ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E2 - 10223

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DYNAMICS OF HEAVY SPHERICALLY-SYMMETRIC PULSONS

1976

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Submitted to "Письма в ЖЭТФ"

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E2 - 10223

Динамика сферически-симметричных пульсонов большой амплитуды

Изучены формирование и дальнейшая эволюция слабоизлучающих сферически-симметричных пульсонов большой амплитуды в рамках синус-уравнения Гордона и уравнения для поля Хиггса, время жизни которых $\tau \sim 10^3$. Обсуждаются n-узловые осциллирующие решения этих уравнений и определяемый ими спектр масс скалярных частиц.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

Препринт Объединенного института ядерных исследований
Дубна 1976

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E2 - 10223

Dynamics of Heavy Spherically-Symmetric Pulsons

Large amplitude long-lived ($\tau \sim 10^3$) stable spherically-symmetric "pulsons" are found to exist. There are three characteristic stages of their evolution. Oscillating solitons with n nodes are obtained defining the spectrum of masses of extended scalar "particles".

The investigation has been performed at the Labotarory of Computing Techniques and Automatization, JINR.

Preprint of the Joint Institute for Nuclear Research
Dubna 1976

Classical soliton solutions of Lorentz-invariant (LI) equations may be used for creation of the quantum theory of elementary particles [1,2]. Since spherically-symmetric (ss) stationary solitons are unstable [3], oscillating ss-solitons ("pulsons") obtained in papers [4,5] seem to be of more interest as realistic spatial models of neutral mesons. SS-pulsons may be regarded as ss-analogues of bound states of two solitons (bions) in the (x,t) case [6,7]. In this paper we study formation and the following evolution of pulsons with the large amplitude $c(t)$ of field oscillations ("heavy" pulsons) searching LI nonlinear equations:

$$u_{tt} - \Delta_{rr} u + \sin u = 0, \qquad (1)$$

$$u_{tt} - \Delta_{rr} u - u + u^3 = 0. \qquad (2)$$

Three characteristic stages of pulson evolution have been observed via computer investigation of Eq. (1) with initial data defined by the formula describing (x,t)-bion of Eq. (1) (see [6]):

$$u(r,0) = 4\,\mathrm{arctg}\left(\frac{\epsilon}{\omega}\,\mathrm{sech}\,\epsilon r\right), \quad \omega = \sqrt{1-\epsilon^2}, \qquad (3)$$

where it was chosen $\epsilon/\omega = 10$. At the first stage the formation of one-scaled field bunch having bell-like shape of oscillating function

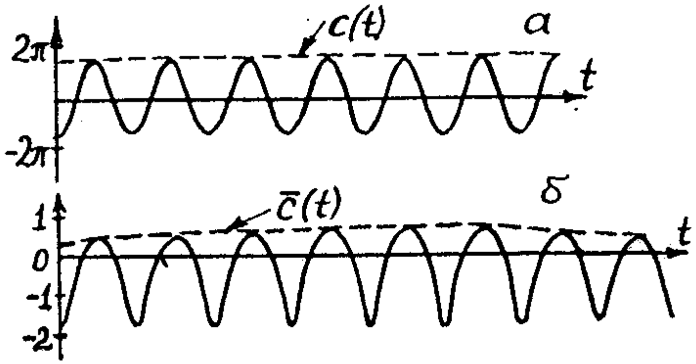

Fig. 1. Functions $u(0,t)$: a) for Eq. (1), b) for Eq. (2).

