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G.M.Sotkov, R.P.Zaikov

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G.M.Sotkov,* R.P.Zaikov

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* Higher Pedagogical Institute, Shumen, Bulgaria.

Сотков Г.М., Зайков Р.П. E2 - 10250

Конформно-инвариантные двух- и трехточечные функции для полей произвольного спина

Найдены конформно-инвариантные двух- и трехточечные функции для "фундаментальных" полей произвольного спина и с произвольной масштабной размерностью в x-пространстве Минковского. Двухточечные функции для дираковских полей и для симметричных и антисимметричных тензорных полей выписаны явно.

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Conformal Invariant Two- and Three-Point Functions for Fields with Arbitrary Spin

The conformal invariant two- and three-point functions for any "fundamental" fields with an arbitrary spin and scale dimensions are found in the Minkowsky x-space. There the two-point functions for Dirac, symmetric and antisymmetric tensor fields are given. The three-point functions for two Dirac fields and one symmetrical tensor field, as well as any other field for which this function is nonvanishing, are given. In the case of conserved currents the Ward identities are considered.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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I. Introduction

Conformal invariant (CI) two- and three-point functions were used in conformal invariant partial wave expansion [5] and in the operator product expansion [2]. The conformal invariant two-point function for scalar and Dirac fields was given in papers [6,7] in the Minkowsky x-space and for fundamental fields with any spin in the momentum space in [10]. The CI three-point function for three scalar, two Dirac and one scalar fields was given in [6] and [7] for arbitrary symmetric tensor fields in papers [3] and [1].

In the present paper we give the CI two-point function for arbitrary "fundamental" fields in the Minkowsky x-space and any nonvanishing three-point functions. For this purpose the formalism of homogeneous functions of the complex spinor $z=(z_1, z_2)$ [8,10], is used.

In section II the general form is given of the CI two-point functions for any "fundamental" fields in the Minkowsky x-space, as corresponding function of those found in momentum space in [10]. The two-point functions for Dirac, symmetric and antisymmetric tensor fields are given explicitly.

The general form of the three-point function of arbitrary "fundamental" fields is given in section III. The explicit form of the three-point function for three scalar, two Dirac spinor and one symmetric tensor fields and for any other possible fields for which the three-point function is nonvanishing are given. The case of the three-point function for tensor fields is considered.

In section IV the case of conserved current and corresponding Ward identities is considered.

Our considerations are given only for a finite component field and may be extended easily to the case of infinite-component fundamental fields.

II. Conformal Invariant Two-Point Functions

Consider the two-point function

$$F(x_1,z_1,\chi_1;x_2,z_2,\chi_2)=\langle 0|\Psi_1(x_1,z_1,\chi_1)\Psi_2(x_2,z_2,\chi_2)|0\rangle, \quad (2.1)$$

where the fields $\Psi_j(x_j,z_j,\chi_j)\,(j=1,2)$ are transformed with respect to some "fundamental" irreducible representation $\chi_j=\{d_j,\ell_0^j,\ell_1^j\}$ of conformal $SU(2,2)$ group. Here d is the scale dimension of $\Psi(x,z)$ and ℓ_0, ℓ_1 are two numbers labelling the corresponding IR of $SL(2,C)$ subgroup (any "fundamental" IR of $SU(2,2)$ is irreducible with respect to the $SL(2,C)$ subgroup).

The field which transforms according to the finite dimensional IR representation $[\ell_0,\ell_1]$ of $SL(2,C)$ is given as a homogeneous polynomial of two component complex spinor $z=(z_1,z_2)$ (see [8]), i.e.,

$$\Psi(x;z)=\Psi^{a_1,\dots,a_{\nu_1};\dot b_1,\dots,\dot b_{\nu_2}}(x)z_{a_1}\dots z_{a_{\nu_1}}\bar z_{\dot b_1}\dots\bar z_{\dot b_{\nu_2}}, \quad (2.2)$$

$$(a,\dot b=1,2)$$

where $\Psi^{\{a\},\{\dot b\}}(x)$ are ordinary field components and

$$\nu_1=\ell_1+\ell_0-1, \qquad \nu_2=\ell_1-\ell_0-1. \quad (2.3)$$

The scale dimensionality of the field $\Psi(x)$ is

$$d_\psi=d+\frac{\nu_1+\nu_2}{2}=d+\ell_1-1,$$

where d is the scale dimensionality of $\Psi(x,z)$[10].

