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E2 - 10227

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FIXED-ANGLE SCATTERING ON THE YUKAWA-TYPE POTENTIALS

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Голоскоков С.В., Кулешов С.П., Митрюшкин В.К. **E2 - 10227**

Рассеяние на фиксированные углы на юкавских потенциалах

В работе предлагается метод нахождения амплитуд высокоэнергетического рассеяния, аналогичный используемому при определении асимптотик фейнмановских диаграмм. Вычисляется главный асимптотический член произвольной борновской амплитуды в пределе $|\vec{p}| \to \infty$, $|\vec{\Delta}|/|\vec{p}|$ - фикс. На примере второго борновского члена демонстрируется способ нахождения младших асимптотических членов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований

Дубна 1976

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Fixed-Angle Scattering on the Yukawa-Type Potentials

A method for finding the amplitudes of high-energy scattering on the Yukawa-type potentials is proposed similar to the one for defining the asymptotics of Feynman diagrams. The main asymptotic term is calculated for an arbitrary Born amplitude in the limit $|\vec{p}| \to \infty$, $|\vec{\Delta}|/|\vec{p}|$ -fixed. A method of calculating lower-order asymptotic terms is demonstrated for the second Born term.

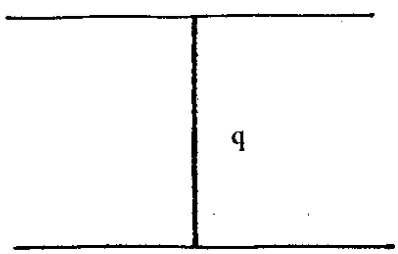
The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research

Dubna 1976

1. Recent experiments (see, e.g., ref.[1]) indicate a possible power dependence of the hadron high-energy scattering amplitude. To interpret this behaviour, composite models of elementary particles have been proposed (see, e.g., ref.[2]). It seems also that it is reasonable to apply a potential approach for studying that problem. Within the relativistic theory it is just the quasipotential approach[3]. In papers[4] power automodel asymptotics have been found for the large-angle high-energy particle scattering on smooth potentials.

However, quasipotentials of the Yukawa type are of a special interest. These arise even in the lowest perturbation expansion order. Indeed, applying the well-known so-called "bracket" operation[3] to the graph



we obtain for the quasipotential in the momentum space $\sim \frac{1}{\vec{q}^{\,2} + \sigma^2}$ and in the coordinate space $\sim \frac{e^{-\sigma r}}{r}$.

The fixed-angle scattering on Yukawa potentials within the nonrelativistic theory is considered in many papers. Among the first papers in this trend are [5], in which three first Born terms have been calculated for the scattering amplitude on potential $e^{-\sigma r}/r$. A further study of this problem,, was based, as a rule, on the eikonal representation (ER) of the amplitude. The ER and several first nondivergent eikonal corrections have been derived from the Born series for large momenta of the incident particle $\vec{p}$ and fixed transfer momenta $\vec{\Delta}$ [16]. The validity of ER in the limit $|\vec{\Delta}| \to \infty$ requires some justification.

In paper [17] it has been shown that the results of ER for the amplitude on the Yukawa potential do not contradict the results of paper [5].

An applicability of ER for large transfer momenta was justified on the basis of numerical comparison with the Born terms (see, e.g., ref. [8]).

The most consistent and rigorous approach is a direct calculation of the Born amplitudes in one or another asymptotical limit with a subsequent summation of all Born terms. Note also that for the intermediate-coupling constant the Born series is well defined.

To obtain the asymptotics of Born amplitudes for scattering on Yukawa-type potentials in the limit $|\vec{p}| \to \infty$, $|\vec{\Delta}|/|\vec{p}|$ - fixed within the Lippmann-Schwinger and Logunov-Tavkhelidze equations, we propose here a method analogous to the one used for finding the asymptotics of Feynman diagrams.

The main asymptotic terms of an arbitrary Born amplitude are calculated. The procedure is easily extended to calculate the lowest asymptotic terms (the so-called lowest logarithms) that is exemplified with the second Born term.

