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ELECTROMAGNETIC CORRECTIONS TO DEEP INELASTIC LEPTON-NUCLEON SCATTERING AT HIGH ENERGIES

II. Corrections to Continuous Spectrum

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II. Corrections to Continuous Spectrum

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Электромагнитные поправки к глубоконеупругому лептон-нуклонному рассеянию при высоких энергиях.
II. Поправки к непрерывному спектру

Рассматриваются электромагнитные поправки к непрерывному спектру в глубоконеупругом мюон-нуклонном рассеянии при высоких энергиях. Сравниваются результаты вычисления электромагнитных поправок по формулам, найденным в данной работе, с результатами, полученными по формулам Peaking Approximation.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований
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Akhundov A.A., Bardin D.Yu.,
Shumeiko N.M. **E2 - 10205**

Electromagnetic Corrections to Deep Inelastic Lepton-Nucleon Scattering at High Energies.
II. Corrections to Continuous Spectrum

Electromagnetic corrections to continuous spectrum in the deep inelastic muon-nucleon scattering at high energies are analysed. The results of calculations by formulae obtained are compared with the ones by formulae derived in the Peaking Approximation.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research
Dubna 1976

Introduction

This paper continuous our preliminary study of electromagnetic corrections (EC) to the lepton current in the deep inelastic lepton-nucleon scattering at high energies initiated by scattering experiments planned at SPS at 50-280 GeV. Preceding paper [1] has dealt with the contribution of the elastic radiative tail to the measured cross section. Here we consider EC to the continuous spectrum.

As is known [2-4], the electromagnetic correction procedure is tightly related to the analysis of concrete experimental data. Prior to experiment, it is possible only to estimate EC that can be performed by using appropriate weighted average data presently available. We apply the experimental results on hadron electroproduction as these are rather vast and well-analysed (see,e.g., [4-6]).

To calculate EC at high energies, it is necessary, in general, to know the behaviour of structure functions in that region. Prior to experiment such an information is, clearly, absent. Therefore we have forced to extrapolate the electroproduction data to the necessary kinematical region.

This paper deals mainly with obtaining exact formulae for the lower-order EC by the method from ref. [7]. To calculate numerically magnitudes and gross features of EC in the deep inelastic region we use a fit of the electroproduction data [8] within a parton model [9].

In a subsequent paper, to obtain more realistic results we shall take a fit based on the analysis of the electroproduction data. Furthermore, we shall take account of a contribution of the radiative tail from the resonance region, the results of analysis of world data performed in paper [6] being used.

Let us discuss now main features of the calculation of EC to the continuous spectrum.

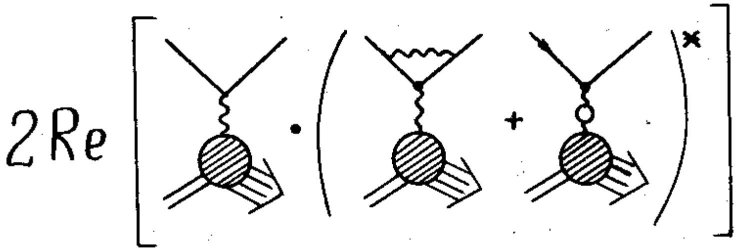
In inclusive-type experiments the process

$$\ell + N \to \ell + \gamma + \text{hadrons} \qquad (1)$$

cannot be distinguished from the main reactions

$$\ell + N \to \ell + \text{hadrons} \qquad (2)$$

therefore the radiative tail from continuous spectrum (1) gives a contribution (R -contribution) to the observed cross section of deep inelastic scattering. Apart from diagram in Fig. 1 corresponding to process (1), also diagrams with virtual-photon exchange give a contribution (V -contribution) to the inclusive cross section of deep inelastic ℓN - scattering in order α^3. The latter can be represented in the following symbolic form:



(3)

The sum of R- and V-contribution to the EC to the continuous spectrum is free of the infrared divergence present in each of the contributions. A standard regularization is

