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QUANTUM FIELD THEORY  
AND PARTON MODEL

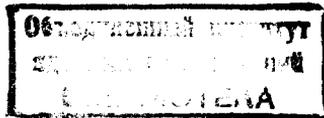
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**QUANTUM FIELD THEORY  
AND PARTON MODEL**

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Квантовая теория поля и партонная модель

Дан обзор работ, посвященных теоретико-полевому обоснованию партонной модели; в частности, рассмотрена связь между операторными разложениями и партонной моделью как в области  $Q^2 \gg M_p^2$ , так и в области  $Q^2 \sim M_p^2$  (области раннего скейлинга).

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Quantum Field Theory and Parton Model

The review is given of the development which have lead to the field theoretic derivation of parton model. In particular, the connection between the operator product expansions and the parton model is discussed both in the region  $Q^2 \gg M_p^2$  and in the region  $Q^2 \sim M_p^2$  (precocious scaling).

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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## Introduction

Theoretical attempts to explain the experimental data on deep inelastic scattering have lead to the parton description of hadrons. The basic ideas of the parton model as formulated by Feynman are the following /1/:

a) In the frame where the 3-momentum of hadron  $|\vec{P}|$  approaches infinity one can consider the hadron as an approximately parallel beam of "partons", each having a definite fraction  $x$  of the hadron momentum  $P$ , i.e. the hadron can be described by the parton distribution functions  $f^P(x)$  characterizing the number of partons with longitudinal momentum  $xP$ , so that

$$\sum_P \int f^P(x) dx = 1$$

, where the summation is taken over possible kinds of partons.

b) The essential assumption is that partons interact as if they were point-like particles, that is, the cross-sections of parton subprocesses do not depend on parameters with the length dimension.

This parton model deals with the limit where all the masses of hadrons and partons could be neglected, e.g. for deep inelastic scattering with the case  $-q^2 \equiv Q^2 \sim (Pq) \gg M_{\text{hadr}}^2, m_a^2$ , where  $P$  is the momentum of a target hadron,  $q$  is the momentum transfer to the hadron,  $m_a$  is the mass of a-type parton. It is usually supposed that  $M_{\text{hadr}}^2 \gg m_a^2$  for light quarks (partons).

Thus an approximate scaling /2/ in deep inelastic scattering is explained by the parton model as a phenomenon due to the asymptotical automodel behaviour at small distances: if only variables

$Q^2$  and  $(Pq)$  are significant, the dimensionless structure functions  $W(Q^2, Pq, M^2)$  of deep inelastic scattering should turn into the functions of single variable  $x = Q^2/2Pq$ .

Due to its attracting simplicity and also due to a qualitative (massive dilepton production, high  $p_T$  hadron inclusive production) and sometimes quantitative (deep inelastic lepto-production) description of high momentum transfer phenomena, the parton model has become widely popular. Our task here is to consider the theoretical status of the parton model, i.e. its validity from the view point of renormalizable quantum field theory (RQFT) rather than achievements and applications of the parton model. RQFT is now the only known theory satisfying all the basic postulates of relativistic elementary particle physics (causality, unitarity, Lorentz- and renormalization-invariance, etc.). We will also briefly review some new developments connected with an explanation of the precocious scaling.

#### 1. Early attempts of RQFT-derivation of the parton model.

An attempt to derive the parton model from RQFT was undertaken by Drell, Levy and Yan <sup>/3/</sup>. But to obtain the Bjorken scaling law they have to add the assumption that the transverse momentum of bare particles is limited. Essentially the same assumption is needed in the covariant parton model of Landshoff, Polkinghorne and Short <sup>/4/</sup> where it is required that the integrals over transverse momentum are convergent. This assumption is considered to be well justified by experiments on high-energy hadron-hadron collisions where the mean square transverse momentum of secondaries is of order  $(300 \text{ MeV})^2$ . But it is by no means justified in realistic renormalizable quantum field theory.

Chang and Fishbane <sup>/5/</sup> have shown in perturbation theory that in renormalizable theories the integration over transverse momentum leads to the logarithmic violation of scaling due to the terms  $\ln^n(-q^2)$  with  $n$  rising with an order of relevant Feynman diagram. A detailed summation of these logarithmic terms in the leading logarithmic approximation was performed by Gribov and Lipatov <sup>/6/</sup>, but the effective coupling constant for the theories they have considered increases with  $q^2$  in this approximation, hence, it is impossible to consider the limit  $q^2 \rightarrow \infty$ . The summation of all the logarithmic terms <sup>/7/</sup> assuming finite charge renormalization leads to a power violation of scaling depending on the value of bare coupling constant.

A lot of papers represent the tensor of deep inelastic scattering through the current commutator

$$W^{\mu\nu}(P, q) = \frac{1}{8\pi} \sum_{\sigma} \int e^{iqy} \langle P, \sigma | [J^{\mu}(y), J^{\nu}(0)] | P, \sigma \rangle. \quad (1)$$

It was first shown by Ioffe <sup>/8/</sup> that the Bjorken limit of this tensor  $Q^2, (Pq) \rightarrow \infty$ ,  $(Pq)/Q^2$  -fixed is determined by singularities of the commutator on the light cone  $y^2 = 0$ . Light cone expansions of a product of two operators have been investigated by Frishman <sup>/9/</sup> and Brandt and Preparata <sup>/10/</sup>. Fundamental investigations of automodel asymptotics in quantum field theory performed by Bogolubov, Vladimirov and Tavkhelidze <sup>/11/</sup> with the help of Dyson-Jost-Lehmann representation have provided a rigorous support for the light-cone analysis. It was shown that to obtain the automodel behaviour one has to add some assumptions concerning the behaviour of the spectral function of the DJL-representation, that

is concerning the singularity on the light cone. In particular, the parton model results were obtained by Fritzsche and Gell-Mann<sup>/12/</sup>, by Frishman<sup>/9/</sup> and by Llewellyn Smith<sup>/13/</sup>, who postulated that the leading singularity is that of the free field theory, i.e. the electromagnetic current  $J^\mu(y)$  is essentially constructed from free quark fields

$$J^\mu(y) \cong \sum_a \bar{\psi}_a(y) \gamma^\mu \psi_a(y) Q_a, \quad (2)$$

where  $Q_a$  is the electric charge of a-quark. This light cone approach is equivalent to the parton model for deep inelastic scattering<sup>/14/</sup>, but from the parton model there follow some predictions for the process  $pp \rightarrow \mu^+ \mu^- X$  (the Drell-Yan mechanism<sup>/15/</sup>) and for the large  $p_T$  inclusive reactions<sup>/16/</sup> whereas the light cone technique says little about these processes<sup>/17/</sup>.

The most interesting is the use of operator product expansions (OPE), first proposed by Wilson<sup>/18/</sup>

$$J^\mu(y) J^\nu(0) = \sum_{i,n} C_{(i),n}^{\mu\nu}(y) O_{\alpha_1 \dots \alpha_n}^{(i)}(0) \{y^{\alpha_1} \dots y^{\alpha_n}\}, \quad (3)$$

where  $C_{(i),n}^{\mu\nu}(y)$  are some singular as  $y^2 \rightarrow 0$  functions describing the behaviour of product  $J^\mu(y) J^\nu(0)$  and  $O_{\alpha_1 \dots \alpha_n}^{(i)}(z)$  are some local operators.

The validity of OPE from the view point of perturbation theory was investigated by many authors<sup>/19/</sup>. In these papers it was shown that the use of OPE is justified in perturbation theory. The paper by Anikin and Zavyalov<sup>/20/</sup> is also very interesting because they have treated the perturbation expansion as a whole.