2. An $(n+1)$-th Born term of the scattering amplitude obtained from the Lippmann-Schwinger equation with the potential

$$V(\vec{q}) = \frac{g}{\sigma^2 + \vec{q}^{\,2}}$$

$$T_{n+1}(\vec{p}\,';\vec{p})' = g\cdot\left(\frac{g}{2\pi^2}\right)^n \int \prod_{i=1}^{n} d\vec{q}_i \, \frac{1}{\sigma^2 + (\vec{p}\,' - \vec{q}_1)^2} \times$$

$$\times \frac{1}{(\vec{q}_n^{\,2} - \vec{p}^{\,2} - i\epsilon)[\sigma^2 + (\vec{q}_n - \vec{p})^2 - i\epsilon]} \cdot \prod_{j=1}^{n} \frac{1}{(q_j^2 - p^2 - i\epsilon)[\sigma^2 + (\vec{q}_j - \vec{q}_{j+1})^2]} .$$

To the amplitude T_{n+1} it is convenient to make correspond the graph analogous to the Feynman diagram

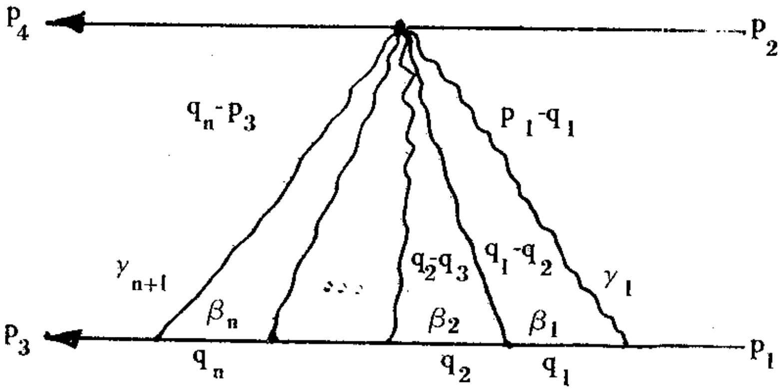


Fig. 1

where

$p_1 = (0; \vec{p})$

$p_2 = (0; -\vec{p})$

$p_3 = (0; \vec{p}\,')$

$$p_4 = (0; -\vec{p}\,')$$

$$q_i = (0; \vec{q}_i\ ) ; \quad i = 1, \dots, n$$

and wave lines are the propagators with mass σ, and internal straight lines are the propagators with $M^2 = p_k^2 = (i\,\vec{p}_k\,)^2$, $k = 1,\dots,4$. Using the Feynman identity

$$\frac{1}{\prod\limits_{i=1}^{N} A_i} = (N-1)! \int_0^1 \dots \int \prod_{i=1}^{N} d\alpha_i \, \frac{\delta\left(\sum\limits_{j=1}^{N} \alpha_j - 1\right)}{\left[\sum\limits_{i=1}^{N} \alpha_i A_i\right]^N}$$

for T_{n+1} the representation similar to the Chisholm representation [9] for Feynman diagrams (see *Fig. 1*)

$$T_{n+1} = g\left(\frac{g}{2\pi^2}\right)^n \cdot \pi^{\frac{3n}{2}} \cdot \Gamma\left(\frac{n}{2}+1\right) \times$$

$$\times \int_0^1 \dots \int \prod_{i=1}^{n} d\beta_i \prod_{i=1}^{n+1} d\gamma_i \cdot \delta\left(\sum_{j=1}^{n+1} \gamma_j + \sum_{j=1}^{n} \beta_j - 1\right) \frac{C_n^{\frac{n}{2}-\frac{1}{2}}}{D_n^{\frac{n}{2}+1}} \tag{2.1}$$

can be obtained, where

$$C_n = \begin{vmatrix} \gamma_1 + \gamma_2 + \beta_1 & -\gamma_2 & 0 & \dots & 0 \\ -\gamma_2 & \gamma_2 + \gamma_3 + \beta_2 & -\gamma_2 & \dots & 0 \\ & & & & \\ 0 & 0 & 0 & \dots & \gamma_n + \gamma_{n+1} + \beta_n \end{vmatrix}$$

and D_n, on the basis of the relation

$$p^2 = \frac{1}{2(1-z)} \Delta^2$$

$$p = |\vec{p}|; \quad \Delta = |\vec{\Delta}|$$

where z is the cosine of the scattering angle, may be written in the form

$$D_n = F_n \cdot \Delta^2 + h_n - i\epsilon$$

The functions F_n and h_n can be calculated with the help of rules formulated for the Feynman diagrams in paper /10/ if the Mandelstam variables are taken as follows