The conformal invariance condition in infinitesimal form for two-point function (2.1) may be written as follows

$$(X_1 + X_2) F(x_1, z_1, \chi_1; x_2, z_2, \chi_2) = 0, \tag{2.4}$$

where $X_j \in \mathcal{L}_j$ $(j = 1,2)$ and $\mathcal{L}_j$ is the Lie algebra of the conformal group acting on the fields Ψ_j. The relativistic invariant two-point function is given by (see /8/ and /10/)

$$F(x_1, z_1, \chi_1; x_2, z_2, \chi_2) = \tag{2.5}$$

$$= F(x^2, z_1 \underset{\sim}{x} \bar{z}_1, z_2 \underset{\sim}{x} \bar{z}_2, z_1 \underset{\sim}{x} \bar{z}_2, z_2 \underset{\sim}{x} \bar{z}_1, z_1 \epsilon z_2),$$

where $x = x_1 - x_2$, $z_j \underset{\sim}{x} \bar{z}_k = x^{\mu} z_j \sigma_{\mu} \bar{z}_k$ $(j, k = 1,2)$, σ_0 - 2x2 unit matrix, $\underset{\sim}{\sigma}$ - the Pauli matrix and $\epsilon = i\sigma_2$.

We shall apply later the irreducibility condition for the function (2.5). From (2.4) for the dilatational and special conformal transformations we have the following conditions:

$$(D_1 + D_2) F(x_j, z_j, \chi_j) = 0, \tag{2.6}$$

$$(K^1_{\mu} + K^2_{\mu}) F(x_j, z_j, \chi_j) = 0, \qquad (\mu = 0,1,2,3), \tag{2.7}$$

where the explicit form of generators D and K_{μ} is given in the paper /10/.

From (2.5), (2.6) and (2.7) we have the following system of Eqs.

$$(d_1 - d_2) F(x_j, z_j, \chi_j) = 0,$$

$$\{d_1 + d_2 + 2x^2 \frac{\partial}{\partial x^2} + 2 \sum_{j,k=1}^{3} z_j \underset{\sim}{x} \bar{z}_k \frac{\partial}{\partial z_j \underset{\sim}{x} z_k} + z_1 \epsilon z_2 \frac{\partial}{\partial z_1 \epsilon z_2}\} F = 0,$$

$$\frac{\partial F}{\partial z_1 \epsilon z_2} = \frac{\partial F}{\partial z_1 \underset{\sim}{x} \bar{z}_1} = \frac{\partial F}{\partial z_2 \underset{\sim}{x} \bar{z}_2} = 0, \qquad (2.8)$$

The irreducibility condition with respect to the SL(2,C) for the two-point function is given by (see (2.2))

$$F(x;\lambda_1 z_1, \lambda_2 z_2) = \lambda_1^{\nu_1^1} \bar{\lambda}_1^{\nu_2^1} \lambda_2^{\nu_1^2} \bar{\lambda}_2^{\nu_2^2} F(x, z_1, z_2), \qquad (2.9)$$

where λ_1 and λ_2 are arbitrary complex numbers, and

$$\nu_1^j = \ell_1^j + \ell_0^j - 1, \quad \nu_2^j = \ell_1^j - \ell_0^j - 1 \ (j = 1,2).$$

The general solution of Eqs. (2.8) and (2.9) is given by

$$F^{[\chi_1, \chi_2]}(x; z_1, z_2) = \qquad (2.10)$$

$$= N^{[\ell_0, \ell_1]} (x^2)^{-d-\nu_1-\nu_2} (z \underset{\sim}{x} \bar{z})^{\nu_1} (z \underset{\sim}{x} \bar{z})^{\nu_2},$$

where

$$d = d_1 = d_2,$$

$$\nu_1 = \nu_1^1 = \nu_2^2 = \ell_1^1 + \ell_0^1 - 1 = \ell_1^2 - \ell_0^2 - 1, \qquad (2.11)$$

$$\nu_2 = \nu_2^1 = \nu_1^2 = \ell_1^1 - \ell_0^1 - 1 = \ell_1^2 + \ell_0^2 - 1:$$

and $N_d^{[\ell_0, \ell_1]}$ is a normalization constant.

