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LAGRANGIAN FIELD THEORY AND QUARK CONFINEMENT

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Лагранжева теория поля и удержание кварка

На языке лагранжевого формализма в рамках квантовой теории поля построено квантованное поле, которое удовлетворяет следующим условиям
(1) поле свободного кварка равно тождественно нулю,
(2) причинная функция этого поля нетривиальна.

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Lagrangian Field Theory and Quark Confinement

In the Lagrangian formalism of quantum field theory we have constructed a quantized field $q(x)$ which we call the virton field satisfying the conditions:
(1) the field of free virton quanta is equal identically to zero,
(2) the causal function of this virton field differs from zero.
We consider this virton field $q(x)$ can be a good candidate for description of quark confinement.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

The progress of the quark theory compels to think that quarks exist objectively, perhaps, in some very unusual form. The fact that they have not yet been discovered experimentally indicates that possibly quarks are such objects of the microworld which we did not yet meet. Therefore the standard methods of local quantum field theory are not, probably, applicable to describe quark fields.

During the last years there appear a lot of models which make attempts to explain the quark confinement. A good review of progress in this field has been made by H.Joos in /1/ where recent (until March 1976) scientific publications on this subject are listed.

The basic idea of these approaches consists in the assumption that quarks are the usual Dirac particles which interact with a "gluon" field. This gluon field creates such a potential between quarks which secures complete quark confinement. The models of bags suppose that inside a bag quarks are almost free but they cannot get out of the bag surface due to boundary conditions.

In this paper we propose another way of solving the quark confinement problem.

We introduce a new quantized field describing particles which do not exist at all as usual physical particles, like electron, proton and so on, and exist in the virtual state only. These nonexisting particles will be called "virtons" and the field $q(x)$ describing these particles the virton field.

This field can be constructed in the following way. The fact that the usual elementary particles are observed in experiment is expressed mathematically in the quantum field formalism that the fields of free particles satisfy the Dirac or Klein-Gordon equations.

We will suppose that virtons are pure quantum field objects of such a kind that the field of free virtons is equal to zero identically, i.e.,

$$q(x) \equiv 0. \tag{1.1}$$

We will use the Lagrangian formalism for describing of elementary particles. Then our hypothesis means that in the Lagrangian of the virton field

$$\mathcal{L}_0(x) = \bar{q}(x) Z(\hat{p}) q(x), \tag{1.2}$$

where $\hat{p} = i\hat{\partial} = i\gamma_\mu \frac{\partial}{\partial x_\mu}$, the operator $Z(\hat{p})$ should be chosen in such a way that the unique solution of the equation

$$Z(\hat{p}) q(x) = 0 \tag{1.3}$$

should be (1.1).

Thus we postulate that the virton field is described by the Lagrangian (1.2) and satisfies the equation (1.3) the solution of which in the classical case is zero (1.1).

On the other hand, we want the Green function of the field $q(x)$ obeying the equation

$$Z(\hat{p}) G(x-y) = i\delta(x-y) \tag{1.4}$$

to be nontrivial

$$G(x-y) = iZ^{-1}(\hat{p})\delta(x-y). \tag{1.5}$$

It means that the virton field which equals zero in the free state can exist nevertheless in the virtual state. If this virton field is connected with fields of usual physical particles, this will lead to the nontrivial interaction of these particles.

In the framework of the standard methods of local quantum field theory it is impossible to satisfy the equation (1.3) with solution (1.1) and the equation (1.4) with solution (1.5) at the same time. However, in the framework of the nonlocal field theory developed in [2] this problem can be solved.

The idea consists in the following. We want to construct a regularized quantum field $q^\delta(x)$ defined on a Fock space $\mathcal{H}$ which satisfies the imposed conditions

$$\lim_{\delta\to 0} q^\delta(x) = q(x) = 0,$$
$$\lim_{\delta\to 0} \langle 0|T(q^\delta(x)\bar{q}^\delta(y))|0\rangle = iZ^{-1}(\hat{p})\delta(x-y). \tag{1.6}$$

In this paper we give a solution of this problem.