$$\tilde{s} = (p_1 + p_2)^2 = 0$$

$$\tilde{t} = (p_1 - p_2)^2 = -\Delta^2$$

$$\tilde{u} = (p_1 - p_4)^2 = -2p^2(1+z).$$

Then we have

$$F_n = \prod_{i=1}^{n+1} \gamma_i - \frac{1}{2(1-z)} \cdot f_n$$

$$f_n = -C_n \sum_{i=1}^{n} \beta_i + \beta_1 \gamma_1 \frac{\partial}{\partial \gamma_1} C_n + \beta_2 \gamma_1 \gamma_2 \frac{\partial^2}{\partial \gamma_1 \partial \gamma_2} C_n + \dots +$$

$$+ \beta_n \gamma_1 \dots \gamma_n + \beta_n \gamma_{n+1} \frac{\partial}{\partial \gamma_{n+1}} C_n +$$

$$+ \beta_{n-1} \gamma_{n+1} \gamma_n \frac{\partial^2}{\partial \gamma_{n+1} \partial \gamma_n} C_n + \dots + \beta_1 \gamma_{n+1} \dots \gamma_2 \quad .$$

$$h_n = \sigma^2 \cdot C_n \sum_{i=1}^{n+1} \gamma_i \tag{2.2}$$

3. The main contribution to integral (2.1) in the limit $\Lambda^2 \to \infty$, z -fixed, comes from the ranges of integration over $\{\beta, \gamma\}$, where $F_n \sim 0$ and those regions are essential where the variables forming the so-called p -paths and t -subgraphs are in the vicinity of zero (see, e.g., refs./10,11/). It is clear that for the diagram in *Fig. 1* the t-paths are the sets of variables $\{\beta_1,\dots,\beta_n,\gamma_i\}$, $i=1,2,\dots,n+1$ and t -subgraphs are the sets of variables $\{\gamma_i,\gamma_{i+1},\beta_i;\beta_1,\dots\beta_{i-1},\beta_{i+1},\dots\beta_n),i=1,2,\dots n.$ Now let us make several changes of variables. We shall perform the successive non-barycentric scaling of t -subgraphs from right to left of the following form

$$\begin{aligned} &\gamma_i \to \lambda_1 \gamma_i \ , \quad i=2,3,\dots,n+1 \\ &\beta_i \to \lambda_1 \beta_i \ , \quad i=2,3,\dots,n \\ &\beta_1 \to \sqrt{\lambda_1}\cdot\beta_1 . \end{aligned} \qquad (3.1)$$

with the subsidiary condition

$$\sum_{i=2}^{n+1} \gamma_i + \sum_{i=2} \beta_i + \beta_i^2 = 1$$

and then

$$\begin{aligned} &\gamma_i \to \lambda_2 \gamma_i \ ; \quad i=3,\dots,n+1 \\ &\beta_i \to \lambda_2 \beta_i \ ; \quad i=3,\dots,n \\ &\beta_j \to \sqrt{\lambda_2}\,\beta_i \ ; \quad j=1,2 \end{aligned} \qquad (3.1)$$

with the subsidiary condition

$$\sum_{i=3}^{n+1} \gamma_i + \sum_{i=3}^{n} \beta_i + \beta_1^2 + \beta_2^2 = 1 \quad \text{and so on.}$$

And the last change of variables is the scaling of t-path $\{\beta_1, \beta_2, \dots \beta_n, \gamma_n\}$:

$$\begin{aligned} &\gamma_n \to \lambda_n \gamma_n \\ &\beta_i \to \sqrt{\lambda_n} \cdot \beta_i \; ; \; i = 1,\dots,n \, . \end{aligned} \tag{3.2}$$