From (2.11) it follows that the CI two-point function is nonvanishing only when

$$\chi_1 = \{d, \ell_0, \ell_1\} \quad \text{and} \quad \chi_2 = \{d_d - \ell_0, \ell_1\}. \qquad (2.12)$$

If we use the identity (see /8/)

$$z_1 \underset{\sim}{x} \bar{z}_1 z_2 \underset{\sim}{x} \bar{z}_2 - z_1 \underset{\sim}{x} \bar{z}_2 z_2 \underset{\sim}{x} \bar{z}_1 = \frac{1}{2} x^2 z_1 \sigma^\mu \bar{z}_1 z_2 \sigma_\mu \bar{z}_2 \qquad (2.13)$$

the two-point function (2.10) may be written down in the following equivalent form (when $\ell_0 > 0$)

$$F^{[\chi_1\chi_2]}(x; z_1, z_2) = N_d^{[\ell_i, \ell_j]} (x^2)^{2-d-2\ell_1} (z_1 \underset{\sim}{x} z_2)^{2\ell_0} \times$$

$$\times (z_1 \underset{\sim}{x} \bar{z}_1 z_2 \underset{\sim}{x} \bar{z}_2 - \frac{1}{2} x^2 z_1 \sigma^\mu \bar{z}_1 z_2 \sigma_\mu \bar{z}_2)^{\ell_1 - \ell_0 - i} \qquad (2.14)$$

For tensor fields ($\ell_0 = 0$ and $\ell_1 = n+1$) from (2.14) we have

$$F^{[d,n]}(x; \xi, \eta) = N_d^n (x^2)^{-d-n} (\frac{x\xi x\eta}{x^2} - \frac{\xi\eta}{2}) \quad , \qquad (2.15)$$

where $\xi_\mu = z_1 \sigma_\mu \bar{z}_1$ and $\eta_\mu = z_2 \sigma_\mu \bar{z}_2$. Formula (2.15) gives the general form of the two-point function for symmetric tensor fields with rank n /2/ in the case when the variables ξ and η are commuting.

Replacing in (2.15) variables ξ and η with anticommuting ζ and ζ', where

$$\{\zeta_\mu, \zeta'_\nu\} = \zeta_\mu \zeta'_\nu + \zeta'_\nu \zeta_\mu = 0 ,$$

we have the CI two-point function for antisymmetric tensor fields with rank $n = 2, 3$ and 4 . The case $n = 2$ was considered in paper /9/.

For the Dirac fields, which transform by the representation $\{d, \frac{1}{2}, \frac{3}{2}\} \oplus \{d, -\frac{1}{2}, \frac{3}{2}\}$

from (2.10) we have

$$F(x; z_1, z_2) = (x^2)^{-d-1} \{N_d^{(1/2;-1/2)} z_1 \underset{\sim}{x} \bar{z}_2 + N_d^{(-1/2,1/2)} z_2 \underset{\sim}{x} \bar{z}_1\}. \quad (2.16)$$

In the case $N_d^{\{1/2,-1/2\}} = N_d^{\{-1/2,1/2\}}$ from (2.15) we have

$$F(x) = N_d \frac{\hat{x}}{(x^2)^{d+1}} , \qquad (2.17)$$

where $\hat{x} = \gamma^{\mu} x_{\mu}$ and γ_{μ} are Dirac matrices. It can be proved that (2.17) is γ_5 -invariant.

In Appendix A it is shown that (2.10) is the Fourier transform of the corresponding two-point function in momentum space /10/. There is shown that the normalized constant N and the corresponding constant $\tilde{N}$ in momentum space /10/ are connected by

$$N_d^{[\ell_0,\ell_1]} = (-1)^{d+\ell_1-1} \, 2^{d+2(\ell_1+1)} \, \pi^2 \frac{\Gamma(d)\Gamma(d+1)}{\Gamma(2-d)\Gamma(d-2\ell_1+3)} \tilde{N}_d^{[\ell_0,\ell_1]} . \tag{2.18}$$

III. Three-Point Function

Let us take three "fundamental" fields $\Psi_j (x_j ; z_j , \chi_j)$ $(j = 1,2,3)$ which are transformed by irreducible representations $\chi_j = \{ d_j , \ell_0^j, \ell_1^j \}$ of conformal group. The three-point function for these fields is

$$G(x_j ; z_j , \chi_j) = \langle 0 | \Psi_1(x_1,z_1,\chi_1)\Psi_2(x_2,z_2,\chi_2)\Psi_3(x_3;z_3,\chi_3)|0\rangle . \tag{3.1}$$

The conformal invariance condition (in infinitesimal form) for the function G is

$$(X_1 + X_2 + X_3) G(x_j ; z_j , \chi_j) = 0 , \tag{3.2}$$

where $X_j \in \mathcal{L}_j$ is the conformal Lie algebra acting on the fields Ψ_j. The irreducibility with respect to SL(2, C) subgroup is given by

$$G(x_j ; \lambda_j z_j , \chi_j) = \prod_{k=1}^{3} \lambda_k^{\nu_1^k} \lambda_k^{-\nu_2^k} G(x_j ; z_j , \chi_j) , \tag{3.3}$$

where λ_j $(j=1,2,3)$ are arbitrary complex numbers and

$$\nu_1^j = \ell_1^j + \ell_0^j - 1, \quad \nu_2^j = \ell_1^j - \ell_0^j - 1 \quad (j = 1,2,3). \qquad (3.4)$$

