ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E2 - 10374

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Submitted to "Physics Letters"

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E2 - 10374

Смешивание лептонов, распад $\mu \to e\gamma$ и осцилляции нейтрино

На основе калибровочной теории со смешиванием и в предположении о существовании тяжелых лептонов рассмотрен распад $\mu \to e\gamma$. Показано, что вероятность этого распада может быть достаточно большой (при массах тяжелых лептонов порядка нескольких ГэВ). Обсуждается связь между распадом $\mu \to e\gamma$ и осцилляциями нейтрино.

Работа выполнена в Лаборатории теоретической физики и Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований
Дубна 1977

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E2 - 10374

Lepton Mixing, $\mu \to e\gamma$ Decay and Neutrino Oscillations

The $\mu \to e\gamma$ decay is investigated in gauge theories with lepton mixing under the assumption that in nature there exist heavy leptons. It is shown that for lepton masses of the order of a few GeV the $\mu \to e\gamma$ decay probability may well be close to its experimentally determined upper limit. The relation between such a decay process and neutrino oscillations is briefly considered.

The investigation has been performed at the Laboratory of Theoretical Physics, and Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research
Dubna 1977

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1. Introduction

It is well known that the charged lepton current in the standard theory of weak interaction has the form:

$$j_\alpha = \bar{\nu}_{eL}\gamma_\alpha e_L + \bar{\nu}_{\mu L}\gamma_\alpha \mu_L, \qquad (1)$$

where ν_e and ν_μ are the operators of the electron and muon neutrino fields, $e_L = \frac{1}{2}(1+\gamma_5)e$, etc. In such a theory the decay $\mu \to e\gamma$ obviously is strictly forbidden.

In references /1,2/ the assumption was made that ν_e and ν_μ in the expression of the current are orthogonal combinations of the fields of two neutrinos ν_1, ν_2 with finite masses m_1, m_2:

$$\begin{aligned} \nu_e &= \nu_1\cos\theta + \nu_2\sin\theta, \\ \nu_\mu &= -\nu_1\sin\theta + \nu_2\cos\theta, \end{aligned} \qquad (2)$$

where θ is a mixing angle.

In such a theory new phenomena arise as neutrino oscillations /2/ and the processes $\mu \to e\gamma$, $\mu \to 3e$, etc., due to neutral asymmetrical lepton currents induced by a high order perturbation theory.

Calculations /3/ based on the above mentioned theory, however, show that the probabilities of the processes $\mu \to e\gamma$, $\mu \to 3e$, etc., are in magnitude by many orders smaller than the experimental upper limits. The ratio of the $\mu^+ \to e^+ + \gamma$ decay probability to the $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ decay probability, according to ref. /3/ is *

$$R_\mu = \frac{\Gamma(\mu^+ \to e^+ + \gamma)}{\Gamma(\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu)} = \frac{3}{32} \frac{\alpha}{\pi} \left(\frac{m_1^2 - m_2^2}{M_w^2}\right)^2 \sin^2\theta \cos^2\theta , \qquad (3)$$

where M_w is the mass of the intermediate W boson ($M_w \geq 37$ GeV).

If the mixing angle θ is close to $\pi/4$, a consideration of possible neutrino oscillations in experiments already performed allows one to conclude /2/ that

$$|m_1^2 - m_2^2| \lesssim 1 \ \mathrm{eV}^2 . \qquad (4)$$

With such a limit for $|m_1^2 - m_2^2|$ the upper limit of R_μ would be smaller than the experimentally established /4,5/ upper limit

$$R_\mu^{exp} < 2.2 \cdot 10^{-8} \qquad (5)$$

by several tens of orders of magnitude.

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*Let us note incidentally that some inaccuracies present in the preprint JINR P2-9595 (1976) were corrected in ref. /3/.

If the mixing angle is small ($\sin^2\theta \lesssim 10^{-2}$) limits on the neutrino mass difference cannot be obtained anymore by considering neutrino oscillations, and

$$m_1 < 35 \ \mathrm{eV} \qquad m_2 < 0.65 \ \mathrm{MeV} \qquad (6)$$

(35 eV and 0.65 MeV are the upper limits of the electron neutrino /6/ and muon neutrino /7/ masses, correspondingly). For such limits the upper limit of R_μ is much larger than in the case (4), but is still much smaller than the experimental limit (5). Assuming M_w = 60 GeV from (3) and (6) we get

$$R_\mu < 3 \cdot 10^{-26}$$

Thus the only method of testing neutrino mixing, if there are only two neutral leptons (i.e., two neutrinos), is to investigate neutrino oscillations*.

It can be shown that also in the case of mixing of n neutrinos of finite masses the ratio R_μ is smaller than the experimental upper limit by many orders of magnitude.

