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CONFORMAL INVARIANT TWO AND THREE-POINT FUNCTIONS IN FLAT SUPERSPACE

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Конформно-инвариантные двух- и трехточечные функции
в плоском суперпространстве

Найдены некоторые представления конформной группы в плоском суперпространстве. Получено, что инвариантные двух- и трехточечные функции имеют аналогичную структуру, как и в обычном пространстве-времени.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Conformal Invariant Two- and Three-Point
Functions in Flat Superspace

Some of the representations of conformal group acting in flat superspace are found. The invariant two- and three-point functions are analogous in form to the corresponding functions in ordinary space-time.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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In the last years the quantum field theories invariant with respect to relativistic supertransformations and to superconformal transformations are investigated [1-6]. It is natural such theories to be considered in the superspace [6] which is an extension of Minkowsky's space-time with some anticommuting (Fermi) variables.

This superspace flat in the space-time sector (if ignoring gravitational effects) is curved in other sectors [7]. In papers [8,9] the superconformal algebra acting on the flat superspace was proposed.

Because of attractive properties of conformal invariant QFT in the ordinary space-time [10] it is interesting to invastigate properties of such a theory in the flat superspace. Indeed here, the total two- and three-point functions up to normalization constant are determined also from the invariance condition only. The four-point function depends only a function of two harmonic ratios.

Determined in such a way two- and three-point Green functions may be used to construct the higher Green functions using the skeleton diagram [11] or the operator product expansion [10] techniques.

Consider the proposed in [8,9] graded Lie algebra of conformal transformations in a flat superspace

$$[M_{AB};M_{CD}] = i\{g_{CB}M_{AD} - (-1)^{\bar{B}\bar{C}} g_{DB} M_{AC} - (-1)^{\bar{A}\bar{B}} g_{CA} M_{BD} + $$
$$+ (-1)^{\bar{A}\bar{B}+\bar{A}\bar{C}} g_{DA}M_{BC}\},$$

$$[P_A;M_{BC}] = i\{g_{BA}P_C - (-1)^{\bar{B}\bar{C}} g_{CA} P_B\},$$

$$[K_A;M_{BC}] = i\{g_{BA}K_C - (-1)^{\bar{B}\bar{C}} g_{CA} K_B\}, \tag{1}$$

$$[D, P_A] = -P_A, \quad [D,K_A] = iK_A,$$

$$[P_A;K_B] = 2i(g_{BA}D - M_{AB}).$$

$$[D, M_{AB}] = [P_A;P_B] = [K_A;K_B] = 0,$$

where

$$[X_{AB};X_{CD}] = (-1)^{(\bar{A}+\bar{B})(\bar{C}+D)} [X_{CD};X_{AB}]$$

$$M_{AB} = -(-1)^{\bar{A}\bar{B}} M_{BA}, \quad A,B,\ldots = \{\mu = 0,1,\ldots,2h-1; \alpha = 1,\ldots,2r\}$$

$$(h = 1,2,\ldots, r = 0,i,\ldots),\ g_{AB} = (g_{\mu\nu}; C_{\alpha\beta}),\ g_{00} = -g_{jj} = 1,\ C_{\alpha\beta} = -C_{\beta\alpha}$$

can be given by

$$C = \left(\begin{matrix} i\sigma_2 & & \\ & \ddots & \\ & & i\sigma_2 \end{matrix} \right)$$

and $\bar{A} = 0$ if $A = \mu$ and $\bar{A} = 1$ if $A = \alpha$.

In the case when $r = 0$ algebra (1) coincides with the conformal algebra in the Minkowsky space with $2h-1$ space-like dimensions.

Consider the field $\Psi(z) = \Psi(x,\Theta)$. As in the case of ordinary superfields [6] $\Psi(z)$ is a polynomial in Θ, i.e., in the Taylor expansion in

$$\Psi(z) = \Psi(x) + \Theta^{\alpha}\left(\frac{\partial\Psi}{\partial\Theta^{\alpha}}\right)_{\Theta=0} + \ldots + \frac{1}{(2r)!}\Theta^{\alpha_+}\ldots\Theta^{\alpha_{2r}}\left(\frac{\partial}{\partial\Theta^{\alpha_{2r}}}\ldots\frac{\partial}{\partial\Theta^{\alpha_+}}\Psi\right)_{\Theta=0} \tag{2}$$

the terms of order $k > 2r$ vanish. Here we use only the left derivative with respect to Grassmann variables.