Using Wilson expansions and assuming scale invariance, Polyakov has obtained a nontrivial sum rule<sup>/21/</sup>, analogous to that derived by Cornwall and Norton in the light cone approach<sup>/21a/</sup>:

$$\int_1^\infty \frac{d\omega}{\omega^{n+1}} W(\omega, q^2) = \sum_i \left(\frac{Q^2}{\lambda}\right)^{\gamma_n^i} M^i(n, \lambda, m^2). \quad (4)$$

The sum rule (4) expresses a very specific violation (of the same pattern as that given by diagram summation with finite charge renormalization assumed<sup>/17/</sup>) of the Bjorken scaling law.

The next important step was performed by Christ, Hasslacher and Müller<sup>/22/</sup>. They have investigated the functions  $C_{(i),n}^{\mu\nu}(y)$ , corresponding to the contribution from the parts of Feynman diagrams with highly virtual momenta, with the help of renormalization group<sup>/23/</sup> methods. We will discuss this in more detail later. By the use of RG they have easily obtained the results of direct summation of Feynman diagram asymptotical forms<sup>/5-7/</sup>. Very important result of the investigation<sup>/22/</sup> is the dependence of Fourier transform  $\tilde{C}_{(i),n}^{\mu\nu}(q^2, \lambda)$  on effective coupling constant<sup>/24/</sup> of RG. The deviation of  $\tilde{C}(q^2, \lambda, \bar{g})$  from its free field value would be small when  $\bar{g}^2(q^2)$  is small. The important fact is also that in the renormalizable theory one should introduce the dimensional parameter  $\Lambda$  of renormalization which remains even in the limiting case  $q^2 \gg M_{\text{had}}^2$ , thus giving the possibility of scaling violation.

The case of nonzero anomalous dimension  $\bar{g}^2(q^2) \rightarrow g_0^2 = \text{const} \neq 0$  as  $q^2 \rightarrow \infty$  is that considered by Efremov<sup>/7/</sup> and by Polyakov<sup>/21/</sup>. The closest to the free field theory are asymptotically free theories where  $\bar{g}^2(q^2) \rightarrow 1/\ln(q^2/\Lambda^2)$  at large  $q^2$ . It was shown by

Politzer and by Gross and Wilczek /25/ that this possibility is realized in nonabelian gauge theories. The predictions of these theories for deep inelastic electroproduction in the region  $Q^2 \gg \gg M^2$  have been investigated by Georgi and Politzer /26/, by Gross and Wilczek /27/ and by Bailin, Love and Nanopoulos /28/. Ahmed and Ross /29/ have considered, in this framework, the spin-dependent deep inelastic scattering. The authors of the present paper have investigated the connection between the sophisticated treatment based on operator product expansions on the light cone and the original "naive" parton model. Let us consider this connection /30/ in more detail.

## 2. Light-cone expansions and parton model at $Q^2 \gg M_{\text{had}}^2$ .

Operator product expansions have appeared to be the most effective tool to provide the field-theoretical basis for the parton model. The connection between the two approaches is based on the sum rules that have been first obtained from the OPE by Polyakov. The standard form for these rules /22/ is the following:

$$\tilde{W}_n(Q^2) \equiv \int_1^\infty \frac{d\omega}{\omega^{n+1}} W(\omega, q^2) = \sum_i \tilde{C}_{(i)}(Q^2/\mu^2, n) M^i(n, \mu^2), \quad (5)$$

where  $\tilde{C}_{(i)}(Q^2/\mu^2, n)$  are, roughly speaking, the Fourier transforms of the functions  $C_{(i),n}(y, \mu^2)$ , appeared in the OPE (3) and  $M^i(n, \mu^2, m^2)$  are the coefficients in the matrix elements of operators

$$\sum_\sigma \langle P, \sigma | O_{\alpha_1 \dots \alpha_n}^{(i)}(0) | P, \sigma \rangle = \{P_{\alpha_1} \dots P_{\alpha_n}\} M^i(n, \mu^2). \quad (6)$$

To obtain  $W(Q^2, Pq, m^2)$  one should perform the standard procedure of analytic continuation of eq. (5) into the complex  $n$ -plane. The problems connected with the Mellin transformation

$$W(\omega, q^2) = \frac{1}{2\pi i} \int_{n_0-i\infty}^{n_0+i\infty} W_n(Q^2) \omega^n dn \quad (7)$$

have been treated by Parisi /31/ and by Gross /32/. Defining the "distribution functions"  $F^i(x, \mu^2)$

$$\int_0^1 F^i(x, \mu^2) x^{n-1} dx = M^i(n, \mu^2) \quad (8)$$

or, in the equivalent form

$$F^i(x, \mu^2) = \frac{1}{2\pi i} \int_{n_0-i\infty}^{n_0+i\infty} M^i(n, \mu^2) x^{-n} dn \quad (9)$$

and using (5), (7) and (9) we obtain the representation for the structure functions  $W(\omega, q^2)$ :

$$W(\omega, q^2) = \int_0^1 \tilde{C}_i(Q^2/\mu^2, x\omega) F^i(x, m^2, \mu^2) \frac{dx}{x} \quad (10)$$

which has just the "parton type":  $F^i$  describes the splitting of the parton with fraction  $x$  of the total hadron momentum, and the function  $\tilde{C}_i(Q^2/\mu^2, x\omega)$  describes an interaction between the parton and a virtual photon with momentum  $q$ . To get the full correspondence with the parton model one has to "untangle" the expressions for  $\tilde{C}_i(Q^2/\mu^2, x\omega)$  and for  $F^i(x, m^2, \mu^2)$ : to single out the dependence on parton charges explicitly, to separate the contributions from particles and antiparticles, to clarify

the meaning of parameter  $\mu^2$  and to determine under what conditions one can neglect the dependence of the functions on  $Q^2$ .

For this purpose let us turn back to the operator product expansion on the light cone (3). For definiteness we consider the standard gauge theory of **strong interactions**.

All singularities of the product  $J(y)J(0)$  are concentrated in functions  $C_{(i),n}(y^2, \mu^2)$  and  $O_{\alpha_1 \dots \alpha_n}^{(i)}(z)$  are operators of the following type

$$O_{\alpha_1 \dots \alpha_n}^{(i)}(z) \sim \int \bar{\psi}_a(z) \gamma_{\alpha_1} \overleftrightarrow{D}_{\alpha_2} \dots \overleftrightarrow{D}_{\alpha_n} \lambda_{ab}^i \psi_b(z), \quad (11)$$

where  $S$  denotes the symmetrization over  $\alpha_1 \dots \alpha_n$ . The notation  $C_{(i),n}(y^2, \mu^2)$  emphasizes the fact that the singularities of the functions  $C(y^2)$  are not in general canonical, hence one must introduce the parameter  $1/\mu$  with the dimension of length serving as a unit for measuring distances, i.e. the quantity  $1/\mu$  can be considered as a boundary between small and large distances. It is reasonable to choose  $\mu \gtrsim M_{\text{hadr}}$ , but the particular choice of parameter  $\mu$  within this region is arbitrary. The fact that  $\langle P | J(y) J(0) | P \rangle$  does not depend on  $\mu$  implies that  $\langle P | O^i | P \rangle$  do depend on this parameter:  $\mu$  is the renormalization parameter for these operators. This can be easily understood by noting that when calculating the matrix elements of operators with a sufficiently large number of derivatives ( $n \geq 3$ ), the divergences can be removed only with the help of counterterms of the new type which are not present in the original Lagrangian, i.e. the divergences from these matrix elements cannot be eliminated by the ordinary R-operation <sup>[24]</sup>. One should add also the receipt

of  $O_{\alpha_1 \dots \alpha_n}^{(i)}$  - renormalization characterized by the new parameter  $\mu$ .