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*As is shown in ref. /8/, experiments designed to investigate neutrino oscillations at artifical neutrino facilities might permit one to test the neutrino mixing hypothesis if $|m_1^2 - m_2^2| \gtrsim 10^{-2} \ \mathrm{eV}^2$. If $|m_1^2 - m_2^2| \ll \ll 10^{-2} \mathrm{eV}^2$, such a hypothesis must be tested in solar neutrino experiments.

2. Heavy Neutral Leptons

The situation might radically change if in nature there exist heavy leptons* (hereafter we shall use the word neutrinos for neutral leptons with masses less than, say, the electron mass, and shall call heavy neutral leptons those with masses larger than the electron mass). Let us consider the $\mu \to e\gamma$ decay in such a case. For illustration we shall choose a concrete scheme. We assume that the lepton charged current is given by the experssion

$$j^{\ell}_{a} = j_{a} + j'_{a} : \qquad (7)$$

The first term is given by expressions (1) and (2), while the second is

$$j'_{a} = \bar{N}_{eR}\gamma_{a} e_{R} + \bar{N}_{\mu R}\gamma_{a}\mu_{R} , \qquad (8)$$

where

$$N_{e} = N_{1}\cos\theta' + N_{2}\sin\theta' ,$$

$$N_{\mu} = - N_{1}\sin\theta' + N_{2}\cos\theta' , \qquad (9)$$

$$e_{R} = \frac{1}{2}(1 - \gamma_{5})\, e , \text{ etc.}$$

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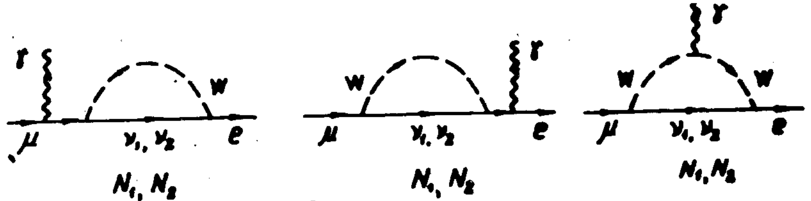
*It is known that experiments performed at colliding beam facilities give serious indications that a heavy charged lepton with mass ~1.8 GeV does exist [9].

In these expressions N_1 and N_2 are the field operators of the heavy leptons with masses M_1 and M_2, θ' is a mixing angle. Obviously $M_1, M_2 \geq M_K$, where M_K is the kaon mass. Thus we have assumed that the field operators of heavy neutral leptons are associated only with the right-handed current. Let us remark that such a current arises in schemes based on exceptional groups [10]. Let us remark also that the so called "high y-anomaly" [11] as well as the increase with energy [11] of the ratio of the antineutrino and neutrino cross sections $\frac{\sigma(\bar{\nu}_{\mu} N \to \mu^{+} X)}{\sigma(\nu_{\mu} N \to \mu^{-} X)}$ give some evidence for right-handed currents [12], although these effects might be interpreted in a different way [13]. The $SU_2 \times U_1$ gauge theory of the weak and electromagnetic interactions with the charged current given by expression (8), clearly implies that the neutral current of charged leptons is a vector current*.

The diagrams for the $\mu \to e\gamma$ decay are presented in the figure (unitary gauge)



Diagrams for the $\mu \to e\gamma$ process.

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*In the scheme which is considered here the right-handed charged hadron current has the form $J'_{a} = \bar{u}_{R}\gamma_{a} b'_{R} + \bar{c}_{R}\gamma_{a} r'_{R}$, where b' and r' are orthogonal combinations of the field operators of the heavy quarks b and r. Clearly, the corresponding neutral hadron current contains vector and axial components as well.

Neglecting the contribution from the diagrams with virtual neutrinos we get for the probability* of the $\mu \to e\gamma$ decay:

$$\Gamma(\mu \to e+\gamma) = \frac{G^2 m_\mu^5}{128\pi^3} \frac{\alpha}{16\pi} \left(\frac{M_1^2 - M_2^2}{M_w^2}\right)^2 \sin^2\theta' \cos^2\theta'. \quad (10)$$

The ratio of the $\mu^+ \to e^+ + \gamma$ to the $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ decay probabilities is

$$R_\mu = \frac{3}{32} \frac{\alpha}{\pi} \left(\frac{M_1^2 - M_2^2}{M_w^2}\right)^2 \sin^2\theta' \cos^2\theta'. \quad (11)$$

Of course R_μ has a maximum value for $\theta' = \pi/4$ and below we shall consider only this case. From (5) and (11) it follows that $\left(\frac{|M_1^2 - M_2^2|}{M_w^2}\right)^{1/2} < 1.4 \cdot 10^{-1}$. Taking $M_w = 60$ GeV we get $\sqrt{|M_1^2 - M_2^2|} < 8.5$ GeV and consequently $|M_1 - M_2| < 8.5$ GeV**.

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*Expression (10) is obtained under the assumption that $\frac{M_i^2}{M_w^2} \ll 1$ $(i = 1,2)$.