The relativistic invariant solution $(X \in ISL(2,C))$ of (3.2) is given by

$$G(x_j; z_j, \chi_j) = G^{\{\chi_1, \chi_2, \chi_3\}}(x_{12}^2, x_{23}^2, x_{13}^2, u_{jk}, v_{jk}, \kappa_{jk}). \qquad (3.5)$$

Here $G^{\{\chi_1 \chi_2 \chi_3\}}$ is an arbitrary function (wee apply irreducibility condition (3.3) later), $x_{jk} = -x_{kj} = x_j - x_k$, $u_{jk} = z_j \underset{\sim}{x}_{12} \bar{z}_k$, $v_{jk} = z_j \underset{\sim}{x}_{23} \bar{z}_k$, $\kappa_{jk} = z_j \epsilon z_k$. From 24 variables x_{12}^2, x_{23}^2, x_{31}^2, u_{jk}, v_{jk}, and κ_{jk} on which $G^{\{\chi_1 \chi_2 \chi_3\}}$ depends, only 14 are independent.

From (3.2) for the dilatational and special conformal transformations we have

$$(D^1 + D^2 + D^3) G^{\{\chi_1, \chi_2, \chi_3\}} = 0,$$
$$(3.6)$$
$$(K_\mu^1 + K_\mu^2 + K_\mu^3) G^{\{\chi_1, \chi_2, \chi_3\}} = 0.$$

Equations (3.6) taking into account (3.5) and the explicit form of D and K_μ from /10/ may be written in the form

$$\frac{\partial G}{\partial u_{23}} = \frac{\partial G}{\partial u_{32}} = \frac{\partial G}{\partial v_{12}} = \frac{\partial G}{\partial v_{21}} = \frac{\partial G}{\partial \kappa_{jk}} = 0,$$

$$\frac{\partial G}{\partial u_{13}} - \frac{\partial G}{\partial v_{13}} = 0,$$

$$\frac{\partial G}{\partial u_{31}} - \frac{\partial G}{\partial v_{31}} = 0,$$

$$x_{12}^2\frac{\partial G}{\partial u_{11}} + (x_{23}^2 + 2x_{12}x_{23})\frac{\partial G}{\partial v_{11}} = 0,$$

$$x_{12}^2\frac{\partial G}{\partial u_{22}} - x_{23}^2\frac{\partial G}{\partial v_{22}} = 0,$$

$$(x_{12}^2 + 2x_{12}x_{23}^2)\frac{\partial G}{\partial u_{33}} + x_{23}^2\frac{\partial G}{\partial v_{33}} = 0,$$

$$\{2(2d_1 - d_2 - d_3) + 2x_{12}^2\frac{\partial}{\partial x_{12}^2} - 4x_{23}^2\frac{\partial}{\partial x_{23}^2} + 2x_{13}^2\frac{\partial}{\partial x_{13}^2} +$$

$$+ 2u_{11}\frac{\partial}{\partial u_{11}} + 2u_{22}\frac{\partial}{\partial u_{22}} + (2(u_{33} + 3v_{33})\frac{\partial}{\partial u_{33}} +$$

$$+ 2u_{12}\frac{\partial}{\partial u_{12}} + 2u_{21}\frac{\partial}{\partial u_{21}} + (2(u_{13} + 3v_{13})\frac{\partial}{\partial u_{13}} +$$

$$+ (2u_{31} + 3v_{31})\frac{\partial}{\partial u_{31}} + 2v_{11}\frac{\partial}{\partial v_{11}} - 4v_{22}\frac{\partial}{\partial v_{22}} -$$

$$- 4v_{33}\frac{\partial}{\partial v_{33}} - v_{13}\frac{\partial}{\partial v_{13}} - v_{31}\frac{\partial}{\partial v_{31}} - 4v_{23}\frac{\partial}{\partial v_{23}} - 4v_{32}\frac{\partial}{\partial v_{32}}\}G=0,$$

$$\{2(d_1 + d_2 - 2d_3) + 4x_{12}^2\frac{\partial}{\partial x_{12}^2} - 2x_{23}^2\frac{\partial}{\partial x_{23}^2} +$$