In the standard gauge theory of quark interactions there are three types of terms in the right-hand side of the Wilson expansion:

$$\begin{aligned} & C^V(y^2, \mu^2, n) S \{ \text{Tr} F_{\alpha_1 \alpha_2} \overleftrightarrow{D}_{\alpha_3} \dots \overleftrightarrow{D}_{\alpha_{n-1}} F_{\alpha_n}^\alpha \}, \\ & \sum_a C^{F,0}(y^2, \mu^2, n) S \{ \psi_a \gamma_{\alpha_1} \overleftrightarrow{D}_{\alpha_2} \dots \overleftrightarrow{D}_{\alpha_n} \psi_a \}, \\ & \sum_{a,b} C^{F,i}(y^2, \mu^2, n) S \{ \bar{\psi}_a \gamma_{\alpha_1} \overleftrightarrow{D}_{\alpha_2} \dots \overleftrightarrow{D}_{\alpha_n} \frac{\lambda_{ab}^i}{2} \psi_b \}, \end{aligned} \quad (12)$$

where  $a, b$  denote the quark flavor. It is convenient to introduce the following functions

$$\begin{aligned} \sum_{\downarrow} C^{F,i}(y^2, \mu^2, n) \lambda_{ab}^i / 2 &= (Q_a^2 - Q_b^2) \delta_{ab} E^{NS}(y^2, \mu^2, n) \\ C^{F,0}(y^2, \mu^2, n) &= \langle Q^2 \rangle E^S(y^2, \mu^2, n) \\ C^V(y^2, \mu^2, n) &= \langle Q^2 \rangle E^g(y^2, \mu^2, n) \end{aligned} \quad (13)$$

to take into account quark charges explicitly.  $E^S$  and  $E^{NS}$  mean singlet and nonsinglet quark contributions. Matrix elements of corresponding operators can be represented in the form

$$\begin{aligned} \frac{i^{n-1}}{4} \sum_{\sigma} \langle P, \sigma | S \{ \bar{\psi}_a \gamma_{\alpha_1} \overleftrightarrow{D}_{\alpha_2} \dots \overleftrightarrow{D}_{\alpha_n} \psi_a \} | P, \sigma \rangle &= \\ = \{ P_{\alpha_1} \dots P_{\alpha_n} \} ( \tilde{f}^a(n, \mu^2) + (-1)^n \tilde{f}^{\bar{a}}(n, \mu^2) ) \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{i^n}{4} \sum_{\sigma} \langle P, \sigma | S \{ \text{Tr} F_{\alpha_1 \alpha_2} \overleftrightarrow{D}_{\alpha_3} \dots \overleftrightarrow{D}_{\alpha_{n-1}} F_{\alpha_n}^\alpha \} | P, \sigma \rangle &= \\ = \{ P_{\alpha_1} \dots P_{\alpha_n} \} \frac{1 + (-1)^n}{2} \tilde{f}^g(n, \mu^2). \end{aligned} \quad (15)$$

Defining Fourier transforms of functions  $E(y^2, \mu^2, n)$  as usual <sup>[27]</sup> one can obtain for  $\mathcal{E} = -\mathcal{M}^2$

$$\begin{aligned}
W(\omega, q^2) &= 2 \operatorname{Im} \sum_{n=0}^{\infty} \frac{\omega^{n+} (-\omega)^n}{4\pi} \{ \langle Q^2 \rangle [E^g(Q^2/\mu^2, n) \tilde{f}^g(n, \mu^2) \\
&+ E^s(Q^2/\mu^2, n) \sum_a \{ \tilde{f}^a(n, \mu^2) + \tilde{f}^{\bar{a}}(n, \mu^2) \}] + \\
&+ E^{NS}(Q^2/\mu^2, n) \sum_a (Q_a^2 - \langle Q^2 \rangle) \cdot \\
&\cdot \{ \tilde{f}^a(n, \mu^2) + \tilde{f}^{\bar{a}}(n, \mu^2) \} \}. \quad (16)
\end{aligned}$$

Note that the sum runs over  $n$  even. The imaginary part ( $\frac{1}{2} W(\omega) = \operatorname{Im} T(\omega) \equiv [T(\omega+i\epsilon) - T(\omega-i\epsilon)]/2i$ ) can be easily obtained with the help of the Sommerfeld-Watson transformation

$$\begin{aligned}
W &= 2 \operatorname{Im} T = \operatorname{Im} \sum_{n=0}^{\infty} a_n \frac{\omega^{n+} (-\omega)^n}{2\pi} = \\
&= \operatorname{Im} \frac{1}{2\pi i} \int_{n_0-i\infty}^{n_0+i\infty} dn \frac{\pi}{\sin \pi n} a_n \frac{\omega^{n+} (-\omega)^n}{2\pi} = \int_{n_0-i\infty}^{n_0+i\infty} \omega^n a_n \frac{dn}{2\pi i}. \quad (17)
\end{aligned}$$

To derive the consequences from the OPE the dispersion relation for  $T(\omega, q^2)$  is usually used. As a result, the moments of  $W(\omega, q^2)$  are proportional to the matrix elements of operators

$$\tilde{W}_n(Q^2) \equiv \int_1^{\infty} \frac{d\omega}{\omega^{n+1}} W(\omega, q^2) = a_n; \quad n \text{ even.} \quad (18)$$

For  $n$  odd the moments are not proportional to the matrix elements of operators, since  $\langle P|O|P \rangle \sim f^a(n, \mu^2) - f^{\bar{a}}(n, \mu^2)$  whereas  $W(n, q^2) \sim f^a(n, \mu^2) + f^{\bar{a}}(n, \mu^2)$  for  $n$  odd also. This reflects the positivity property of  $W(\omega, q^2)$ .

All the singularities of  $a_n$  are on the left from the line of integration in (17).

Now we can introduce the parton distribution functions  $f^p(x, \mu^2)$ :

$$f^p(x, \mu^2) = \frac{1}{2\pi i} \int_{n_0-i\infty}^{n_0+i\infty} \tilde{f}^p(n, \mu^2) x^{-n} dn, \quad (19)$$

where  $p = a, \bar{a}, g$ . The continuation into the complex  $n$ -plane is unique due to the Karlson theorem [33] often used in the complex angular momentum technique. The absence of singularities on the right from the line of integration leads to the property  $f^p(x, \mu^2) = 0$  when  $x \geq 1$ . The inverse Mellin transformation gives the formula

$$\tilde{f}^p(n, \mu^2) = \int_0^1 \frac{dx}{x} f^p(x, \mu^2) x^n. \quad (20)$$

Formulae of this type were used for parton distribution functions also by Parisi and Petronzio [34] and by Georgi and Politzer [35].