**Here the analogy between quarks and leptons is clearly apparent in the possibility of drawing conclusions on the upper limit of heavy lepton mass differences from the low rate of a process, - the $\mu \to e\gamma$ decay, in which induced asymmetrical lepton neutral currents are acting: this reminds us of the well known prediction that the charmed quark mass could not be much higher than a few GeV, obtained from the small amplitudes of processes induced by asymmetrical hadron neutral currents, such as $K^\circ \leftrightarrow \bar{K}^\circ$, $K^+ \to \pi^+ e^+ e^-$, etc.

Some values of R_μ for various values of $\sqrt{|M_1^2 - M_2^2|}$ are given in the table:

Table

Values of R_μ for different values of the parameter $\sqrt{|M_1^2 - M_2^2|}$ (($M_w = 60$ GeV))

$\sqrt{\lvert M_1^2 - M_2^2 \rvert}$ (GeV)	R_μ
1	$4.2 \cdot 10^{-12}$
2	$6.7 \cdot 10^{-11}$
3	$3.4 \cdot 10^{-10}$
4	$1.1 \cdot 10^{-9}$

Thus we conclude that, if lepton mixing occurs and neutral leptons with masses of few GeV do exist, the probability of the $\mu \to e\gamma$ decay might be close to its experimental upper limit /4,5/ and could be measured in a range of values ($R_\mu \geq 10^{-11}$), which can be investigated by subtle experimental methods as the facility ARES /14/.

We have considered above the case of two heavy neutral leptons. Now we shall give the expression for R_μ in a scheme /15/ with n charged leptons $(e, \mu, L_1, \ldots)$, n neutrinos and n neutral heavy leptons. The right-handed charged current in such a scheme is given by the expression

$$j''_\alpha = \bar{N}_{eR} \gamma_\alpha e_R + \bar{N}_{\mu R} \gamma_\alpha \mu_R + \bar{N}_{L_1 R} \gamma_\alpha L_{1R} + \ldots ,$$

where $N_\ell = \sum_{i=1}^{n} O_{\ell i} N_i$ ($\ell = e, \mu, L_1, \ldots$) and O is an orthogonal matrix, N_i is the field operator of the heavy neutral lepton with mass M_i. The ratio R_μ in this case is

$$R_\mu = \frac{3}{32}\frac{\alpha}{\pi}\left(\sum_{i=1}^{n} O_{ei}\, O_{\mu i}\, M_i^2/M_w^2\right)^2 .$$

3. Heavy Charged Leptons

In the case considered above the $\mu \to e\gamma$ decay is due to the charged lepton current, but one may consider a different mechanism for it. For example, this decay might be due to a neutral current, if in the weak interaction Hamiltonian there is a (neutral current) term inducing transitions $\mu \to L_i \to e$ (L_i is a heavy charged lepton) with the emission (or absorption) of a Z-boson or a Higgs boson.

4. Neutrino Oscillations and $\mu \to e + \gamma$ Decay

Below we discuss the relation between the hypothetical phenomena of neutrino oscillations and of the $\mu \to e\gamma$ decay. The observation of such phenomena would show that lepton mixing takes place indeed. In such a general sense the observation of either of these effects would make the existence of the other more likely; in particular, the neutrino masses probably would be finite. Apart from this general connection, it should be emphasized, however, that neutrino oscillations and

processes as the $\mu \to e\gamma$ decay, etc., might well be entirely unconnected.

First one can think of the case in which neutrino oscillations might be observable, but the process $\mu \to e\gamma$ is in fact unobservable: this is the situation discussed at the beginning of this note when there are only neutrinos, but not heavy neutral leptons.

Second, one can imagine a situation in which the $\mu \to e\gamma$ decay is perfectly observable (let us say that there are heavy neutral leptons of sufficiently high masses) but neutrino oscillations are unobservable, for example, because of a small mixing angle and/or a small difference in the neutrino masses*.

Third, one can imagine cases similar to the one presented in the previous paragraph, when the $\mu \to e\gamma$ decay is observable (heavy charged lepton and asymmetrical neutral current in the Hamiltonian) but the neutrino oscillations may be completely absent (not only unobservable in practice), because the process $\mu \to e + \gamma$ could have no relation whatever to the oscillations.

5. Conclusion

Let us remark here that among many possible schemes only one concrete was considered. The general conclusion is that lepton mixing and the existence of heavy leptons with masses of a few GeV would imply that the

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* An example, where neutrino oscillations would be difficult to observe in the presence of heavy leptons, was discussed in ref. /16/.

$\mu\to e\gamma$ decay may well be in the range of possible experimental observations. From our discussion it follows that both the process of neutrino oscillations and the $\mu\to e\gamma$ decay (as well as all the other processes connected with induced asymmetrical lepton neutral current) should be investigated experimentally even at an accuracy slightly better than the existing one.

We are grateful to S.M.Korenchenko and G.V.Micelmacher for valuable discussions.

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Received by Publishing Department
on January 7, 1977.