2. EQUATION FOR THE VIRTON FIELD

In the Lagrangian formalism the noninteracting fermion field $q(x)$ is described by the Lagrangian density

$$\mathcal{L}_0(x) = \bar{q}(x)Z(\hat{p})q(x), \qquad (2.1)$$

where $Z(\hat{p})$ is an operator. For instance, for the Dirac and Klein-Gordon equations it has respectively the forms

$$Z(\hat{p}) = \hat{p} - m, \quad Z(\hat{p}) = p^2 - m^2.$$

By the variational principle the field $q(x)$ for Lagrangian (2.1) obeys the equation

$$Z(\hat{p})q(x) = 0. \qquad (2.2)$$

The equation for the free Green function $G(x-y)$ of the field $q(x)$ is written in the form

$$Z(\hat{p})G(x-y) = i\delta(x-y). \qquad (2.3)$$

For the Dirac or Klein-Gordon equation the solution and quantization of eq. (2.2) is the well studied problem.

Our first task is as follows: to find, within the standard Lagrangian formalism, classes of such operators $Z(\hat{p})$ defining equations which could pretend to desctibe the virtons in the framework of our hypothesis.

As stated above, we proceed from the assumption that the virtons do not exist as usual physical particles. This hypothesis can be realized in the following way. We suppose that the field describing the free virton field satisfying the equation (2.2) is identically equal to zero

$$q(x) \equiv 0. \qquad (2.4)$$

In other words, in Lagrangian (2.1) the operator $Z(\hat{p})$ should be chosen in such a way that the unique solution eq. (2.2) should be the zeroth solution (2.4).

Consider now that conditions on the form of $Z(\hat{p})$ follow from our requirements. First, from the requirement that the Lagrangian should be real and the action

$$S = \int dx \mathcal{L}_0(x)$$

should exist as a functional on rather smooth functions in the Minkowski and Euclidean spaces it follows that the function $Z(z)$ should be an entire analytic function of variable z and $[Z(z)]^* = Z(z^*)$.

Second, the requirement that eq. (2.2) possesses the unique solution (2.4) implies that the function $Z(z)$ has no zeros at any values of z.

The general form of entire functions of a finite order N satisfying these requirements is as follows

$$Z(z) = Ce^{P_N(z)}, \qquad (2.5)$$

where $P_N(z)$ is a polynomial of degree N with real coefficients, C is a constant.

We will use the methods of nonlocal quantum field theory. It means that the Green functions (1.5) should decrease in the Euclidean region. This requirement leads to the following condition on the function $Z(z)$:

$$Z(z) = O(e^{(-z^2)^{N/2}}) \qquad \text{when } z^2 \to -\infty. \qquad (2.6)$$

Further, if we introduce a principle of minimum in the sense, that we take the lowest degree of polynomial $P_N(z)$ in exponential function (2.6) which allows all the above requirements to be satisfied, then we obtain

$$Z(z) = -M\exp\{-\ell z - \frac{L^2}{4} z^2\}, \quad (2.7)$$

where M, ℓ and L are constants.

The constant M of the dimensionality of mass in this approach gives the scale of the virton field $q(x)$. It is not independent variable because no physical characteristics depend on it directly. In fact, this constant will enter only into definite combinations with the coupling constants of interactions of our virtons with other elementary particles we will consider below.

The constants ℓ and L are fundamental in our approach. They will define the dynamics of all possible virton interactions. The meaning of these constants will be considered in Section 5.

Thus our requirements permit us to find the operator $Z(\hat{p})$ with two independent parameters ℓ and L only and to avoid any functional arbitrariness.

Finally, the operator

$$Z(\hat{p}) = -M\exp\{-\ell\hat{p} - \frac{L^2}{4} \hat{p}^2\} \quad (2.8)$$

obeys the above conditions.