$$+ 4u_{11}\frac{\partial}{\partial u_{11}} + 4u_{22}\frac{\partial}{\partial u_{22}} - 2u_{33}\frac{\partial}{\partial u_{33}} + 4u_{12}\frac{\partial}{\partial u_{12}} +$$

$$+ 4u_{21}\frac{\partial}{\partial u_{21}} + u_{13}\frac{\partial}{\partial u_{13}} + u_{31}\frac{\partial}{\partial u_{31}} - 2(3u_{11} + v_{11})\frac{\partial}{\partial v_{11}} -$$

$$-2v_{22}\frac{\partial}{\partial v_{22}} - 2v_{33}\frac{\partial}{\partial v_{33}} - (3u_{13}+2v_{13})\frac{\partial}{\partial v_{13}} -$$

$$-(3u_{31}+2v_{31})\frac{\partial}{\partial v_{31}} - 2v_{23}\frac{\partial}{\partial v_{23}} - 2v_{32}\frac{\partial}{\partial v_{32}} \} = 0,$$

$$\{ d_1 + d_2 + d_3 + 2x_{12}^2\frac{\partial}{\partial x_{12}^2} + 2x_{23}^2\frac{\partial}{\partial x_{23}^2} + 2x_{13}^2\frac{\partial}{\partial x_{13}^2} +$$

$$+ 2\sum_{j,k=1}^{3} u_{jk}\frac{\partial}{\partial u_{jk}} + 2\sum_{j,k=1}^{3} v_{jk}\frac{\partial}{\partial v_{ij}} \}G = 0. \qquad (3.7)$$

The general solution of the irreducibility condition (3.3) and the system of partial differential Eqs. (3.7) is given by

$$G^{[\chi_j]}(x_j; z_j) = \sum_{p,q,r,s=0} C^{pqrs}_{\{\chi_1,\chi_2,\chi_3\}}(x_{12}^2)^{\frac{d_3-d_1-d_2}{2}} \times$$

$$\times (x_{23}^2)^{\frac{d_1-d_2-d_3}{2}} (x_{13}^2)^{\frac{d_2-d_1-d_3}{2}} \times$$

$$\times\left(\frac{z_1 \underset{\sim}{x}_{12}\bar{z}_2}{x_{12}^2}\right)^{\nu_2^2-\nu_1^2+p+q-r} \left(\frac{z_2 \underset{\sim}{x}_{12}\bar{z}_1}{x_{12}^2}\right)^p \left(\frac{z_2 \underset{\sim}{x}_{23}\bar{z}_3}{x_{23}^2}\right)^q \times$$

$$\times\left(\frac{z_3 \underset{\sim}{x}_{23}\bar{z}_2}{x_{23}^2}\right)^r \left(\frac{z_1 \underset{\sim}{x}_{13}\bar{z}_3}{x_{12}^2}\right)^{\nu_2^3-\nu_1^3+r+s-q} \left(\frac{z_3 \underset{\sim}{x}_{13}\bar{z}_1}{x_{13}^2}\right)^s \times$$

$$\times (\lambda_{23}^1)^{\nu_2^1-p-s} (\lambda_{31}^2)^{\nu_1^2-p-q} (\lambda_{12}^3)^{\nu_1^3-r-s} \qquad (3.8)$$

and

$$\nu_1^1+\nu_1^2+\nu_1^3=\nu_2^1+\nu_2^2+\nu_2^3, \tag{3.9}$$

where

$$\lambda^j_{k\ell}=\frac{z_j\, \underset{\sim}{x}_{jk}\, \bar{z}_j}{x^2_{jk}}-\frac{z_j\, \underset{\sim}{x}_{j\ell}\, \bar{z}_j}{x^2_{j\ell}}, \quad (j,k,\ell = 1,2,3).$$

Here C^{pqrs} are arbitrary constants and the upper limits in the sums are determined from the polynomiality conditions of G with respect to z_j (for finite-dimensional representations of SL(2,C)).

From (3.9) taking into account (3.3) we have

$$\ell_0^1+\ell_0^2+\ell_0^3=0, \tag{3.10}$$

which gives the restriction for the irreducible representations of SL(2,C) for which the three-point function is nonvanishing.