$u(r,t)$ takes place. Here more than a half of initial energy $E = \int_0^{r_{max}} 4\pi r^2 H dr$, $H = \frac{1}{2}[u_t^2 + u_r^2 + 2(1-\cos u)]$ is radiated to infinity. Practically, just from the beginning of pulson formation the quasiperiodic character of function $u(0,t)$ (fig. 1a) with period $T \simeq 7{,}4$ is observed. Then the period decreases slowly (to $T \approx 2\pi$) together with pulsation amplitude $c(t)$ decrease. At the second stage ($t \approx 200 \div 630$) the amplitude of a formed slightly radiating pulson is modulated with period $T_1 \sim 10T$ and decreases slowly from $c(t) \approx 2\pi$ to $c(t) \approx \frac{4}{3}\pi$ (Fig. 2a). The evolution of a formed pulson practically does not depend on initial data $u(r,0)$ which have led to its formation. The fact of formation of a long-lived heavy pulson of Eq. (1) from various initial data means its "stability" at amplitude range $\frac{4}{3}\pi \lessapprox c(t) \leq 2\pi$. Besides, these results allow us to suggest, that even the sine-Gordon equation which is completely integrable in the (x,t) case, has no stable non-radiating ss-solutions analo-

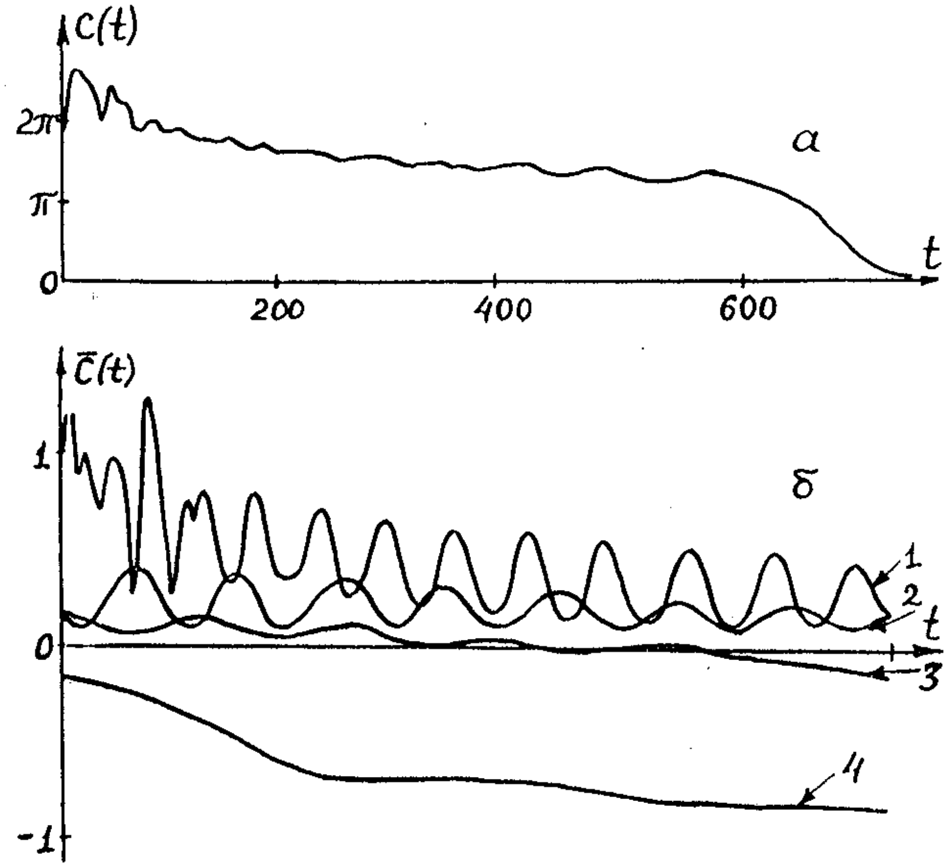

Fig. 2. Pulson amplitude vs t. a) equation (1), b) equation (2) ($1 - t = 0 \div 520$; $2 - t = 520 \div 1040$; $3 - t = 1040 \div 1560$).

gous to plane multisoliton ones (in particular, to bion solutions), see also[4].

At the third stage, beginning from $t \approx 630$ a relatively rapid ($\Delta t \sim 80$) decrease of $c(t)$ up to $c(t) \ll 1$ takes place accompanied by "swelling" of field bunch.