3. QUANTIZATION OF THE VIRTON FIELD $q(x)$

Our further problem is to quantize the system described by Lagrangian (2.1) with operator $Z(\hat{p})$. This task is rather peculiar as the corresponding classical solution of eq. (2.2) is identically equal to zero.

To solve the problem we use methods developed in the quantum field theory with nonlocal interaction [3]. The idea of our method of quantization is as follows: in Lagrangian (2.1) the operator $Z(\hat{p})$ is changed by a regularized operator $Z^{\delta}(\hat{p})$ such that, first, the function $Z^{\delta}(z)$ has an infinite number of zeros

$$Z^{\delta}(z) \sim \prod_{j=1}^{\infty} (1 - \frac{z}{M_j(\delta)}) \quad (3.1)$$

at points

$z_j = M_j(\delta) > 0 \qquad (j = 1,2,\dots)$

which in the limit of removing the regularization $(\delta \to 0)$

$$M_j(\delta) \to \infty \quad (3.2)$$

and, second,

$$\lim_{\delta\to 0} Z^{\delta}(z) = Z(z). \quad (3.3)$$

In our case this can be achieved, for example, in the following way:

$$Z^{-1}(z) = -\frac{1}{M}\exp\{\ell z + \frac{L^2}{4} z^2\} =$$

$$= -\frac{e^{-\frac{1}{2}\ell\mu}}{M} \sum_{n=0}^{\infty} \frac{(z+\mu)^{2n}}{n!} \cdot (\frac{L^2}{4})^n \to \quad (3.4)$$

$$\to [Z^{\delta}(z)]^{-1} =$$

$$= -\frac{e^{-\frac{1}{2}\ell\mu}}{M} \sum_{n=0}^{\infty} \frac{(\frac{L^2}{4})^n}{n!} \cdot \frac{(z+\mu)^{2n}}{\prod_{j=1}^{2n+n_0} (1 - \frac{\delta}{j^{\sigma}}(z+\mu)L)} =$$

$$= \sum_{j=1}^{\infty} \frac{(-1)^j A_j(\delta)}{M_j(\delta) - z}.$$

Here

$$M_j(\delta) = (\frac{j^\sigma}{\delta} - \mu L)\frac{1}{L} \quad (j = 1,2,\dots), \qquad (3.5)$$

$\mu = \frac{2\ell}{L^2}$, the parameter $\sigma < ½$ and $A_j(\delta)$ are positive coefficients which can be determined easily. A parameter n_0 defines the decreasing of the regularized function $[Z^\delta(z)]^{-1}$ in the complex z-plane:

$$[Z^\delta(z)]^{-1} = O(\frac{1}{|z|^{n_0}}) \quad \text{when } |z| \to \infty .$$

Let us introduce the system of fields

$$q_j^\delta(x) = \sqrt{A_j(\delta)}\,\frac{Z^\delta(\hat{p})}{\hat{p} - M_j(\delta)}\,q^\delta(x), \quad (j = 1,2,\dots), \qquad (3.6)$$

$$q^\delta(x) = \sum_{j=1}^{\infty} (-1)^j \sqrt{A_j(\delta)}\,q_j^\delta(x) \qquad (3.7)$$

and

$$\mathcal{L}_0(x) = \bar{q}(x)Z(\hat{p})q(x) \to$$

$$\to \mathcal{L}_0^\delta(x) = \bar{q}^\delta(x)Z^\delta(\hat{p})q^\delta(x) = \sum_{j=1}^{\infty}(-1)^j\,\bar{q}_j^\delta(x)(\hat{p} - M_j(\delta))q_j^\delta(x). \qquad (3.8)$$

These fields $q_j^\delta(x)$ correspond to the fictitious or "ghost" nonphysical quanta with mass $M_j(\delta)$ and have no real physical sense. They play an auxiliary role and should disappear in the limit $\delta \to 0$.