The transformation properties of $\Psi(z)$ with respect to superconformal transformations are the following:

$$[P_A,\Psi(z)] = i\partial_A\Psi(z),$$

$$[M_{AB},\Psi(z)] = \{i(z_A\partial_B - (-1)^{\bar{A}\bar{B}} z_B\partial_A) + \Sigma_{AB}\}\Psi(z),$$

$$[D,\Psi(z)] = (iz^A\partial_A + \Delta)\Psi(z),$$

$$[K_A,\Psi(z)] = \{i[2z_A z^B\partial_B - z^2\partial_B + $$
$$+ 2iz^B(g_{AB}\Delta - \Sigma_{BA})] + k_A\}\Psi(z). \tag{3}$$

where Σ_{AB}, Δ and k_A are generators of the stability subgroup, i.e., the one which leaves $z=0$ /12/. We restrict ourselves to the case $k_A=0$ /12/. For the scalar fields $\Sigma_{AB}=0$ and $\Delta = \mathrm{id}$. For the generalized graded tensor fields we apply the formalism of homogeneous polynomials in superspace, i.e.,

$$\Psi(z,\xi) = \Psi^{A_1,\dots,A_\ell}(z)\xi_{A_1}\dots\xi_{A_\ell}, \tag{4}$$

where $\Psi^{\{A\}}$ is the graded symmetric traceless tensor, i.e.,

$$\Psi^{A_1,\dots,A_j,A_k,\dots,A_\ell} = (-1)^{\bar{A}_j\bar{A}_k}\,\Psi^{A_1,\dots,A_j,A_k,\dots,A_\ell},$$

$$g_{A_1A_2}\Psi^{A_1,A_2,\dots} = 0 \qquad \text{and} \quad \xi^2 = \xi^A\xi_A = 0.$$

In terms of the homogeneous variables ξ, , Σ_{AB} and Δ have the following form

$$\Sigma_{AB} = i(\xi_A\nabla_B - (-1)^{\bar{A}\bar{B}}\xi_B\nabla_A), \tag{5}$$

$$\Delta = i(d + \xi^A\nabla_A).$$

where $\nabla_A = \frac{\xi}{\partial\xi^A}$ and d is the scale dimension of the corresponding scalar field.

From (3) and (5) it follows that the Fermi components Θ of z have the same dimensions as the space-time components.

Consider the n-point function

$$F_n(z_1,\xi_1;\dots;z_n,\xi_n) = \langle 0|\Psi_1(z_1,\xi_1)\dots\Psi_n(z_n,\xi_n)|0\rangle. \tag{6}$$

Assuming that the vacuum is superconformal invariant, i.e.,

$$J_{ab}|0\rangle = 0$$

the invariance condition (in the infinitesinal form) for is given by the following system of equations:

$$\sum_{j=1}^{n} J^j_{ab}F_n(z_k,\xi_k) = 0, \tag{7}$$

where J^i_{ab} (see /12/) are superconformal generators acting on the field Ψ_j according to (3).

The solution of Eqs. (7) for $n=2$, i.e., the invariant two-point function F_2 is given by

$$F_2(z_1,\xi_1;z_2,\xi_2) = N^\ell_d(z^2_{12})^{-d}\,[\xi_1\xi_2 - \frac{(z_{12}\xi_1)(z_{12}\xi_2)}{z^2_{12}}]^\ell, \tag{8}$$

where $z_{12}=z_1-z_2$, $z^2=z^A z_A=z^A g_{AB} z^B$, $d=d_1=d_2$, $\ell=\ell_1-\ell_2$ and N_d^ℓ is a normalization constant. The form of (8) is analogous to the corresponding conformal invariant two-point function in the ordinary space-time given elsewhere [10].

Taking into account (4), from (8) we have

$$F_2^{\{A,B\}}(z)=N_d^\ell (z^2)^{-d} \underset{\{A\}\{B\}}{S} z^{A_1B_2}(z)\ldots r^{A_\ell B_\ell}(z) - \text{traces}, \tag{9}$$

where $r^{AB}(z)=g^{AB}-(-1)^{AB}\dfrac{z^A z^B}{z^2}$ is graded extension of the well known conformal tensor and $\underset{\{A\}\{B\}}{S}$ is the graded symmetrization operator, i.e. symmetrization with respect to Bose indices and antisymmetrization in Fermi ones.