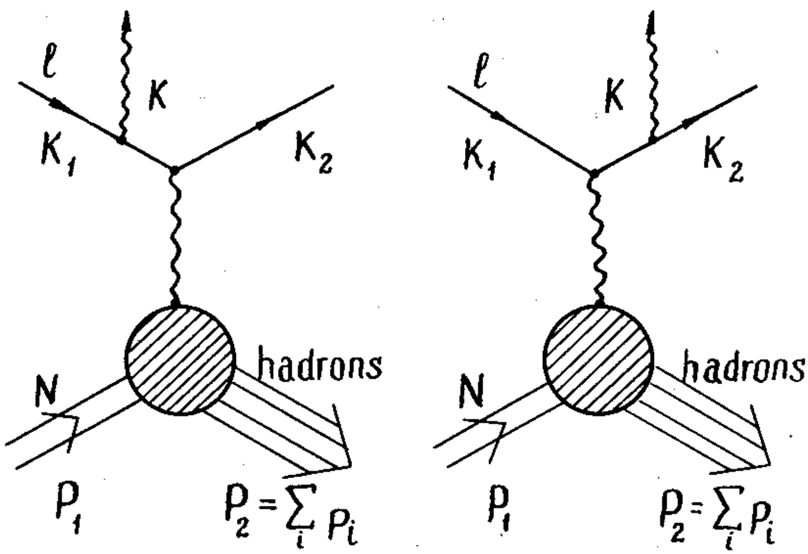


Fig. 1. Diagrams giving the main contribution to process (1).

performed by introducing an infinitesimal photon mass λ . Cancelling λ is usually achieved by splitting the R-contribution into two parts: that from soft photons ($k_0 \leq \Delta$), S -contribution, and the one from hard photons ($k_0 > \Delta$) , H-contribution.

A similar procedure has been performed in paper by L.M.Mo and Y.S.Tsai [2]. Formulae obtained there (see (B.6)-(B.8)) naturally depend on Λ that breaks their relativistic invariance.

We calculate EC to the continuous spectrum without introducing the "softness" parameter Λ and in the Lorentz-invariant form. To separate the infrared-divergent part of the cross section we apply a procedure described in paper [7]. We also do not neglect the lepton mass m in order to have a possibility to calculate EC in the whole kinematical region.

Section 2 describes the kinematics of process (1). In section 3 the inclusive cross section of (1) is calculated. In section 4 we remove the infrared divergence. In section 5, with the scaling hypothesis, the finite part of the cross section of process (1) is found in the form of a single integral. A discussion of results completes the paper.

2. Kinematics of the Process

Reaction (1) is characterized by six independent invariants which in the problem under consideration are taken as follows

$$S=-2p_1k_1\,,\ X=-2p_1k_2\,,\ Y=\Lambda^2,\ T=-2p_1q\,,\ t=q^2,\ z=-2k_2k, \quad (4)$$

where

$$\Lambda = k_1 - k_2, \quad q = p_2 - p_1 .$$

Let us introduce the λ -functions

$$\lambda_S \equiv \lambda(-(p_1 + k_1)^2, -p_1^2, -k_1^2) = S^2 - 4m^2 M^2 ,$$

$$\lambda_X \equiv \lambda(-(p_1 + k_2)^2, -p_1^2, -k_2^2) = X^2 - 4m^2 M^2 ,$$

$$\lambda_Y \equiv \lambda(-(p_1 + \Lambda)^2, -p_1^2, -\Lambda^2) = S_X^2 + 4M^2 Y , \qquad (5)$$

$$\lambda_t \equiv \lambda(-(p_1 + p_2)^2, -p_1^2, -p_2^2) = T^2 + 4M^2 t ,$$

$$\lambda_k \equiv \lambda(-(p_1 + k)^2, -p_1^2, -k^2) = (S_X - T)^2 ,$$

where M is the nucleon mass; $S_X = S - X$.