With the help of eq.(20) the function  $W(\omega, q^2)$  can be written in the "parton" form

$$\begin{aligned}
W(\omega, q^2) &= \int_{1/\omega}^1 \frac{dx}{x} \{ E^{NS}(Q^2/\mu^2, x\omega) \sum_{p=a, \bar{a}, g} (Q_p^2 - \langle Q^2 \rangle) f^p(x, \mu^2) \\
&+ \langle Q^2 \rangle E^s(Q^2/\mu^2, x\omega) \sum_{p=a, \bar{a}} f^p(x, \mu^2) + \\
&+ \langle Q^2 \rangle E^g(Q^2/\mu^2, x\omega) f^g(x, \mu^2) \} \quad (21a)
\end{aligned}$$

which is in full correspondence with the hard scattering formula (see, e.g. [36] dictated by the parton model:  $f^p(x, \mu^2)$  describes the splitting of the parton with momentum  $xP$  from the hadron with momentum  $P$  and the functions  $E(Q^2/\mu^2, x\omega)$  describe the interaction between the parton and a virtual photon. The Born approximation for  $E$  is  $E_B^S = E_B^{NS} = \delta(1-x\omega)$ ,  $E_B^g = 0$  and leads to the well-known parton formula

$$W(\omega, q^2) = \sum_a Q_a^2 (f^a(1/\omega) + f^{\bar{a}}(1/\omega)). \quad (21b)$$

The functions  $f^p(x, \mu^2)$  satisfy the normalization conditions of the parton model<sup>1/1</sup> due to the conservation of corresponding operators: operator  $S\{i\bar{\psi}\gamma_{\alpha_1}\overleftrightarrow{D}_{\alpha_2}\psi - \text{Tr}F_{\alpha_1}F_{\alpha_2}\}$  is the energy-momentum tensor, consequently

$$\sum_a \{ \tilde{f}^a(2, \mu^2) + \tilde{f}^{\bar{a}}(2, \mu^2) \} + \tilde{f}^g(2, \mu^2) = 1 \quad (22)$$

or  $\int_0^1 x dx [f^g(x, \mu^2) + \sum_a \{f^a(x, \mu^2) + f^{\bar{a}}(x, \mu^2)\}] = 1$

for any choice of  $\mu^2$ .

Operator  $\bar{\psi}_a \gamma_\mu \psi_b$  corresponds to the vector current and its conservation leads to the sum rules

$$\int_0^1 dx \sum_a \{ f^a(x, \mu^2) - f^{\bar{a}}(x, \mu^2) \} c_a = c_N, \quad (23)$$

where  $c_a$  is some quantum number, conserved in strong interactions, of an a-parton (its electric charge, strangeness, the 3rd component of isospin) and  $c_N$  is that of a hadron.

For the spin dependent deep inelastic scattering there appear new operators

$$S\{ \bar{\psi} \gamma_\sigma \gamma_5 \overleftrightarrow{D}_{\alpha_1} \dots \overleftrightarrow{D}_{\alpha_n} \psi \}, \quad (24)$$

where  $S$  denotes the symmetrization over  $\sigma_{\alpha_1} \dots \alpha_n$ , and corresponding matrix elements

$$\begin{aligned} & \frac{i^{n-1}}{4} S \langle P, s | \bar{\psi} \gamma_5 \gamma_\sigma \overleftrightarrow{D}_{\alpha_1} \dots \overleftrightarrow{D}_{\alpha_n} \psi | P, s \rangle = \\ & = \sum_w \{ \tilde{f}^a(\omega, s; n, \mu^2) + (-1)^n \tilde{f}^{\bar{a}}(\omega, s; n, \mu^2) \} S[w_\sigma P_{\alpha_1} \dots P_{\alpha_n}], \end{aligned}$$

where  $w_\sigma = \bar{u}(p, w) \gamma_\sigma \gamma_5 u(p, w) / m$  is the vector of parton polarization (with the normalization  $\bar{u}(p, w) u(p, w') = 2m \delta_{ww'}$  assumed) and the sum runs over two possible states of polarization, i.e. the distribution functions appear in the combination

$$\tilde{f}^a(\omega, s; n, \mu^2) - \tilde{f}^a(-\omega, s; n, \mu^2) \equiv \tilde{h}^a(\omega, s; n, \mu^2). \quad (26)$$

There are also twist 3 operators<sup>29,37/</sup>

$$S' \{ \bar{\psi} \gamma_\lambda \gamma_5 \overleftrightarrow{D}_\sigma \overleftrightarrow{D}_{\alpha_1} \dots \overleftrightarrow{D}_{\alpha_n} \psi \},$$

where  $S'$  denotes antisymmetrization over  $\lambda\sigma$  and symmetrization over  $\sigma_{\alpha_1} \dots \alpha_n$ . One must introduce the new distribution function  $\tilde{d}(\omega, s; n, \mu^2)$

$$\begin{aligned} & \langle P, s | S' \{ \bar{\psi}_a \gamma_\lambda \gamma_5 \overleftrightarrow{D}_\sigma \overleftrightarrow{D}_{\alpha_1} \dots \overleftrightarrow{D}_{\alpha_n} \psi_a \} | P, s \rangle = \\ & = \frac{1}{2} \sum_w (\omega_\lambda P_\sigma - \omega_\sigma P_\lambda) P_{\alpha_1} \dots P_{\alpha_n} \tilde{d}^a(\omega, s; n, \mu^2). \end{aligned} \quad (27)$$

For a free particle  $\tilde{f}(n+1, \dots) = \tilde{d}(n, \dots)$ . The difference  $\tilde{f}(n+1, \dots) - \tilde{d}(n, \dots)$  nevertheless exists in general case, and unlike the longitudinal form factor  $W_L$  (which is also zero in free theory and  $W_L = O(\bar{g}^2(Q^2)) + O(M^2/Q^2)$  in general case), there is no smallness in form factor  $g_2(\omega, q^2)$ :  $g_2 = O(1)$  even in the case of asymptotically free theory because the properties of distribution functions are determined by large distance dynamics (note, that  $W_L$  does not require new distribution functions).

The functions  $E(q^2/\mu^2, \omega)$  can be calculated in perturbation theory in the region where  $\bar{g}^2(Q^2)$  is small. The most effec -

tive there are renormalization group methods based in the case considered here on the independence of  $\tilde{W}(n, q^2)$  both of  $\mu^2$  and of the renormalization parameter  $\lambda$  of the ordinary R-operation. The equations are simplified by equating  $\lambda$  and  $\mu^2$ , and are simply the equality  $dW/d\ln(\mu)$  expressed in terms of explicit dependences

$$\left\{ -\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right\} E^{(i)}(n, \mu^2, g) = \gamma_{ji}^{(i)}(n, g) E^{(j)}(n, \mu^2, g), \quad (28)$$

where  $\beta(g) = -\mu \frac{\partial}{\partial \mu} g$ , and  $\mu \frac{\partial}{\partial \mu} \langle P | O_n^i | P \rangle = \gamma_{ij}^{(i)}(g, n) \langle P | O_n^j | P \rangle$ . Anomalous dimensions  $\gamma_{ij}^{(i)}(g^2, n)$  form a matrix allowing for mixing between  $E^S$  and  $E^E$ . The solution to the RG equation (28) is

$$E_i(q^2/\mu^2, n, \mu^2, g(\mu^2)) = \sum_j E_j(1, n, Q^2, \bar{g}(Q^2)) \times \left\{ T \exp \left[ \frac{1}{2} \int_{\mu^2}^{Q^2} \hat{\gamma}(\bar{g}(t), n) \frac{dt}{t} \right] \right\}_{ij}, \quad (29)$$

where  $T$  means the exponential to be  $t$ -ordered.

For detailed higher order calculations it is more suitable to use the  $\mu$ -scheme of renormalization<sup>/38/</sup> for the function, e.g.,  $E^S(q^2/\mu^2, n, \mu^2, g(\mu^2))$ :

$$E^S(q^2/\mu^2, n, \mu^2, 0) = 1 \quad (30a)$$

rather than the  $\lambda$ -scheme, where

$$E^S(1, n, \mu^2, g(\mu^2)) = 1. \quad (30b)$$

It is possible, however, to use the  $\lambda$ -scheme and moreover to attribute all the  $q^2$ -dependence to the parton distribution functions taking  $\mu^2 = q^2$ . It seems impossible for the time being to calculate  $f^P(x, q^2)$  from the theory because it is the strong coupling problem, but if one knows  $f^P(x, q_0^2)$  for some value  $q_0^2$  such

that  $\bar{g}^2(q_0^2)/4\pi^2 \ll 1$ , then one can calculate  $f^P(x, Q^2)$  for higher  $Q^2$ . The essence of the parton model is the possibility to factorize the large distance contribution  $f^P(x, Q^2)$  from the short distance one  $E(q^2/\mu^2, \omega)$  which in the case of weak coupling at large  $Q^2$  can be calculated in the leading logarithmic approximation. In the paper by Lipatov<sup>/39/</sup> parton distribution functions were also calculated in the leading logarithmic approximation which is equivalent here to setting  $f(n, 0) = 1$ , or  $f(x, 0) = \delta(1-x)$ . Although the investigation<sup>/39/</sup> is, of course, very valuable heuristically, it is the weak coupling result and it hardly can be considered as a rigorous support for the parton model in the real case of strong coupling at large distances.