with the subsidiary condition

$$\gamma_n + \sum_{i=1}^{n} \beta_i^2 = 1 \, .$$

In this case the contribution to the main asymptotic term comes from λ_i around zero. Note that another, alternative, order of the change of variables is possible. For instance, in (3.2) instead of $\gamma_n \to \lambda_n \gamma_n$ one can make the change $\gamma_{n+1} \to \lambda_n \cdot \gamma_{n+1}$ and the scaling in t – subgraphs from left to right, etc. It can be easily verified that the whole number of such alternative possibilities of the change variables equals 2^n and all these should be considered. Due to the fact that variables λ_i are small we may retain in C_n, F_n, h_n only terms of the lowest order in λ_i. Then (see *Appendix A*)

$$\begin{aligned} C_n &\simeq \prod_{i=1}^{n} \lambda_i \\ h_n &\simeq \sigma^2 \cdot \prod_{i=1}^{n} \lambda_i \\ F_n &\simeq \prod_{i=1}^{n} \lambda_i^2 \cdot \tilde{F}_n = \prod_{i=1}^{n} \lambda_i^2 \; [\gamma_n - \frac{1}{2(1-z)} \sum_{i=1}^{n} \beta_i^2 \,] \, . \end{aligned}$$

After the change (3.1-2) the leading asymptotic term of the amplitude in the limit $\Delta^2 \to \infty$ takes the form

$$g \left(\frac{g}{2\pi^2}\right)^n 2^n \cdot \pi^{\frac{3n}{2}} \cdot \Gamma\left(\frac{n}{2} + 1\right) \times$$

$$\times \int_0^1 d\gamma \cdot \int_0^1 \cdots \int \prod_{i=1}^{n} d\beta_i \cdot \delta(\gamma_n + \frac{1}{2(1-z)} \cdot \sum_{i=1}^{n} \beta_i^2 - 1) \times$$

$$\times \int_0^{\epsilon} \cdots \int \frac{\prod_{i=1}^{n} d\lambda_i \cdots \lambda_i^{n/2}}{[\prod_{i=1}^{n} \lambda_i \, \tilde{F}_n \cdot \Delta^2 + \sigma^2 - i\epsilon]^{\frac{n}{2}+1}} . \qquad (3.3)$$

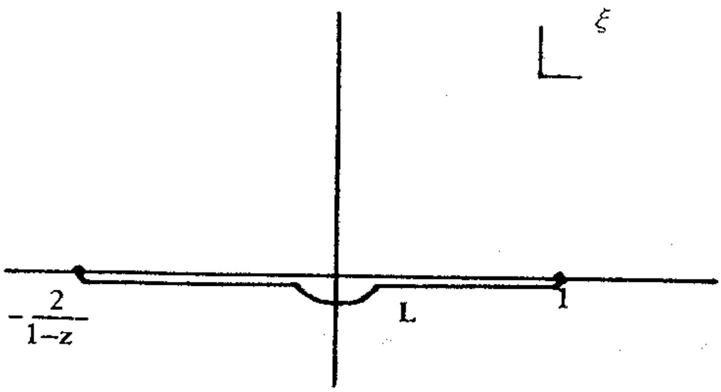
It is not hard to see that the function $\tilde{F}_n$ vanishes only on the so-called "regular manifold" [10]. Therefore expression (3.3) can be represented in the form

$$g\left(\frac{g}{2\pi^2}\right)^n \; 2^n \cdot \pi^{\frac{3n}{2}} \cdot \Gamma\left(\frac{n}{2}+1\right) \times$$

$$\times \int_L d\xi \cdot \int_0^1 d\gamma_n \cdot \int \cdots \int \prod_{i=1}^{n} d\beta_i \cdot \delta(\xi - \tilde{F}_n) \cdot \delta\left(\gamma_n + \frac{1}{2(1-z)} \sum_{i=1}^{n} \beta_i^2 - 1\right) \times$$

$$\times \int_0^{\epsilon} \cdots \int \frac{\prod_{i=1}^{n} d\lambda_i \cdot \lambda_i^{n/2}}{[\prod_{i=1}^{n} \lambda_i \, \xi \cdot \Delta^2 + \sigma^2 - i\epsilon]^{n/2+1}}$$

where the contour L has the form



It is easy to see (*Appendix B*) that

$$\int_0^{\epsilon} \!\!\ldots\! \int \frac{\prod\limits_1^n d\lambda_i \cdot \lambda_i^{n/2}}{[\prod\limits_{i=1}^n \lambda_i \; \xi \; \Delta^2 + \sigma^2 - i\epsilon]^{\frac{n}{2}+1}} =$$

$$= \frac{1}{\Delta^{n+2} \cdot \xi^{\frac{n}{2}+1}} \left[\frac{\ln^n \frac{\Delta^2}{\sigma^2}}{n!} + 0(\ln^n \frac{\Delta^2}{\sigma^2}) \right].$$