The solution of these equations can be written in the usual form

$$q_j^\delta(x) = \frac{1}{(2\pi)^{3/2}}\int d\vec{k}[v^\delta_{j\vec{k}}\,d_{j\vec{k}}\,e^{-ikx} + w^\delta_{j\vec{k}}\,h^+_{j\vec{k}}\,e^{ikx}], \qquad (3.9)$$

$$\bar{q}_j^\delta(x) = \frac{1}{(2\pi)^{3/2}}\int d\vec{k}[\bar{v}^\delta_{j\vec{k}}\,d^+_{j\vec{k}}\,e^{ikx} + \bar{w}^\delta_{j\vec{k}}\,h_{j\vec{k}}\,e^{-ikx}], \qquad (3.10)$$

where $v^\delta_{j\vec{k}}$ and $w^\delta_{j\vec{k}}$ are the Dirac spinors and

$$k_{j0} = E_{j\vec{k}}(\delta) = \sqrt{M_j^2(\delta) + \vec{k}^2}:$$

The Hamiltonian of this system of fields has the form

$$H_0^\delta = \sum_{j=1}^{\infty}(-1)^j\int d\vec{k}\,E_{j\vec{k}}(\delta)[d^+_{j\vec{k}}d_{j\vec{k}} - h_{j\vec{k}}h^+_{j\vec{k}}]. \qquad (3.11)$$

As the energy of our system must be positive the spinor fields $q_j^\delta(x)$ should be quantized according to the canonical procedure of quantization with the indefinite metrics:

$$\{d_{j\vec{k}}, d^+_{j'\vec{k}'}\}_+ = \{h_{j\vec{k}}, h^+_{j'\vec{k}'}\}_+ = (-1)^j\,\delta_{jj'}\,\delta(\vec{k} - \vec{k}').$$

The rest of anticommutators equals zero.

The space of states $\mathcal{H}$ containing all ghost particles is a vector space with the indefinite metrics. It consists of (1) a vacuum state $|0\rangle$, that is unique, defined by the conditions

$$d_{j\vec{k}}|0\rangle = h_{j\vec{k}}|0\rangle = 0$$

and normalized to $\langle 0|0\rangle = 1$, (2) one-particle and many-particle states which can be constructed with the help of the basic vectors

$$|jn, im\rangle = \frac{1}{\sqrt{n!m!}}\,d^+_{j_1\vec{k}_1}\cdot\ldots\cdot d^+_{j_n\vec{k}_n}\cdot h^+_{i_1\vec{p}_1}\cdot\ldots\cdot h^+_{i_m\vec{p}_m}|0\rangle.$$

All the one-particle, many-particle states and vacuum generate a complete system of eigenstates in the vector space $\mathcal{H}$. It is essential that vacuum $|0\rangle$ and operators $d_{j\vec{k}}$ and $h_{j\vec{k}}$ are independent of the parameter of regularization δ.

We define the space $\mathcal{H}(E)$ which consists of normalized physical states of this system with the energy nonexceeding an energy E:

$$\Psi(E)=\sum_{\{jn,im\}}\int d^n\vec{k}\int d^m\vec{p}\,\tilde{f}_{\{jn,im\}}(\vec{k},\vec{p})\theta(E-\Sigma E_{j\vec{k}}(\delta)-\Sigma E_{i\vec{p}}(\delta))|jn,im>,$$

where

$$d^n\vec{k}=d\vec{k}_1\cdot\ldots\cdot d\vec{k}_n,\quad \Sigma E_{j\vec{k}}(\delta)=\sum_{\nu=1}^{n}E_{j_\nu\vec{k}_\nu}(\delta),$$

$$\tilde{f}_{\{jn,im\}}(\vec{k},\vec{p})=\tilde{f}_{j_1\ldots j_n,i_1\ldots i_m}(\vec{k}_1,\ldots,\vec{k}_n;\vec{p}_1,\ldots,\vec{p}_m)\in\tilde{Z}_2.$$

Let us define the space of test functions $\tilde{Z}_2$. We say that the function of N variables $u_1,\ldots,u_N$ $\tilde{f}(u_1,\ldots,u_N)\in\tilde{Z}_2$ if $\tilde{f}(u_1,\ldots,u_N)$ is differentiable and

$$|\tilde{f}(u_1,\ldots,u_N)|\le C_\epsilon\exp\{-\frac{1}{\epsilon}\sum_{\nu=1}^{N}|u_\nu|^2\}$$

for any $\epsilon>0$ and a constant $C_\epsilon>0$.