One may write from (3.8) the explicit form of the three-point function for some special fields:

a) The scalar fields, i.e., $\nu_1^j=\nu_2^j=0$ $(j=1,2,3)$ /2/

$$g(x_1,d_1,x_2,d_2;x_3,d_3)=$$

$$=C(x^2_{12})^{\frac{d_3-d_1-d_2}{2}}(x^2_{23})^{\frac{d_1-d_2-d_3}{2}}(x^2_{13})^{\frac{d_2-d_1-d_3}{2}} \tag{3.11}$$

b) One tensor field $\nu_1^3=\nu_2^3=n$ and two fields with an arbitrary spin

$$G(x_1,z_1,\chi_1;x_2,z_2,\chi_2;x_3,z_3,\chi_3)=g(x_1,d_1;x_2,d_2;x_3,d_3)\times$$

$$\times\sum_{pqrs}C^{p,q,r,s}_{\chi_1\chi_2\chi_3}\left(\frac{z_1\,\underset{\sim}{x}_{12}\,\bar{z}_2}{x^2_{12}}\right)^{\nu_2^2-\nu_1^2+p+q-r}\times$$

$$\times \left(\frac{z_2 \underset{\sim}{x}_{12} \bar{z}_1}{x_{12}^2}\right)^p \left(\frac{z_2 \underset{\sim}{x}_{23} \bar{z}_3}{x_{23}^2}\right)^q \left(\frac{z_3 \underset{\sim}{x}_{23} \bar{z}_2}{x_{23}^2}\right)^r \left(\frac{z_1 \underset{\sim}{x}_{13} \bar{z}_3}{x_{13}^2}\right)^{r+s-q} \left(\frac{z_3 \underset{\sim}{x}_{13} \bar{z}_1}{x_{13}^2}\right)^s \times$$

$$\times (\lambda^1_{23})^{\nu^1_2 - p - s} (\lambda^2_{31})^{\nu^2_1 - p - q} (\lambda^3_{12})^{n-r-s} \quad . \tag{3.12}$$

Here from (3.9) or (3.10) we have the following restrictions for the ℓ^1_0 and ℓ^2_0

$$\ell^1_0 + \ell^2_0 = 0$$

and consequently

$$\chi_1 = \{d_1, \ell_0, \ell^1_1\} \qquad \text{and} \quad \chi_2 = \{d_2, -\ell_0, \ell^2_1\} \tag{3.13}$$

and $g(x_i; d_i)$ is the three point function of scalar fiedls given by (3.11).

c) In the case when $\chi_1 = \{d_1, \frac{1}{2}, \frac{3}{2}\}$ and $\chi_2 = \{d_2, -\frac{1}{2}, \frac{3}{2}\}$ from (3.12) we have

$$G_n^{\{½, -½\}}(x_1, z_1; x_2, z_2; x_3, z_3) = g(x_1, d_1; x_2, d_2; x_3, d_3) \times$$

$$\times \{ C_1 \frac{z_1 \underset{\sim}{x}_{12} \bar{z}_2}{x_{12}^2} (\lambda^3_{12})^n + C_2 \frac{z_3 \underset{\sim}{x}_{23} \bar{z}_2}{x_{23}^2} \frac{z_1 \underset{\sim}{x}_{13} \bar{z}_3}{x_{13}^2} (\lambda^3_{12})^{n-1} \} =$$

$$= g(x_1, d_1; x_2, d_2; x_3, d_3) \{ C_1 \frac{z_1 \underset{\sim}{x}_{12} \bar{z}_2}{x_{12}^2} \lambda^3_{12} +$$

$$+ \frac{C_2}{2} \frac{z_1 \underset{\sim}{x}_{13} \tilde{\zeta} \underset{\sim}{x}_{23} \bar{z}_2}{x_{12}^2 x_{23}^2} \} (\lambda^3_{12})^{n-1} , \tag{3.14}$$

where C_1 and C_2 are two arbutrary constants, $\zeta_\mu = z_3 \sigma_\mu \bar{z}_3$, $\tilde{\sigma}_\mu = g_{\mu\mu} \sigma_\mu$. Here the identity

$$z_1 \underset{\sim}{x} \bar{z}_3 \, z_3 \underset{\sim}{y} \bar{z}_2 = \frac{1}{2} z_1 \underset{\sim}{x} \tilde{\sigma}^\mu \underset{\sim}{y} \bar{z}_2 z_3 \sigma_\mu \bar{z}_3$$ following from

$$g_{\mu\nu}(\sigma^{\mu})^{a\dot{b}}(\sigma^{\nu})^{c\dot{d}} = 2\epsilon^{ac}\epsilon^{\dot{b}\dot{d}}$$

was used.