It is necessary to note that we have no need to consider the parton distribution functions  $F(x, k_t)$  over transverse momentum  $k_t$ . It is usually supposed that  $F(x, k_t)$  are very fast vanishing functions of  $k_t$ , so that  $\langle k_t^2 \rangle = \int k_t^2 F(x, k_t) d^2 k_t$  exists and moreover  $\langle k_t^2 \rangle \simeq (300 \text{ MeV})^2$ , i.e. the mean square momentum is determined by the hadron size. One may expect  $F_{ld}(x, k_t) \sim \exp(-k_t^2/\langle k_t^2 \rangle)$  for small  $k_t^2 \ll M_p^2$ , i.e., in the region of strong coupling (large distance contribution). But for  $k_t^2 \gtrsim M_p^2$  a short distance contribution  $F_{sd}(x, k_t) \simeq \bar{g}^2(k_t^2)/k_t^2$  becomes essential, which leads to the divergences in the integrals over  $k_t$  (cf.<sup>/3,39/</sup>). It is evident that  $\int k_t^2 F_{sd}(x, k_t) d^2 k_t \rightarrow \infty$  both in the asymptotically free ( $\bar{g}^2(k_t^2) \simeq 1/\ln(k_t^2/\Lambda^2)$ ) and in the scale invariant ( $\bar{g}^2(k_t^2) \rightarrow g_0^2 \neq 0$ ) theories. The contribution from small distance parton interactions is small numerically at  $k_t^2 = M_p^2$  as compared to  $F_{ld}$  because of  $\bar{g}^2(M_p^2)/4\pi^2 \ll 1$ .

One can, consequently, consider  $f(x, Q^2)$  as the sum

$$f(x, Q^2) = f_{ed}(x) + f_{sd}(x, Q^2)$$

$$f_{ed}(x) = \int F_{ed}(x, k_t) d^2k_t; \quad f_{sd}(x, Q^2) = \int F_{sd}(x, k_t) d^2k_t \cdot \theta(Q^2(1/x-1) - k_t^2) \theta(k_t^2 - M_p^2) \quad (31)$$

where  $\theta(Q^2(1/x-1) - k_t^2)$  comes from spectrality properties of E-functions. The singular behaviour of  $F_{sd}(x, k_t)$  at large  $k_t^2$  leads to the logarithmic corrections to the Bjorken scaling law, or, in another language, to the dependence of parton distribution function on the new parameter - wave length of virtual photon which probes the hadron structure, just as it was predicted by Kogut and Susskind /40/ in their "scale invariant parton model." In the region of small effective coupling the function  $f(x, Q^2)$  slowly varies with  $Q^2$  and this evolution (the second term in the sum) can be exactly calculated in perturbation theory whereas  $f_{ed}(x)$  must be taken from experimental data.

The behaviour of the function  $F_{sd}(x, k_t)$  with account of the fact that the integrals over  $k_t$  are cut off at the value of an order  $Q^2$  (due to spectrality properties of E-functions) leads to the value of  $\langle k_t^2 \rangle$  rising as  $Q^2$ . In this connection the hypothesis of Levin and Ryskyn /41/ that  $\langle k_t \rangle \sim 1 - 2$  GeV seems to be temporary.

Thus we have seen that the parton model has a strong quantum field theoretical support: it can be derived from operator product expansions, which are valid at least in perturbation theory. We can summarize: in the region  $Q^2 \gg M_{\text{had}}^2$  but  $\bar{g}^2(Q^2) \ln(Q^2/M^2) / 4\pi^2 \ll 1$  where the Born approximation for the functions  $E(Q^2/M^2, \omega)$  is justified, RQFT leads to the standard parton picture with Bjorken scaling and parton distribution functions independent of  $k_t$ .

Beyond this region new subprocesses become essential which lead to scaling violation. The character of the violation depends on the character of charge renormalization, hence, there immediately arises the question about experimental investigation of charge renormalization. Very sensitive here seems to be the ratio  $R(x, Q^2) \equiv \frac{\sigma_L}{\sigma_T}$  of longitudinal to transverse form factors of deep inelastic scattering. In the region  $Q^2 \gg M^2$  and  $x$  fixed, the value of  $R$  is proportional to the effective coupling constant  $\bar{g}^2$  which is logarithmically vanishing for asymptotically free theories and tends to a constant value for scale invariant theories. The accuracy of existing experimental data, however, does not allow the discrimination between the two possibilities.

### 3. A new approach to the precocious scaling

Up to here we have dealt with the case  $Q^2 \gg M^2$ , thus leaving the question about the precocious scaling. There, indeed, follow no suggestions in favour of this property from our preceding discussions. But an approximate scaling is observed even at  $Q^2 \sim M_p^2$  when  $x$ -variable is slightly modified by nonasymptotic terms. These problems were carefully investigated by Georgi and Politzer /35/. This paper, in our view, opens a new period in the study of deep inelastic phenomena. Starting from OPE and asymptotic freedom they have got a parton-like description in the region  $\bar{g}^2(Q^2) / 4\pi^2 \ll 1$ , i.e. even for  $Q^2 \sim M^2$ . Georgi and Politzer aimed not to neglect masses in their analysis. Parton distribution functions are determined in ref. /35/ by matrix elements of ope-

rators with definite spin, i.e. symmetric and traceless

$$\sum_{\sigma} \langle P, \sigma | O_{\alpha_1 \dots \alpha_n}^{(P)} | P, \sigma \rangle = \pi_{\alpha_1 \dots \alpha_n} \times \{ \tilde{f}^P(n, \mu^2) + (-1)^n \tilde{f}^{\bar{P}}(n, \mu^2) \}, \quad (32)$$

where  $\pi_{\alpha_1 \dots \alpha_n} = \{ P_{\alpha_1 \dots \alpha_n} - \text{traces} \}$  and  $g^{\alpha_i \alpha_j} \pi_{\alpha_1 \dots \alpha_n} = 0$ .

One can construct  $\pi_{\alpha_1 \dots \alpha_n}$  in the explicit form for any number  $n$ . It is necessary to reexpand OPE in terms of traceless operators using the equations of motion for quark fields. The weakness of coupling constant at small distances justifies, in opinion of authors of ref. /35/, the use of free field equations of motion  $(i \delta + m_q) \psi_q = 0$  as the zero order approximation.