The space Z_2 which is the space of Fourier transformations of functions $\tilde{f}\in\tilde{Z}_2$ consists of entire analytical functions $f(\zeta_1,\ldots,\zeta_N)$ for which there exists $C_\epsilon>0$ such that

$$|f(\zeta_1,\ldots,\zeta_N)|\le C\exp\{\epsilon\sum_{\nu=1}^{N}|\zeta_\nu|^2\}$$

for any $\epsilon>0$ and

$$\int_{-\infty}^{\infty}d\xi_1\ldots\int_{-\infty}^{\infty}d\xi_N|f(\xi_1+i\eta_1,\ldots,\xi_N+i\eta_N)|<\infty$$

for any $\eta_1,\ldots,\eta_N$.

Then for $\Psi(E)\in\mathcal{H}(E)$ we have

$$(\Psi(E),\Psi(E))=\sum_{\{jn,im\}}\int d^n\vec{k}\int d^m\vec{p}\,\theta(E-\Sigma E_{j\vec{k}}(\delta)-\Sigma E_{i\vec{p}}(\delta))\times$$

$$\times(-1)^{\Sigma j+\Sigma i}|\tilde{f}_{\{jn,im\}}(\vec{k},\vec{p})|^2<\infty.$$

We will consider the vector space

$$\mathcal{H}=\bigcup_E\mathcal{H}(E)$$

as the inductive limit of the spaces $\mathcal{H}(E)$ relative to the imbedding $\mathcal{H}(E)\to\mathcal{H}$.

Thus we have the field operators $q_j^\delta(x)$ and $q^\delta(x)$ and the vector space $\mathcal{H}$ where these operators act.

4. REMOVAL OF REGULARIZATION

What does happen with the field $q^\delta(x)$ and the Green functions of this field in the limit $\delta\to 0$? Physically it is clear that in this limit any physical states characterized by a definite value of energy cannot contain ghost quanta because the masses of all ghost quanta increase according to (3.5). In order to treat this problem mathematically we define the convergence on space $\mathcal{H}$ as the convergence on spaces $\mathcal{H}(E)$ for any fixed $E>0$. Then

$$\lim_{\delta\to 0}(\Psi_1(E),:q^\delta(x_1)\cdot\ldots\cdot q^\delta(x_n):\Psi_2(E))=0 \qquad (4.1)$$

for any $\Psi_1(E),\Psi_2(E)\in\mathcal{H}(E)$ and n, because for any fixed E there exists $\delta(E)$ such that for all $\delta<\delta(E)$

$$(\Psi_1(E),:q^\delta(x_1)\cdot\ldots\cdot q^\delta(x_n):\Psi_2(E))=0.$$

It means that according to our definition

$$q(x)=\lim_{\delta\to 0}q^\delta(x)=0. \qquad (4.2)$$

Thus the quantized field $q(x)$ of the free virton is equal to zero.

The Green functions in the limit $\delta\to 0$ are distributions defined on Z_2. Therefore we have to consider the improper limit

$$\lim_{\delta\to 0} \int dx\, G^{\delta}(x) f(x) = \lim_{\delta\to 0} \int dp\, \tilde{G}^{\delta}(p)\tilde{f}(p). \qquad (4.3)$$

Let us introduce the Green functions

$$G^{\delta}(x-y) = \{q^{\delta}(x), \bar{q}^{\delta}(y)\}_{+}, \qquad (4.4)$$

$$G^{\delta}_{(-)}(x-y) = \langle 0|q^{\delta}(x)\bar{q}^{\delta}(y)|0\rangle , \qquad (4.5)$$

$$G^{\delta}_{c}(x-y) = \langle 0|T(q^{\delta}(x)\bar{q}^{\delta}(y))|0\rangle . \qquad (4.6)$$

It is possible to show that

$$\lim_{\delta\to 0} \int dx G^{\delta}(x)f(x) = \lim_{\delta\to 0} \int dx\, G^{\delta}_{(-)}(x)f(x) = 0 . \qquad (4.7)$$

We will not perform here these calculations because they are the same as in /3/ for the scalar case.