The function $G_n^{\{-1/2,\,1/2\}}$ given by

$$G_n^{\{-1/2,1/2\}}(x_1,z_1;x_2,z_2;x_3,\zeta) = g(x_1,d_1;x_2,d_2;x_3,d_3)\times$$

$$\times\{C_1^1\frac{z_2\underset{\sim}{x}_{12}\bar{z}_1}{x_{12}^2}\lambda_{12}^3 + \frac{C_2^1}{2}\frac{z_2\underset{\sim}{x}_{23}\tilde{\zeta}\underset{\sim}{x}_{13}\bar{z}_1}{x_{23}^2x_{13}^2}\}(\lambda_{12}^3)^{n-1}, \tag{3.15}$$

where C_1' and C_2' are new constants, is also nonvanishing.

For the Dirac fields $\chi = \{d;\frac{1}{2},\frac{3}{2}\}\oplus\{d,-\frac{1}{2},\frac{3}{2}\}$ if we take $C_1 = C_1$, $C_2 = C_2$, (3.14) and (3.15) may be combined as follows:

$$G(x_1,x_2,x_3,\zeta) = g(x_1,d_1;x_2,d_2;x_3,d_3)\times$$

$$\times\{C_1\frac{\hat{x}_{12}}{x_{12}^2}\lambda_{12}^3 + \frac{C_2}{2}\frac{\hat{x}_{12}\hat{\zeta}\hat{x}_{23}}{x_{13}^2x_{23}^2}\}(\lambda_{12}^3)^{n-1}, \tag{3.16}$$

where the variables z_1 and z_2 are omitted. It can be proved that the function (3.16) is invariant with respect to γ_5-transformations, i.e., $\Psi\to\gamma_5\Psi$, , $\bar{\Psi}\to\bar{\Psi}\gamma_5$ and $\Phi\to\Phi$..

For fields with $s = 1/2$, i.e., $\chi_j = \{d\ ,\pm 1/2, 3/2\}$ $(j=1,2)$ the following nonvanishing functions $G(x_1,z_1,\chi_1^{\pm};$ $x_2,z_2,\chi_2^{\pm};x_3,z_3,\chi_3^{\mp})$ exist, where $\chi_3^{\mp} = \{d_3,\mp 1,\ell^3\}$ are determined from (3.10).

From (3.8) for the last two cases we have

$$G(x_1,x_1,\chi_1^+;x_2,z_2,\chi_2^+;x_3,z_3,\chi_3^-) =$$

$$= C_1^+ g(x_1,d_1,x_2,d_2;x_3,d_3)\frac{z_2\underset{\sim}{x}_{23}\bar{z}_3}{x_{23}^2}\frac{z_1\underset{\sim}{x}_{13}\bar{z}_3}{x_{13}^2}(\lambda_{12}^3)^{\ell_1^3-2}, \tag{3.17}$$

and

$$C(x_1, z_1, \chi_1^- ; x_2, z_2, \chi_2^- ; x_3, z_3, \chi_3^+) =$$
$$= C^- g(x_1, d_1; x_2, d_2; x_3, d_3) \frac{z_3 \tilde{x}_{13} \bar{z}_1}{x_{13}^2} \frac{z_3 \tilde{x}_{23} \bar{z}_3}{x_{23}^2} (\lambda_{12}^3)^{\ell_1^3} . \tag{3.18}$$

It may be proved that the functions (3.14), (3.15), (3.17) and (3.18) are all nonvanishing three-point functions for two spin 1/2 fields and one "fundamental" finite-component field.

d) In the case of three tensor fields $\nu_1^j = \nu_2^j = n_j$ (j=1,2,3) from (3.8) taking into account (2.11) we have

$$G(x_j, \xi_j, \chi_j) = g(x_j, d_j) \sum_{p,q,r=0} C^{pqr} (\lambda_{23}^1)^{n_1 - p - r} \times$$
$$\times (\lambda_{31}^2)^{n_2 - p - q} (\lambda_{12}^3)^{n_3 - q - r} (\mu_{12})^p (\mu_{23})^q (\mu_{31})^r , \tag{3.19}$$

where

$$\mu_{jk} = \xi_j \xi_k - 2 \frac{(x_{jk} \xi_j)(x_{jk} \xi_k)}{x_{jk}^2}$$

and C^{pqr} are arbitrary constants.