And finally, we are left to calculate the integral

$$\int_L \frac{d\xi}{\xi^{\frac{n}{2}+1}} \cdot \int \ldots \int d\gamma_n \prod_{i=1}^n d\beta_i \cdot \delta(\xi - \tilde{F})\, \delta(\gamma_n + \frac{1}{2(1-z)} \sum_{i=1}^n \beta_i^2 - 1).$$

As can be shown, it equals

$$i^n \cdot \pi^{n/2} (1-z)^n \frac{1}{\Gamma(\frac{n}{2}+1)} .$$

As a result, the leading asymptotic term of $(n+1)$ Born amplitude takes the form

$$\frac{g}{\Delta^2} \cdot \frac{1}{n!} \left[\frac{ig}{p} \ln \frac{\Delta}{\sigma} \right]^n . \qquad (3.4)$$

The sum of the main asymptotic terms results in the expression

$$\frac{g}{\Delta^2} \cdot \exp\left[\frac{ig}{p} \ln \frac{\Delta}{\sigma} \right].$$

It is interesting that this expression coincides with the one obtained from the eikonal in the limit $\Delta \gg \sigma$.

In the limit $\sigma \to 0$ for the cross section we arrive at the Rutherford formula for scattering on the Coulomb potential.

4. By the method employed in the paper also the asymptotic terms can be obtained for the lower-order Born amplitudes. In this case the values of λ_i should be taken into account not only around zero. Thus, to calculate the next to the leading asymptotic term in amplitude (2.1) it is necessary to consider also the integration region

$$\lambda_{i_1} \geq \epsilon$$

under the previous assumption that all other parameters $\{\lambda_i\}$, $i \neq i_1$ are small.

For illustration, in *Appendix* C some details are given of calculation of the second Born amplitude T_2, which equals $\frac{ig^2}{\Delta^2} \cdot \frac{1}{p}[\ln\frac{\Delta}{\sigma}+o(1)]$ that coincides with the result of paper [5].

5. Analogously the scattering of the relativistic particle is investigated in the quasipotential approach [3]. For the potential

$$U(s;\vec{q}) = \frac{g(s)}{\vec{q}^{\,2} + \sigma^2}$$

the main asymptotic term of the (n+1) Born amplitude has the form

$$U(s;\Delta)\frac{1}{n!}\left[\frac{4ig(s)}{s} \cdot \ln\frac{\Delta}{\sigma}\right]^n. \tag{5.1}$$

Details of the calculation are given in *Appendix D*.

It is interesting to compare (5.1) with (3.4). These expressions differ from each other by the change $g \to 2g/\sqrt{s}$.

Note than (5.1) can be derived from the eikonal representation for the Yukawa scattering amplitude with the eikonal phase which differs from that for the amplitude of the nonrelativistic particle with the same potential by the factor $2/\sqrt{s}$.

The sum of terms (5.1) can be written in the form

$$U(s;\Delta) e^{i\chi_{eik}(\rho\sim\frac{1}{\Delta})}. \tag{5.2}$$

For the smooth potentials in (5.2) one may put $\chi_{eik}(\rho\sim\frac{1}{\Delta}) \simeq \chi_{eik}(0)$ that results in the expression found in /4/ .

We wish to express our gratitude to A.N.Tavkhelidze, V.A.Matveev, A.N.Sissakian, M.A.Smondyrev for useful discussions.

Appendix A

Here we obtain $\tilde{F}_n$ from representations (3.1) and (3.2).