Consider now the causal function $G^{\delta}_{c}(x)$. We have

$$G^{\delta}_{c}(x-y) = \langle 0|T(q^{\delta}(x)\bar{q}^{\delta}(y))|0\rangle =$$

$$= \sum_{j=1}^{\infty} A_j(\delta)\langle 0|T(q^{\delta}_{j}(x)\bar{q}^{\delta}_{j}(y))|0\rangle = \qquad (4.8)$$

$$= \frac{1}{(2\pi)^4 i} \int dp\, e^{-ip(x-y)} \sum_{j=1}^{\infty} \frac{(-1)^j A_j(\delta)}{M_j(\delta) - \hat{p} - i\epsilon} .$$

Then in the limit $\delta \to 0$ one can get

$$\lim_{\delta\to 0} \int dx G^{\delta}_{c}(x)f(x) = \lim_{\delta\to 0} \frac{1}{(2\pi)^4 i} \int dp \tilde{f}(p) \sum_{j=1}^{\infty} \frac{(-1)^j A_j(\delta)}{M_j(\delta) - \hat{p} - i\epsilon} =$$

$$= \frac{1}{(2\pi)^4 i} \int dp \tilde{f}(p) \frac{1}{M} \exp\{\ell \hat{p} + \frac{L^2}{4} p^2\} \qquad (4.9)$$

according to (4.4). This means that the causal function $G^{\delta}_{c}(x)$ changes in the limit $\delta \to 0$ into a non-local nontrivial propagator

$$\lim_{\delta\to 0} \tilde{G}^{\delta}_{c}(p) = \tilde{G}_{c}(p) = \frac{1}{M} \exp\{\ell \hat{p} + \frac{L^2}{4} p^2\} . \qquad (4.10)$$

Thus we satisfy the conditions (1.6) formulated above.

The limiting causal Green function $G_c(x)$ can be written in the form

$$G_c(x) = \frac{1}{(2\pi)^4 i} \int dp\, e^{-ipx} \frac{1}{M} \exp\{\ell \hat{p} + \frac{L^2}{4} p^2\} . \qquad (4.11)$$

This representation is formal in the Minkowski metrics and can be understood as a distribution defined on Z_2. But in the Euclidean metrics the causal function is a well-defined continuous function:

$$G_c(x_E) = \frac{1}{(2\pi)^4} \int dp_E\, e^{-ip_E x_E} \tilde{G}_c(p_E) , \qquad (4.12)$$

where

$$\tilde{G}_c(p_E) = \frac{1}{M} \exp\{-\ell \hat{p}_E - \frac{L^2}{4} p_E^2\} =$$

$$= \frac{1}{M} \{\cos \ell \sqrt{p_E^2} - \hat{p}_E \frac{\sin \ell \sqrt{p_E^2}}{\sqrt{p_E^2}}\} e^{-\frac{L^2}{4} p^2} \qquad (4.13)$$

In the Euclidean x-space the function $G_c(x_E)$ can be represented in the form

$$G_c(x_E) = A(x_E^2) + i\hat{n}_E B(x_E^2) , \qquad (4.14)$$

where

$$A(x_E^2) = \frac{1}{(2\pi)^4 M} \int dp_E \cos \ell \sqrt{p_E^2} \exp\{-\frac{L^2}{4} p_E^2 - ip_E x_E\} ,$$

$$B(x_E^2) = \frac{i}{(2\pi)^4 M} \int dp_E \frac{(p_E x_E)}{\sqrt{x^2}} \cdot \frac{\sin \ell \sqrt{p_E^2}}{\sqrt{p^2}} \exp\{-\frac{L^2}{4} p_E^2 - ip_E x_E\} \qquad (4.15)$$

$\gamma^E = (\gamma_4 = -i\gamma_0, \vec{\gamma})$ matrices in the Euclidean metrics so that

$$\gamma^E_\mu \gamma^E_\nu + \gamma^E_\nu \gamma^E_\mu = -2\delta_{\mu\nu}, \quad \hat{n}_E = \gamma^E_\mu n^E_\mu, \quad n^E_\mu = \frac{x^E_\mu}{\sqrt{x^2_E}}$$

and $\hat{n}^2_E = -1$.

