$$F_p(t) = \frac{y}{\operatorname{sh} y} \sum_V \frac{a_V}{1 - t/\mu_V^2} . \qquad (7)$$

It is clear that formulae (7) together with (3) result in the correct "almost dipole" asymptotic behaviour of the proton form factor at large $|t|$

$$F_p(|t| \gg M_p^2) \sim \frac{\ln|t|/M_p^2}{t^2} . \qquad (8)$$

At small $|t| < 1(\mathrm{GeV}/c)^2$ the factor $y/\operatorname{sh} y \approx 1$, i.e., the pure VDM is valid, and at large $|t|$ this factor decreases like $\ln(|t|/M^2)/t$. An idea of necessary modification of the VDM through introducing an extra factor was suggested by other authors [4], as well. However, their choice of the propagator as such a factor is pure phenomenological and has been argued only by the wide use of this function in particle physics (see also [7]).

Besides (7) we will also employ another, accepted in the VDM, parametrization of nucleon form factors

$$F_p(t) = F(y)\left[1 - \sum_V a_V + \sum_V \frac{a_V}{1 - t/\mu_V^2}\right], \quad F(y) = \frac{y}{\operatorname{sh} y} \qquad (9)$$

which in the conventional approach, i.e., at $F(y)=1$ corresponds to a possible existence of the nucleon kern. With our modification of the VDM at short distances, i.e., at $F(y) = y/\operatorname{sh} y$ the nucleon form factor in parametrization (9) does not turn into a constant at large $|t|$ and decreases as $\ln(|t|/M^2)/t$.

To take into account a possible contribution of the ρ-meson width we use here the following modification of the ρ-meson propagator [8]

$$\frac{1}{1 - t/\mu_\rho^2} \to \frac{\mu_\rho^2 + 8\Gamma_\rho M_\pi/\pi}{(\mu_\rho^2 - t) + (4M_\pi^2 - t)\Gamma_\rho\, \alpha(t)/M_\pi} , \qquad (10)$$

$$\alpha(t) = \frac{2}{\pi}\sqrt{\frac{4M_\pi^2 - t}{-t}}\ \ln\left[\frac{\sqrt{-t} + \sqrt{4M_\pi^2 - t}}{2M_\pi}\right]. \qquad (11)$$

Keeping the ideas of the quark models, the picture obtained here may be interpreted as follows. By estimations of the models, the relative motion of three quarks constituting the proton is limited to the region with the radius of the proton Compton wave length. Hence, the introduction of the central part contribution (3) is an approximate geometrical consideration of the three-quark contribution.

This geometrical approach is reasonable because the wave function of three-quark motion in this region is unknown. Virtual quark-antiquark pairs, being excited by virtual photon produce vector mesons which due to (4-6) compose the proton structure at distances larger than the proton Compton wave length.

3. RESULTS OF THE ANALYSIS OF EXPERIMENTAL DATA

We extract information on the nucleon electromagnetic form factors directly from the data on differential cross sections. From the very beginning we take a definite dependence of the form factors on variables. Free parameters are obtained from the available experimental data by minimizing the functional

$$\chi^2 = \sum_{i,k} \frac{1}{\Delta^2_{i,k}} (\sigma_{i,k} - N_k \sigma_i^{\text{theor.}})^2. \qquad (12)$$

Here σ_{ik} is the differential cross section in i-th point in k-th experiment, $\Delta_{i,k}$ is the error of $\sigma_{i,k}$, $\sigma_i^{\text{theor.}}$ is the i-th point cross section calculated by the Rosenbluth formula with the corresponding form factors. Factors N_k are varied parameters and are introduced to take into account possible systematic errors of a k-th experiment.

Minimization of χ^2 was performed by the linearization method proposed in ref. [10] by the program FUMULI [11].

The method we use for processing of data, is analogous to that used in the phase-shift analysis and described in ref. [12].

In this work we analyse almost all the data available (299 points) on elastic ep -scattering in the range $0.012 \le -t \le 25.03 (GeV/c)^2$. For the charge and magnetic form factors we assume the relation

$$G_p^E(t) = G_p^M(t)/\mu_p , \qquad (13)$$

where μ_p is the proton total magnetic moment in nuclear magnetons. This assumption agrees with the experimental data [13].

The results of analysis are collected in *Table 1*. Columns 1,3 show the values of varied parameters when form factors in (12) are taken in the form (7) and also in (7) and (10) when the ρ -meson width is taken into account. For comparison columns 2,4 show the results of calculation by VDM-like formulae, i.e., for $F(y)=1$. As is seen from the *Table*, allowing for the central part contribution through introducing the factor $F(y)=y/\mathrm{sh}\,y$ lowers the value of χ^2 by 20-40 that improves the confidence level by an order and more. We also have made the processing of data on ep -scattering cross section by parametrizing the form factors by the sum of five poles (ρ , ω , ϕ , $\rho'(1250)$, $\rho''(1550)$). The introduction of a new pole does not improve the description and some parameters were found with errors exceeding their values. The data processing has been made also with the form factor in form (9). In this case accuracy of the description does not change. Thus, the data do not indicate to the existence of the nucleon kern * .