Two-point function (8) is a polynomial in anticommuting variables. In the simplest case (scalar fields) from (8) (or (9)) we have

$$F_2(z)=N_d^0(z^2)^{-2}=N_d^0(x^2+\Theta C\Theta)^{-2}= N_d^0\{\frac{1}{(x^2)^d}-$$

$$-d\frac{\Theta C\Theta}{(x^2)^{d+1}}+\frac{d(d+1)}{2!}\frac{(\Theta C\Theta)^2}{(x^2)^{d+2}}+\ldots+(-1)^r\frac{d(d+1)\ldots(d+r-1)}{r!}\frac{(\Theta C\Theta)^r}{(x^2)^{d+r}}\}. \tag{10}$$

It may be checked that for

$$N_d^0=(2\pi)^{-h}(4)^d\frac{\Gamma(d)}{\Gamma(h-r-d)}$$

the two-point function (10) satisfies the following normalization condition [10]

$$\int (dz) F_2^d(z_1-z) F_2^{(2h-2r-d)}(z-z_2)=\delta(z_1-z_2), \tag{11}$$

where $F_2^{(2h-2r-d)}$ is the two-point function for the field with scale dimension $\tilde{d}=2(h-r)-d$. We shall prove that the fields with scale dimensions d and $\tilde{d}$ are transformed according to the equivalent representation of superconformal group. (In the case of $r=0$ this is well known). Indeed, the second Casimir operator of superconformal group is given by

$$C_{11}=\frac{1}{2}M^{AB}M_{BA}+K^AP_A-D^2+2i(h-r)D=$$

$$=\frac{1}{2}\Sigma^{AB}\Sigma_{BA}+k^AP_A-\Delta^2+2i(h-r)\Delta.$$

Consequently, for scalar fields we have

$$C_{11}\sim d(2h-2r-d).$$

In the case of scalar fields the invariant three-point function is given by

$$F_3(z_1,z_2,z_3)=$$

$$=C^{(d_j)}(z_{12})^{d_3-d_1-d_2}(z_{23}^2)^{d_1-d_2-d_3}(z_{31}^2)^{d_2-d_1-d_3}, \tag{12}$$

where $z_{jk}=z_j-z_k$ and $C^{\{d_j\}}$ is a normalization constant. For two scalar and one tensor field from (7) we have

$$F_3^{\ell}(z_1,z_2,z_3,\xi) =$$

$$= C_{\ell}^{\{d_j\}} (z_{12}^2)^{d_3-d_1-d_2} (z_{23}^2)^{d_1-d_2-d_3} (z_{31}^2)^{d_2-d_1-d_3} \times \tag{13}$$

$$\times \left(\frac{z_{13}\xi}{z_{13}^2} - \frac{z_{23}\xi}{z_{23}^2}\right)^{\ell} .$$

The transition of (13) to the index formalism can be performed by replacing the latter factor in (13) by the following (graded) symmetric traceless tensor

$$\left(\frac{z_{13}\xi}{z_{13}^2} - \frac{z_{23}\xi}{z_{23}^2}\right)^{\ell} \to \underset{\{A\}}{S} (\lambda_{12}^{3})^{A_1} \ldots (\lambda_{12}^{3})^{A_\ell} - \text{traces}, \tag{14}$$

where $(\lambda_{ij}^{k})^{A} = \dfrac{z_{jk}^{A}}{z_{jk}^{2}} - \dfrac{z_{jk}^{A}}{z_{jk}^{2}}$.

From (8), (12) and (13) it follows that the conformal invariant two- and three-point functions in flat superspace are formally identical in form with the corresponding functions in ordinary space /10/. In the superconformal theory considered in papers /5/ these functions have a more complicated form. The same concerns higher functions. As an example we consider the four-point function for scalar fields. In this case the solution of Eqs. (7) is given by

$$F_4(z_1,\ldots,z_4) = (z_{12}^2)^{-d_1} (z_{23}^2)^{\frac{1}{2}(d_1-d_2-d_3+d_4)} (z_{34}^2)^{\frac{1}{2}(d_2-d_1-d_3-d_4)} \times$$

$$\times (z_{24}^2)^{\frac{1}{2}(d_1-d_2+d_3-d_4)} f\left(\frac{z_{13}^2 z_{24}^2}{z_{12}^2 z_{34}^2}, \frac{z_{14}^2 z_{23}^2}{z_{12}^2 z_{34}^2}\right), \tag{15}$$

where f is an arbitrary function of two harmonic ratios.

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