Evidently, all terms in (2.2) linear in β_i cancel. Indeed,

$$C_n^{(0)} = C_n(\{\beta_i\} = 0) = \prod_1^{n+1} \gamma_i \sum_1^{n+1} \frac{1}{\gamma_j} .$$

Terms in f_n linear in β_i have the form

$$f_n^{(1)} = - C_n^{(0)} \sum_1^n \beta_i + \beta_1 \gamma_1 \frac{\partial}{\partial\gamma_1} C_n^{(0)} + \ldots + \beta_n \prod_1^n \gamma_i +$$
$$+ \beta_n \gamma_{n+1} \cdot \frac{\partial}{\partial\gamma_{n+1}} C_n^{(0)} + \ldots + \beta_1 \prod_2 \gamma_i \tag{A.1}$$

and due to the clear equalities

$$\gamma_1 \frac{\partial}{\partial\gamma_1} C_n^{(0)} = C_n^{(0)} - \prod_2^{n+1} \gamma_i$$

$$\gamma_1\gamma_2 \frac{\partial^2}{\partial\gamma_1\,\partial\gamma_2} C_n^{(0)} = C_n^{(0)} - \prod_1^{n+1} \gamma_i \left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right)$$

expression (A.1) vaniches. The part of f_n quadratic in β_i has the form

$$f_n^{(2)} = -\sum_1^n \beta_i \sum_1^n \beta_j \frac{\partial}{\partial\beta_j} C_n^{(0)} + \beta_1\gamma_1 \frac{\partial}{\partial\gamma_1} \sum_2^n \beta_i \frac{\partial}{\partial\gamma_i} C_n^{(0)} + \ldots +$$

$$+\beta_n \cdot \gamma_{n+1} \cdot \frac{\partial}{\partial\gamma_{n+1}} \sum_1^{n-1} \beta_i \frac{\partial}{\partial\gamma_i} C_n^{(0)} + \ldots + \beta_2\beta_1 \cdot \prod_3^{n+1} \gamma_i ;$$

After the changes (3.1) and (3.2) for $\lambda_i \sim 0$ the main contribution to $f_n^{(2)}$ comes only from terms diagonal in $\beta_i\beta_j$.

Appendix B

Now let us obtain the main asymptotic term of the integral (see also ref./10/):

$$\int_0^1 \ldots \int \frac{\prod_1^n d\lambda_i \cdot \lambda_n^{n/2}}{[\prod_1^n \lambda_i \cdot r + 1]^{n/2+1}} \quad \text{for } r \gg 1 . \qquad \text{(B.1)}$$

The integral (B.1) is reduced by an $[\frac{n}{2}]$ -tuple differentiation to the following form

$$J_n = \int_0^1 \ldots \int \frac{\prod_1^n d\lambda_i \cdot \lambda_i^m}{[\prod_1^n \lambda_i \cdot r + 1]^m} ,$$

where $[\frac{n}{2}]$ is an integer part of the number $\frac{n}{2}$ and $m=\frac{n}{2}-[\frac{n}{2}]$. With the change $\zeta=\lambda_n\tau$

$$J_n(\tau)=\frac{1}{\tau^{m+1}}\int_0^\tau d\zeta\cdot\zeta^m\int_0^1\cdots\int\frac{\prod_1^{n-1}d\lambda_i\cdot\lambda_i^m}{[\prod_1^n\lambda_i\cdot\zeta+1]^{m+1}}$$

$$\frac{d}{d\tau}[J_n(\tau)\cdot\tau^{m+1}]=\tau^m\cdot J_{n-1}(\tau) \qquad (B.2)$$

through the direct calculation we obtain

$$J_1(\tau)=\frac{\ln\tau}{\tau^{m+1}}+O(\frac{1}{\tau^{m+1}}). \qquad (B.3)$$

Then from (B.2) and (B.3) it follows that

$$J_n(\tau)=\frac{1}{\tau^{m+1}}[\frac{\ln^n\tau}{n!}+o(\ln^n\tau)].$$

Appendix C

Let us demonstrate now some details of calculation of the second Born term of the amplitude by the method indicated above:

$$T_2=\frac{g^2}{4}\iiint_0^1 d\gamma_1 d\gamma_2 d\beta\frac{\delta(\gamma_1+\gamma_2+\beta-1)}{[F_1\Delta^2+h_1-i\epsilon]^{3/2}}$$