If the coupling of ω and ϕ -mesons with nucleons is assumed to be relatively small, the form factor can be represented as a sum of the contributions of two poles

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* The same conclusion has been made by authors of ref. [14] within another approach to the form factor parametrization.

Table 1

Model	four poles (ρ,ω,ϕ,ρ'')		four poles (ρ,ω,ϕ,ρ'') and with ρ-meson width		Fitting by formula (14)
	F = y/shy	F – 1	F = y/sh y	F = 1	F = y/sh y
a_1	26.8I±I.86	I0.82±I.5I	0.80±0.06	0.28±0.05	-0.I6±0.0I
a_2	-27.86±2.08	-9.52±I.68	0.32±0.I4	I.89±0.I0	(0.38±0.02)$(GeV/c)^{-2}$
a_3	33.30±0.29	0.06±0.23	0.04±0.09	-I.I5±0.06	(2.55±0.05)$(GeV/c)^{-2}$
N_1	0.963±0.0II	0.9II±0.0II	0.930±0.0I0	0.889±0.009	0.963±0.0II
N_2	I.0I9±0.007	0.999±0.006	1.002±0.006	0.990±0.006	I.02I±0.007
N_3	I.023±0.0I7	0.980±0.0I6	0.993±0.0I5	0.960±0.0I5	I.026±0.0I7
N_4	0.832±0.029	0.795±0.027	0.807±0.028	0.777±0.026	0.834±0.029
N_5	0.99I±0.0I3	0.943±0.0I2	0.960±0.0I2	0.92I±0.0II	0.993±0.0I4
N_6	0.92I±0.020	0.883±0.020	0.893±0.018	0.864±0.0II	0.923±0.020
N_7	0.867±0.0I5	0.82I±0.0I4	0.837±0.0I4	0.802±0.0I3	0.868±0.0I5
N_8	0.978±0.020	0.928±0.0I9	0.945±0.0I9	0.905±0.0I8	0.979±0.02I
N_9	I.00I±0.0II	0.957±0.0I0	0.97I±0.0I0	0.939±0.009	I.003±0.0I2
N_{10}	0.953±0.0I2	0.9I0±0.0II	0.922±0.0II	0.892±0.0I0	0.955±0.0I3
N_{11}	0.960±0.0I2	0.9I2±0.0II	0.929±0.0I0	0.889±0.009	0.96I±0.0I2
N_{12}	0.95I±0.0I3	0.9I3±0.0I2	0.923±0.0I2	0.894±0.0I3	0.953±0.0I3
χ^2	344	370	344	396	343
C.L.	0.85%	0.033%	0.85%	0.003%	0.95%

$$F_p(t) = F(y)\left(\frac{a_1}{1-a_2 t} + \frac{1-a_1}{1-a_3 t}\right). \qquad (14)$$

In column 5 the values of parameters are shown for minimum χ^2. In this case from (14) we obtain

$$(a_2)^{-1/2} = 1.620 \pm 0.04 \quad (\mathrm{GeV/c})$$

$$(a_3)^{-1/2} = 0.63 \pm 0.01 \quad (\mathrm{GeV/c}).$$

Note that the first value within errors coincides with the mass of the discovered ρ''-meson (1550). If the parameters a_2 and a_3 are fixed by the values of masses of vector mesons, the description spoils essentially both for $F(y)=1$ and for $F(y)=y/\mathrm{sh}\, y$.

In the *Table* the normalization factors are given for various parametrizations of the form factors. It is seen that the values of norms for $F(y)=y/\mathrm{sh}\, y$ do not change within errors, differ from unity by 5% in most cases, and the errors of norms are smaller than 5%.

Note that the values of normalization factors within errors coincide with those of norms given in ref. [14].

Thus, the present analysis of data on elastic ep-scattering reveals that the proton central part contribution resulting in the modification of the VDM at distances of order of the proton Compton wave length $(\hbar/Mc)^2 = 0.04 \mathrm{fm}^2$ permits a satisfactory description in the model with the contribution of 4 mesons (including the ρ-meson width) and in the two pole model defined by expression (14). Note that the considered parametrizations (7) of the form factors give their correct behaviour for $|t| \to \infty$ as $1/t^2$ while the VDM cannot describe the experimentally observed decrease of the proton form factor. Note also that in contrast to the phenomenological factors [2], introduced earlier for modifying the VDM, the factor $F(y) = y/\mathrm{sh}\, y$ contains no new parameters and is of a "geometrical" relativistic nature.

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