IV. The Ward Identities

Consider the conserved tensor current $J(x; \xi)$ $(\xi^2 = \xi^\mu \xi_\mu = 0)$ of rank n, i.e.,

$$J(x:\xi) = J^{\mu_1 \dots \mu_n}(x) \xi_{\mu_1} \dots \xi_{\mu_n}$$

and

$$\frac{\partial^2 J(x;\xi)}{\partial x^\mu \partial \xi_\mu} = 0 . \tag{4.1}$$

The two-point function for an arbitrary tensor field is given by (2.15). From (4.1) we have the following Eqs. for this function

$$\frac{\partial^2 F(x_{12},\xi\eta)}{\partial x^{\mu}_{12}\partial \xi_{\mu}} = - \frac{\partial^2 F}{\partial x^{\mu}_{12}\partial \eta_{\mu}} = 0. \qquad (4.2)$$

These Eqs. give that the scale dimension of conserved tensor current is canonical, i.e.,

$$d = 2 \qquad \text{or} \quad d_J = 2 + n , \qquad (4.3)$$

Consider the three-point function (3.12), when $\chi = \{d, \ell_0, \ell_1\}, \chi = \{d, -\ell_0 \ell_1\}$ and $\chi = \{2, 0, n\}$, , i.e., when the last field is the conserved vector current. In this case from the generalized Ward identity (see /4/)

$$\frac{\partial^2 G^T_3}{\partial \xi^{\mu}\, \partial x^3_{\mu}} = -e[\delta(x_1 - x_3) - \delta(x_2 - x_3)]\, S(x_1, x_2) \qquad (4.4)$$

we obtain that the only nonvanishing constant in (3.12) is

$$C^{\ell_1, \ell_0 - 1, 0,0,0}_{\chi_1, \chi_2, \chi_3} = 2e N^d_{\ell_0 \ell_1}. \qquad (4.5)$$

Here $N^d_{\ell_0 \ell_1}$ is the normalization constant of the two-point function in right-hand side of (4.4) given by (2.10).

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Appendix A

The CI two-point function for the fields with arbitrary spin in momentum space is given in /5/. It has the following form:

$$\tilde{F}(p;z_1,z_2) = \tilde{N}_d^{[\ell_0,\ell_1]} (p^2)^{d-2} (z_1 p \bar{z}_2)^{2\ell_0} (z_1 p \bar{z}_1 z_2 p \bar{z}_2)^{\ell_1-\ell_0-i} \times$$

$$\times P^{(d-2,\,2\ell_0)}_{\ell_1-\ell_0-1}\left(1 - \frac{2p^2 z_1 \epsilon z_2 \bar{z}_1 \epsilon \bar{z}_2}{z_1 p \bar{z}_1 \, z_2 p \bar{z}_2}\right). \qquad (A.1)$$

The corresponding two-point function in the x-space may be given by the Fourier transform of (A.1)

$$F(x;z_1,z_2) = \tilde{N}_d^{[\ell_0,\ell_1]} \int d^4p\, e^{-ipx} \tilde{F}(p;z_1 z_2) =$$

$$= \tilde{N}_d^{[\ell_0,\ell_1]} 2^{-n} (i z_1 \partial \bar{z}_2)^{2\ell_0} \times$$

$$\times \sum_{m=0}^{\ell_1-\ell_0-1=n} \binom{n+d-2}{m} \binom{n+2\ell_0}{n-m} (-z_1 \epsilon z_2 \bar{z}_1 \epsilon \bar{z}_2 \Box)^{n-m} \times$$

$$\times (z_1 \epsilon z_2 \bar{z}_1 \epsilon \bar{z}_2 \Box - z_1 \partial \bar{z}_1 \, z_2 \partial \bar{z}_2)^m \int d^4p (p^2)^{d-2} e^{-ipx} =$$

$$= \frac{\pi^2 \Gamma(d)}{2^n \Gamma(2-d)} (i z_1 \partial \bar{z}_2)^{2\ell_0} \sum_{m=0}^{n} \binom{n+d-2}{m} \binom{n+2\ell_0}{n-m} \times$$

$$\times (-z_1 \epsilon z_2 \bar{z}_1 \epsilon \bar{z}_2 \Box)^{n-m} (z_1 \epsilon z_2 \bar{z}_1 \epsilon z_2 \Box - z_1 \partial \bar{z}_1 \, z_2 \partial \bar{z}_2)^m (x^2)^{2-d} =$$

$$= (-1)^{\ell_1 - 1} (-2)^{d+2(\ell_1 - 1)} \frac{\pi^2 \Gamma(d) \Gamma(d+1)}{\Gamma(2-d) \Gamma(d - 2\ell_1 + 3)} (x^2)^{-d - 2\ell_1 + 2} \times$$

$$\times (z_1 \underset{\sim}{x} \bar{z}_2)^{\ell_1 + \ell_0 - 1} (z_2 \underset{\sim}{x} \bar{z}_1)^{\ell_1 - \ell_0 - 1} . \qquad \textbf{(A.2)}$$

After comparison of (A.2) with (2.10) we have the formula (2.18).

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