The pulson dynamics of Eq. (2) is rather similar to that described above. Note, however, the following differences. Function $u(0,t)$ (its period gradually decreases from $T \approx 5.3$ to $T = \sqrt{2}\,\pi$) is not of sine-type even at the second regular stage $(t \approx 100 \div 1360)$ (Fig.1b). Besides, just after the formation of slightly radiating heavy pulson of Eq. (2) the modulation amplitude of function $\bar{c}(t)$ (upper envelope of $u(0,t)$) is relatively large (see Fig. 2b). When $\bar{c}(t)$ goes to zero, it vanishes. The modulation period increases monotonously from $T_2 \approx 8T$ at $t \approx 100$ to $T_2 \approx 17T$ at $t \approx 1200$. At the third stage $\bar{c}(t)$ changes during $\Delta t \sim 300$ from $\bar{c}(t)=0$ at $t \sim 1360$ to $\bar{c}(t) \approx -0{,}8$, where oscillations become practically symmetric with respect to vacuum $u_v = -1$.

Stretching the method developed in[8] to ss-geometry we obtain the denumerable set of small amplitude pulsons of Eq. (1) (for details see[5])

$$u = u_m A_i(\epsilon r)/A_i(0)\cdot\cos(\sqrt{1-\epsilon^2}\,t) + O(\epsilon^3),$$
$$u_m = \sqrt{8}\,\epsilon A_i(0), \quad u_m^2 \ll 1, \qquad (4)$$

and of Eq. (2)

$$v = 1 + u = v_m A_i\left(\sqrt{\frac{2}{1+\epsilon^2}}\,\epsilon r\right)/A_i(0)\cdot\sin\omega t + O(\epsilon^2),$$
$$v_m = \frac{2}{\sqrt{3}}\,\epsilon A_i(0) \ll 1, \quad \omega = \sqrt{\frac{2}{1+\epsilon^2}}\,. \qquad (5)$$

Here $A_i(r)$ are $(i-1)$-node solutions[9] of equation

$$A_{rr} + \frac{2}{r} A_r - A + A^3 = 0. \qquad (6)$$

Masses $m_i = E_i$ of pulsons (4), (5) for different i relate at equal amplitudes $u_m(v_m)$ when

$u_m \to 0\ (v_m \to 0)$ as $I_i A_i^{-1}(0)$ $(\approx 1:2:3:4:9:..)$ where $I_i = \int_0^\infty A_i^2(r) r^2 dr$ (compare[5]). At $u_m \to 0 (v_m \to 0)$ distributions $H_i(r)$ of pulsons (4), (5) are stationary, unlikely heavy pulsons having essentially non-stationary $H(r,t)$ (Fig. 3).

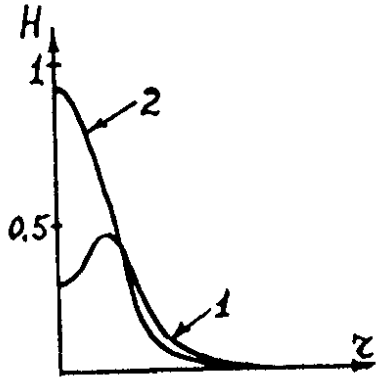

Fig. 3. Distribution $H(r)$ of Eq. (2) pulson. 1) $t=310$, 2) $t=370$.

Besides heavy pulsons without nodes described above, we suppose the existence of long-lived heavy pulsons having nodes of field function $u(r,t)$ $(v(r,t))$.

The instability of pulsons of Eqs.(1), (2) at amplitudes smaller than some critical one (the third stage of heavy pulson evolution) may be explained qualitatively by dominating attraction of pulson by the vacuum distinguished by the boundary conditions.

We should underline that pulsons of Eqs. (1), (2) turn out to be stable with respect to angular perturbations.

The authors consider their pleasant duty to thank I.S.Shapiro for his stimulating interest to this work, B.S.Getmanov and A.E.Kudryavtsev for useful remarks.

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Received by Publishing Department
on November 15 1976.