The result in the case  $\bar{g}^2 (m_p^2) / 4 \pi^2 \ll 1$ ,  $m_q = 0$  is /35/

$$-W_M^M = \frac{x F(\xi)}{\sqrt{1 + 4x^2 m_p^2 / Q^2}} \quad (33)$$

$$2x W_L = 4 \left\{ \frac{m_p^2}{Q^2} \frac{x^3}{1 + 4x^2 m_p^2 / Q^2} \int_{\xi}^1 d\xi' F(\xi') + \frac{2m_p^4}{Q^4} \frac{x^4}{(1 + 4x^2 m_p^2 / Q^2)^2} \int_{\xi}^1 d\xi' \int_{\xi'}^1 d\xi'' F(\xi'') \right\}, \quad (34)$$

where  $F(\xi) = \sum_a Q_a^2 (f_a(\xi) + f_{\bar{a}}(\xi))$  and  $\xi = \frac{2x}{1 + \sqrt{1 + 4x^2 m_p^2 / Q^2}}$  is the new scaling variable. One can obtain the  $\xi$ -variable in the old parton model from delta-function  $\delta((\xi p + q)^2)$  if one does not neglect  $\xi^2 m_p^2$  term, ( $\xi$  was used by Greenberg and Bhau-mik /43/ and by other authors /44/). Reexpansion in traceless operators is essentially  $O(4)$  analysis performed by Nachtmann /45/ who also has obtained the  $\xi$ -variable.  $O(4)$ -analysis was also used

by Baluni and Eichten /46/ and by Ninomiya and Watanabe /47/.

In the case  $m_I, m_P \neq 0$ , where  $m_I, m_P$  are the masses of struck and produced quarks respectively, the variable  $\xi$  is /35/

$$\xi = \frac{(Q')^2}{(Pq) [1 + \sqrt{1 + 4x^2 m_p^2 / Q^2}]} \quad (35)$$

where  $2(Q')^2 = Q^2 + m_F^2 - m_I^2 + \sqrt{Q^4 + 2Q^2(m_F^2 + m_I^2) + (m_F^2 - m_I^2)^2}$ .

It is shown by Frampton /48/ that in this case also the  $\xi$ -variable has a simple parton interpretation:  $\xi = P_I^+ / P_P^+$  where  $P_I = (P_I^0, P_I^t, P_I^3)$ ,  $P_P = (P^0, 0, 0, P^3)$  are the momenta of the struck quark and the target proton, respectively,  $P^+ = P^0 + P^3$  is the light cone variable, and  $P_I^2 = m_I^2$ ,  $P_P^2 = (P_I + q)^2 = m_P^2$ ,  $P_P^t = 0$ . Thus, the approximation needed for  $\xi$ -scaling is clear: struck and produced quarks are on their mass shells; the transverse momentum of the struck parton is negligible; partons are non-interacting particles. The assumption  $P_t^2 = 0$  really was not used in ref. /35/ (cf. our discussion at the end of part 2). The corrections  $\langle P_T^2 \rangle_{1d} / Q^2$  due to the contribution from operators  $\bar{\psi} \sigma_{\alpha\beta} F^{\alpha\beta} \bar{D}_{\alpha_1} \dots \bar{D}_{\alpha_n} \psi$  have been taken into account in the paper by De Rujula, Georgi and Politzer /49/. These contributions are shown to be connected with the resonance bumps in preasymptotical region of  $Q^2$ . This is an explanation of the Bloom-Gilman local duality /50/. The first order logarithmic corrections to the  $\xi$ -scaling were also calculated in ref. /49/. Theoretical predictions for  $N(\omega, Q^2)$  are in good agreement with experimental data from ref. /42/. It is necessary to emphasize that the only free parameter in the theory /49/ is  $\bar{g}^2(m_p^2)$ . But the

predicted value of  $R = \sigma_L / \sigma_T$  is much more smaller than that given by experiment (fig.1). The experimental data are very inaccurate, indeed, and the authors of ref./49/ hope that more precise measurements will show the coincidence between theory and experiment.

Some assumptions leading to  $\xi$ -scaling<sup>/35/</sup> have been criticized by Barbieri, J.Ellis, Gaillard and Ross<sup>/51/</sup>, who have also derived  $\xi$ -variable from the light cone approach and parton model, and by R.Ellis, Parisi and Petronzio<sup>/52/</sup>, who have derived the formulae (33),(34) in the covariant parton model. A considerable part of the criticism was anticipated in ref./49/ where the constructive solutions of some problems are given. But the whole situation with  $\xi$ -scaling is not quite clear. In our opinion, the assumption that the struck quark is on its mass shell is not obvious.

#### 4. Parton model and inelastic hadron-hadron processes

Parton model<sup>/1/</sup> pretends not only to the description of deep inelastic lepton-hadron processes, but also to the description of massive dilepton production (the Drell-Yan mechanism<sup>/15/</sup>) and large  $p_t$  lepton and hadron production in high energy hadron-hadron collisions (the BBK-mechanism<sup>/16,36/</sup>) where it is hard to use the Wilson expansion formalism. Is it possible to invalidate the parton picture in these cases? The positive answer to this question can be obtained in the approach based on the investigation of asymptotical forms of Feynman diagrams (see, e.g. the review <sup>/53/</sup> and references therein and refs.<sup>/54,55/</sup>). The modi-

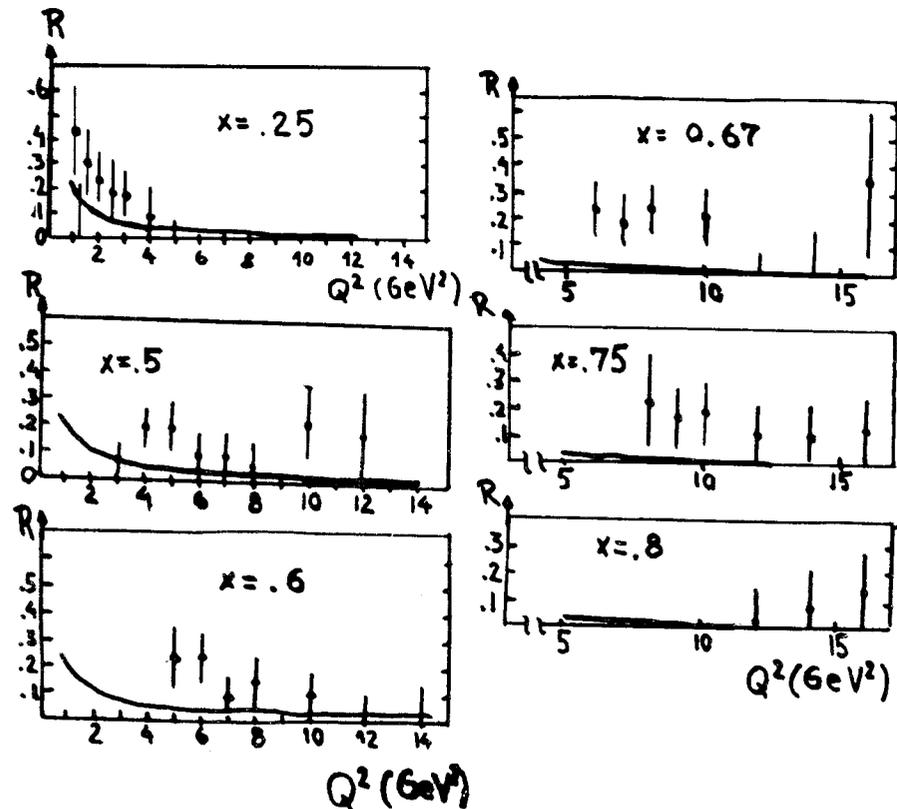


Fig.1. Comparison between the theoretical predictions for  $R = \sigma_L / \sigma_T$  (ref.49) and experimental data (ref.42). Figures taken from ref.49.

fied parton model described in part 2 was first obtained just with the use of these methods<sup>/56/</sup>. The use of Wilson expansions has given the same result<sup>/30/</sup>. An attempt to modify the parton model in order to allow a scaling violation was undertaken by Polkinghorne<sup>/57/</sup>. The starting point of his investigation was the covariant parton model rather than model independent (and more rigorous) renormalizable quantum field theory. Therefore the renormalization problems concerning the normalization parameter  $\mu^2$  and effective coupling constant have not been discussed in ref.<sup>/57/</sup>.