$$F_1=\gamma_1\gamma_2-\frac{1}{2(1-z)}\beta^2$$

$$h_1=\sigma^2(\gamma_1+\gamma_2)(\gamma_1+\gamma_2+\beta).$$

It is convenient to make the following change of variables

$$\gamma_1 \to \lambda \cdot \gamma_1$$

$$\gamma_2 \to (1-\lambda)\,\gamma_2 \qquad \gamma_1 \gamma_2 + \beta^2 = 1\,.$$

$$\beta \to 2\sqrt{\lambda(1-\lambda)}\,\beta$$

Then T_2 takes the form

$$T_2 = \frac{g^2 \pi^2}{4\pi^2} \iiint d\gamma_1 d\gamma_2 d\beta \cdot \delta(\gamma_1 \gamma_2 + \beta^2 - 1) \times$$

$$\times \int_0^1 d\lambda \frac{|\lambda - \frac{1}{2}|\sqrt{\lambda(1-\lambda)}}{[\lambda(1-\lambda)F_1 \cdot \Delta^2 + \tilde{h}_1 - i\epsilon]^{3/2}} \delta[\lambda\gamma_1 + (1-\lambda)\gamma_2 +$$

$$+ 2\sqrt{\lambda(1-\lambda)}\,\beta - 1]$$

$$\tilde{h}_1 = \sigma^2 [\lambda \cdot \gamma_1 + (1-\lambda)\gamma_2][\lambda \cdot \gamma_1 + (1-\lambda)\gamma_2 + 2\sqrt{\lambda(1-\lambda)}\beta - 1].$$

The integral over λ is rather easily calculated by dividing the integration region into three parts $[0,\epsilon]$, $[\epsilon, 1-\epsilon]$ and $[1-\epsilon, 1]$. In each region the integral over λ is calculated asymptotically with any accuracy.

Appendix D

To obtain an α-parameter representation of the quasipotential amplitude we apply the relation

$$\frac{1}{\prod_1^N A_i^{x_i}} = \frac{\Gamma(\sum_1^N x_i)}{\prod_1^N \Gamma(x_i)} \cdot \int_0^1 \ldots \int \prod_1^N d\alpha_i\, \alpha_i^{x_i - 1} \cdot \frac{\delta(\sum_1^N \alpha_i - 1)}{[\sum_1^N \alpha_i A_i]^{\sum_1^N x_i}}. \tag{D.1}$$

The representation for the (n+1) th Born amplitude is

$$T_{n+1}(\vec{p}\,';\vec{p}\,)=g^{n+1}\cdot\pi^{n}\cdot n!\int_0^1\!\!\dots\!\int\prod_1^n d\alpha_i d\beta_i \prod_1^{n+1} d\gamma_i \frac{1}{\prod_1^n\sqrt{\alpha_i}}\times$$

$$\times\delta\Big(\sum_1^n \alpha_i+\sum_1^n\beta_i+\sum_1^{n+1}\gamma_i-1\Big)\frac{C_n^{n-1/2}}{(f_n\cdot p^2+g_n\cdot\Delta^2+h_n-i\epsilon)^{n+1}};\qquad \text{(D.2)}$$

where all values of the parameters α_i , β_i , γ_i are shown in *Fig. 2*.

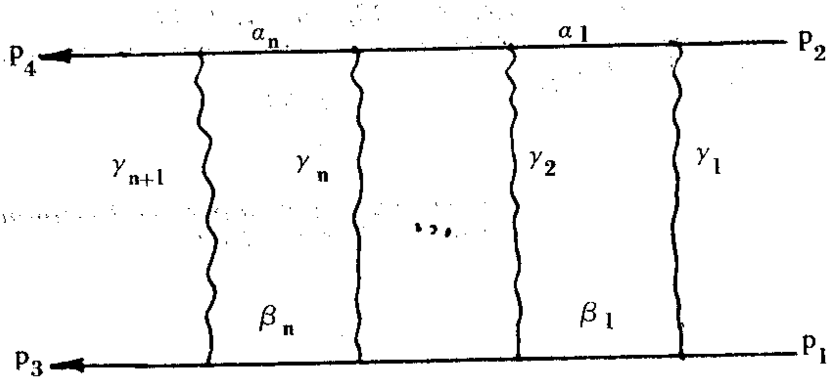


Fig. 2

With the help of the notations introduced one can obtain C_n, f_n, g_n, h_n, (cf. with the case of the Lippmann-Schwinger equation).

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Received by Publishing Department on November 17, 1976.