It is just the representation we shall use in what follows.

Later on it will not be important for us to know the explicit form of regularized expressions. It is essential only that the regularization, first, does exist, second, provides the transition to the Euclidean metrics and, third, can be removed.

5. INTERACTION OF VIRTONS WITH OTHER FIELDS

There exist two different possibilities to consider the interactions of our virtons with fields of stable particles (mesons, baryons and so on). One way is the following. We do not need any gluon fields to "glue" virtons in our approach, therefore we can introduce the Lagrangian of virton field of the type

$$\mathcal{L}(x) = \bar{q}(x) Z(\hat{p}) q(x) + \lambda (\bar{q}(x)\Gamma q(x))(\bar{q}(x)\Gamma q(x)) \qquad (5.1)$$

and look for bound states of systems of virtons. Along this way usual stable particles should be bound states of virtons. This idea deserves an independent research and is not simple because the problem of finding the bound states in quantum field theory is not solved yet.

Another way consists in that the usual particles (mesons, baryons and so on) are considered as elementary particles and are

described by standard quantized fields satisfying a Klein-Gordon or Dirac equation. However, fields (for example, a meson field $\pi(x)$ or a baryon field $B(x)$) cannot interact with each other directly but through an intermediate virton field $q(x)$. For example, the interaction Lagrangian can be as follows

$$\mathcal{L}_I(x) = g\pi(x)(\bar{q}(x)\gamma_5 q(x)) + f[(\bar{B}(x)\gamma_\mu q(x))(\bar{q}^c(x)\gamma_\mu q(x)) + \text{h.c.}]. \qquad (5.2)$$

Just this second possibility will be considered further.

The S-matrix for the interaction Lagrangian (4.2) can be constructed in accordance with methods of nonlocal quantum field theory. Instead of $\mathcal{L}_I(x)$ in (5.2) let us introduce the regularized interaction Lagrangian $\mathcal{L}^\delta_I(x)$ dependent on the regularized field $q^\delta(x)$:

$$\mathcal{L}^\delta_I(x) = g\pi(\bar{q}^\delta \gamma_5 q^\delta) + f[(\bar{B}\gamma_\mu q^\delta)(\bar{q}^{\delta c}\gamma_\mu q^\delta) + \text{h.c.}]. \qquad (5.3)$$

The regularized S-matrix is defined in the usual way

$$S^\delta = T\exp\{i\int dx\, \mathcal{L}^\delta_I(x)\}. \qquad (5.4)$$

In papers /2/ it has been shown that in the limit $\delta \to 0$ there exist a finite unitary causal S-matrix in each perturbation order

$$S = \lim_{\delta\to 0} S^\delta, \qquad (5.5)$$

if the causal Green function is an entire analytical function and decreases in the Euclidean region. All details can be found in /2/.

Thus we can consider any interactions of mesons and baryons through an intermediate virton field $q(x)$. The essential point is that these virton cannot create because the field of free virton is equal to zero.