The essence of the diagrammatic approach based on the consideration of asymptotical forms of Feynman diagrams is as follows:

A) Take the well-known  $\alpha$ -representation of a diagram (see, e.g. ref.<sup>/23/</sup>):

$$T(p_1, \dots, p_n) = H \int_0^{i\infty} \frac{\prod d\alpha_\sigma}{D^2(\alpha)} G(\alpha, p) e^{\left\{ \frac{Q(\alpha, p)}{D(\alpha)} - \sum_{\sigma} (m_{\sigma}^2 - i\epsilon) \right\}}, \quad (36)$$

where the functions D, G and Q are determined by the diagram topology, and H is the product of coupling constants.

B) Consider the Mellin transform with respect to large variables

$$t_1, \dots, t_k \gg s_1, \dots, s_l$$

$$\Phi(j_1, \dots, j_k; s_1, \dots, s_l) = H \sum_{n_i} \int_0^{i\infty} \frac{\prod d\alpha_\sigma}{D^2(\alpha)} g^{n_1 \dots n_k}(\alpha, s) \prod_i \Gamma(n_i - j_i) \cdot \left( \frac{A_i(\alpha)}{D(\alpha)} \right)^{j_i} \exp\{I(\alpha, s, m^2)\}, \quad (37)$$

where  $A_i(\alpha)$  is the coefficient corresponding to  $t_i$ -variable in the function  $Q(\alpha, p)$  and  $I(\alpha, s, m^2)$  is the remaining part

of the exponent depending only on small variables. The asymptotical behaviour in  $t_i$  is determined by the rightmost singularity in the complex  $j_i$ -plane.

C) The singularities in  $j_i$  can appear only from integration over the region where the coefficient  $A_i(\alpha)$  vanishes. In the euclidean region  $A_i(\alpha) \gg 0$  and the possibility  $A_i(\alpha) = 0$  can be realized only when  $\alpha_\sigma = 0$  for the lines  $\sigma$  of subgraph V.

Because  $\alpha_\sigma = 0$  topologically means the contraction of the corresponding line  $\sigma$  into the point, the subgraph V should possess the property that the contraction of V into the point "kills" the dependence of the diagram on large variables  $t_i$ . The examples of such subgraphs are shown in fig.2 for the processes of deep inelastic scattering, massive dilepton production, and large  $p_T$  hadron production in high energy hadronic collisions.

D) The integration over  $\alpha_\sigma$  ( $\sigma \in V$ ) is divided into two parts:

$$\left| \sum_{\sigma \in V} \alpha_\sigma \right| < 1/\mu^2 \quad (\text{scale regime of V-subgraph}) \text{ and}$$

$$\left| \sum_{\sigma \in V} \alpha_\sigma \right| > 1/\mu^2 \quad (\text{nonscale regime of V-subgraph}).$$

The scale regime corresponds to small distances between the coordinates of all the vertices in V.

E) In many cases one can choose the variables  $t_i$  in such a way that

$$\Phi(j, s) = \Psi_V(j, \mu^2) \prod_m \chi_m(j, s, \mu^2, m^2) + R_V(j, s) \quad (38)$$

where  $\Psi_V$  is the contribution from the scale regime of V-subgraph and  $\chi_m$  are the contributions from weak-coupled parts of the diagram that would appear when V-subgraph is contracted into the point, and  $R_V$  is the nonscale contribution. The factorization (38) plays the same role as Wilson expansions do in the case of

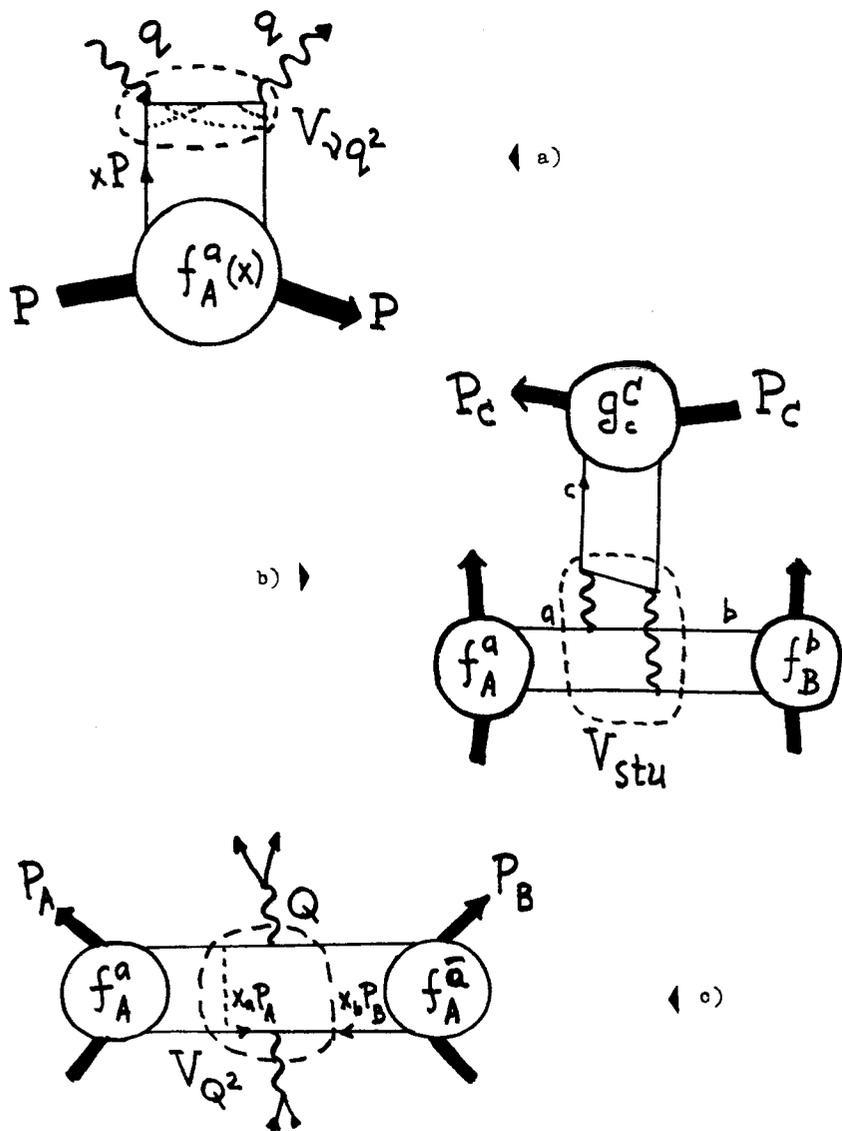


Fig. 2

deep inelastic scattering, but it has a much more wider application.

F) The scale contribution of V-subgraph is the sum of poles

$$\Psi_V = (1/\mu^2)^{j_1 + \dots + j_n - j_0} \sum_{k=0}^N C_k (j_1 + \dots + j_n - j_0)^{-k-1} \quad (39)$$

(i.e. it gives the sum  $t^{j_0} \sum C_k (\ln(t/\mu^2))^k$ ) where  $j_0$  is scale dimension of V-subgraph. For a theory with dimensionless coupling constant  $j_0$  is determined by the number of external lines

$$j_0 = \frac{1}{2} (4 - \ell_V). \quad (40)$$

Hence it is clear that for leading singularities the subgraphs with minimal number of external lines (t-subgraphs) are responsible.

Note, that eq.(38) is valid for any t-subgraph. Summing over all the possible t-subgraphs and over all the diagrams one obtains that the expression (38) is true for the whole amplitude

$$\Phi(j, s) = \Psi(j, \mu^2) \prod_m \chi_m(j, s, m^2, \mu^2) + R(j, s, m^2), \quad (41)$$

where the first term accumulates all the leading singularities of all the diagrams (that is, all the logarithms for senior power  $j_0$ ). For deep inelastic scattering this leads to the expression (fig.2a)

$$\Phi_{eA \rightarrow eX}(j_s, j_{Q^2}, m_A^2) = (1/\mu^2)^{j_s + j_{Q^2}} \cdot \sum_a \Psi_{eA \rightarrow eX}(j_s + j_{Q^2}, j_s) \tilde{f}_A^a(j_s, \mu^2, m_p^2). \quad (42)$$

For the process  $A + B \rightarrow C + X$  where C-particle has a large transverse momentum (fig.2b) the similar representation is valid:

$$\Phi_{AB \rightarrow CX}(j_s, j_t, j_u, m^2) = \left(\frac{1}{\mu^2}\right)^{j_s + j_t + j_u + 2} \times$$

$$\sum_{a,b,c} \Psi_{ab \rightarrow cx}(j_s, j_t, j_u) \tilde{f}_A^a(j_s + j_t, m_A^2, \mu^2) \times$$

$$\tilde{f}_B^b(j_s + j_u, m_B^2, \mu^2) \tilde{g}_C^c(j_t + j_u, m_C^2, \mu^2). \quad (43)$$

The use of momentum variables immediately gives hard scattering formulas of the parton model /16,36/

$$W(\omega, q^2) = \int_0^1 \sum_a E_a(Q^2/\mu^2, x, \omega) f_A^a(x, \mu^2, m_A^2)$$

$$\varepsilon_c \frac{d^3\sigma}{d^3p_c} = \int_0^1 \frac{dx_a dx_b dy_c}{x_a x_b y_c} \left\{ \varepsilon_c \frac{d^3\sigma}{d^3p_c} \right\} f_A^a(x_a, \mu^2) \times$$

$$\times f_B^b(x_b, \mu^2) g_C^c(y_c, \mu^2) \quad \left. \begin{array}{l} s' = x_a x_b s; \quad t' = (x_a/y_c)t \\ u' = (x_b/y_c)u. \end{array} \right\} \quad (44)$$

One can easily show that the functions  $f_A^a(x, \mu^2)$  possess all the properties of parton distribution functions /53/ and the functions  $E$  and  $(d^3\sigma/d^3p_c)_{\mu^2}$  describe small distance parton subprocesses. To calculate these functions, one can use the independence of observables both of the splitting parameter  $\mu$  and of the renormalization point  $\lambda$  of ordinary R-operation /23/, that is, use the renormalization group equation (cf. with the eq.(28)):

$$\left\{ -2 \sum_i j_i + 2j_0 - \beta(g) \frac{\partial}{\partial g} + \sum_n \gamma^T(j, n, g) \right\} \Psi(j, g) = 0. \quad (45)$$

Taking  $\mu^2 = p_T^2 = ut/s$  one can get the expression

$$\left\{ \frac{d\sigma}{dt'} \right\}_{ab \rightarrow cd} \sim \frac{1}{p_T^4} \bar{g}^4(p_T^2) \quad (46)$$

for the parton subprocess contributing to the production of particle C with high  $p_T$ . The similar expression for massive dilepton production is

$$\left( \frac{d\sigma}{dt'} \right)_{ab \rightarrow c\gamma^*} \sim \frac{1}{Q^4} \bar{g}^2(Q^2) \quad (47)$$

which differs from the Drell-Yan prediction. This is due to the fact that the Drell-Yan mechanism in this approach corresponds to the so-called pinch singularity. The mechanism given by eq. (47), unlike the Drell-Yan one, does not contradict the experimentally observed dependence of  $(Q_t)_{\mu^+ \mu^-}$  on  $Q^2$ .

Because of the small value of  $\bar{g}^2(m_p)$  we believe /56/ that the **scaling** behaviour  $(d\sigma/dQ^2) \cdot Q^4 \sim \text{const.}$  sets in later than the scaling law in deep inelastic scattering, but earlier than for large  $p_T$  hadron production. It is possible that the observed deviation from scaling powers can be connected with the power-law variation of effective coupling constant  $\bar{g}^2(p_T^2)$ . At very high  $p_T$  (or  $m_{\mu^+ \mu^-}$ ) values we expect the violation of scaling powers of the same type as it was observed in deep inelastic scattering.

Electromagnetic form factors of composite hadrons are, nevertheless, of zero order in effective coupling constant, e.g.  $F_\pi(Q^2) = \text{const}/Q^2$  where  $\text{const} = 0$  (1) /55/.

## 5. Conclusion

Thus, we have seen that the use of parton ideas is justified in the renormalizable quantum field theory with small effective coupling constant in the region  $\bar{g}^2(Q^2)\ln(Q^2/M^2) \ll 1$ . Beyond this region some modification of the parton model is necessary: one should take into account parton interactions, or, in other words, the dependence of parton distribution function on  $Q^2$ .

We emphasize that one of the basic assumptions of the diagrammatic approach is the consideration of momenta in the euclidean region. In the noneuclidean region there appears a new source of singularities in the complex  $j$ -plane connected with the large distance interaction, the so-called "pinch mechanism", the Drell-Yan mechanism being the first example of such a singularity mechanisms predicting the distribution over transverse momentum of the dilepton pair  $Q_t$  independent of dilepton mass  $Q^2$ ; Landshoff mechanism in large angle elastic scattering being the second one, it predicts the behaviour  $(d\sigma/dt)_{pp \rightarrow pp} \sim 1/t^8$  instead of quark counting power<sup>/58/</sup>  $t^{-10}$ . In the diffractive region pinch singularities contribute only to the positive signature and, hence, they can, in principle, violate the signature degeneracy of Regge trajectories. **None** of these possible effects have yet been observed; the pinch mechanism seems to be suppressed for an unknown reason, just because nobody knows how to sum up these singularities.

An application of the diagrammatic approach to the quark field model gives in the region  $\bar{g}^2(t)\ln(-t/M^2) \ll 1$  quark counting rules for deep elastic scattering processes.

In conclusion we should state with regret that the most mysterious is the problem of quark confinement. To assert that the parton model can be derived from RQFT under the condition that only ordinary hadrons are in the final state, one must prove that the sum of contributions of diagrams fig. 3b, where the "wee" partons<sup>/1/</sup>  $a, b$  compose a bound state  $A$ , is equal to the contribution of diagram fig. 3a, at least at large  $Q^2$ . These problems have been investigated in particular by Preparata<sup>/59/</sup> who has proposed a new approach to high energy phenomena. He has obtained, nevertheless, all the ordinary results (like scaling, etc.). We believe that the equality expressed by fig. 3 is justifiable in field theory, i.e. the summation over the hadrons can be performed by the summation over the partons, but the rigorous proof of this statement is to be given.

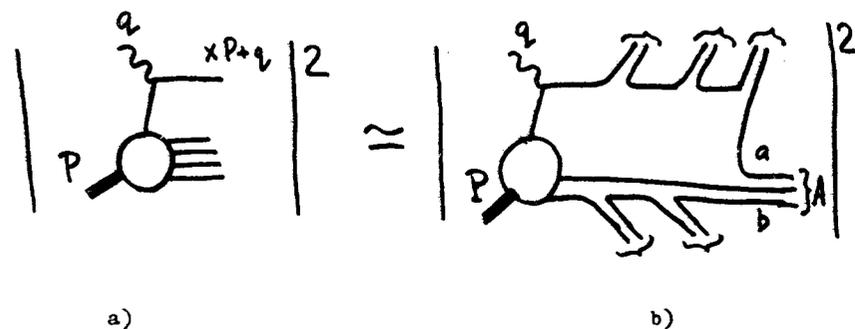


Fig.3

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