6. PHYSICAL MEANING OF PARAMETERS ℓ AND L

In this section let us consider the physical meaning of the constants ℓ and L in (2.8). For this aim we examine the potential corresponding to the causal Green function $G_c(x)$ (4.10) and (4.11) as usual when the Yukawa potential is deduced. We introduce an interaction between the field $q(x)$ and two fermion sources $\Psi_1(x)$ and $\Psi_2(x)$ of the type

$$\mathcal{L}_I(x) = g[(\bar{\Psi}_1(x)q(x)) + (\bar{\Psi}_2(x)q(x)) + h.c.] \qquad (6.1)$$

and calculate the energy of interaction between them due to the exchange of the quanta $q(x)$:

$$W = g^2 \iint dx_1\, dx_2\, [(\bar{\Psi}_1(x_1)G_c(x_1 - x_2)\Psi_2(x_2)) + h.c.]. \qquad (6.2)$$

Let us suppose that these sources are point-like, i.e.,

$$\Psi_j(x) = u\delta^{(3)}(\vec{x} - \vec{r}_j), \quad (j = 1,2), \qquad (6.3)$$

where u is the Dirac spinor describing the spin of our source at rest so that $\bar{u}u = 1$ and $\bar{u}\gamma_\mu u = 0$. Then we obtain

$$W(r) = g^2 \bar{u} G_c(\vec{r}) u = \\ = \mathrm{const}\left[(1 - \frac{\ell}{r})e^{-\frac{(r+\ell)^2}{L^2}} + (1 + \frac{\ell}{r})e^{-\frac{(r-\ell)^2}{L^2}}\right], \qquad (6.4)$$

where $r = |\vec{r}_1 - \vec{r}_2|$ and

$$G_c(\vec{r}) = \int d\vec{p}\, e^{i\vec{p}\vec{r}}\, \tilde{G}_c(\vec{p}),$$
$$\tilde{G}_c(\vec{p}) = \frac{1}{M}\left[\cos\ell\sqrt{\vec{p}^{\,2}} - \gamma\vec{p}\frac{\sin\ell\sqrt{\vec{p}^{\,2}}}{\sqrt{\vec{p}^{\,2}}}\right] e^{-\frac{L^2}{4}\vec{p}^{\,2}} . \qquad (6.5)$$

The potential $W(r)$ decreases with $r \to \infty$ as $\exp\{-\frac{r^2}{L^2}\}$ in contrast to the Yukawa potential $\frac{1}{r}\exp\{-mr\}$.

Let us define the average value $\langle \vec{r}^{\,2} \rangle$ of the distribution described by the function $W(r)$, then one can obtain

$$\langle \vec{r}^{\,2} \rangle = \frac{\int d\vec{r}\,\vec{r}^{\,2} W(r)}{\int d\vec{r}\, W(r)} = \frac{3}{2}(2\ell^2 + L^2). \qquad (6.6)$$

This means that the Green function $G_c(x)$ considerably decreases at distances of order $\sim\sqrt{2\ell^2 + L^2}$. Let us compare the behaviour of this Green function with that of the Green function of usual particles. We can see that the latter noticeably decreases at distances of the order of the particle Compton wave length $\lambda = \frac{1}{m}$ (the Yukawa potential).

Now we may suppose that the "mass" of our virton described by eq. (2.2) with operator (2.8) is defined by the expression

$$M_q^2 = \frac{6}{\langle \vec{r}^{\,2} \rangle} = \frac{4}{2\ell^2 + L^2}. \qquad (6.7)$$

It should be noted that it is not real mass of the virton because our virtons do not not exist as real particles.

7. CONCLUSION

Within the proposed method we satisfy the following condition:

(i) the equation for the virton classical field can be chosen under a reasonable assumption with two independent parameters only and there is no functional arbitraness;

(ii) the solution of the free virton equation is identically equal to zero;

(iii) the virton field $q(x)$ exists as the second-quantized regularized operator $q^{\delta}(x)$ in a Fock space $\mathcal{H}$ with indefinite metrics;

(iv) in the limit of removing the regularization the free virton field tends to zero $q^{\delta}(x) \to 0$;

(v) the causal Green function in that limit remains nontrivial;

(vi) the virton field can interact with other fields and give rise to a nontrivial interaction of the latter. Thus we consider that this virton field is a good candidate to be the quark field.

In a subsequent paper we consider the mass splitting of mesons and baryons within the broken $SU(3)$ symmetry in the framework of the proposed quark theory.

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