On changing to variables (4) the phase space element of process (1) becomes

$$d\Gamma = \frac{d\vec{k}_2}{2k_{20}} \frac{d\vec{k}}{2k_0} \prod_i \frac{d\vec{p}_i}{2p_{i0}} \delta(k_2 + k + \sum_i p_i - k_1 - p_1) =$$

$$= \frac{\pi}{4\sqrt{\lambda_S}} dX dY \frac{dT dt dz}{\sqrt{R_z}} d\Gamma_h . \qquad (6)$$

Here $d\Gamma_h = \Pi_i \frac{d\vec{p}_i}{2p_{i0}} \delta(p_2 - \Sigma_i p_i)$ is the phase space element of a finite hadron system; $R_z = A_z z^2 + 2B_z z + C_z$ is the positive definite quadratic trinomial

$$A_z = -\lambda_Y, \; B_z = 2M^2 Y(Y-t) + X(S_X t - YT) + SY(S_X - T), \qquad (7)$$
$$-C_z = [Xt - Y(S-T)]^2 + 4m^2[(S_X - T)(S_X t - YT) - M^2(t-Y)^2].$$

with the discriminant

$$D_z = B_z^2 - A_z C_z = \frac{1}{64M^4}\lambda(\lambda_S, \lambda_X, \lambda_Y)\cdot(\lambda_Y, \lambda_t, \lambda_k). \qquad (8)$$

In the physical region of invariants X and Y [1]

$$\lambda(\lambda_S, \lambda_X, \lambda_Y) \le 0, \; \lambda(\lambda_Y, \lambda_t, \lambda_k) \le 0, \; \text{i.e.,} \; D_z \ge 0.$$

Because of the positive definiteness of z and $(-A_z)$ we have that $B_z \ge 0$, $-C_z \ge 0$.

To calculate the lepton inclusive spectrum of reaction (1) it is necessary to establish the physical region of variables T, t and z at fixed S, X and Y.

One can easily obtain [10] the physical z-region

$$z_{min,max} = (-B_z \pm \sqrt{D_z})/A_z . \qquad (9)$$

Invariants T and t are limited by the conditions

$$0 \le T - t \le W^2 - M^2 , \qquad (10)$$

$$\lambda(\lambda_Y, \lambda_t, \lambda_k) \le 0, \qquad (11)$$

where $W^2 = -(p_1+\Lambda)^2 = M^2 + S_X - Y$ is the squared invariant mass of a nondetected system of particles.

Inequality (10) follows from the relations for limiting values of the squared invariant mass of finite hadrons $M_f^2 = -(p_1+q)^2 = M^2 + T - t$: $M^2 \le M_f^2 \le W^2$. Inequality (11) is the condition of existence of the triangle of momenta $\vec{\Lambda}$, $\vec{p}_2$ and $\vec{k}$ of process (1) in the lab. system /1/.

The boundary $\lambda(\lambda_Y, \lambda_t, \lambda_k) = 0$ is the pair of intersecting straight lines

$$t_{1,2} = Y + \frac{1}{2M^2}(S_X \pm \sqrt{\lambda_Y})(S_X - T). \qquad (12)$$

Thus the region of T and t we are looking for is placed between lines (12) and t = T (Fig. 2).

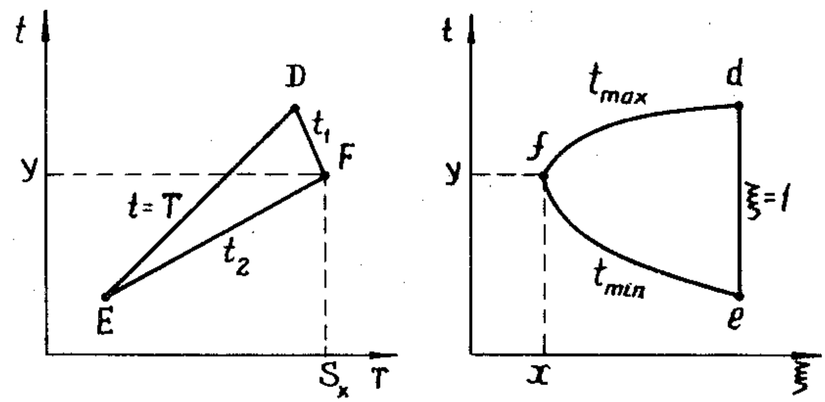


Fig. 2. Physical region of invariants T, t and ξ, t at fixed S, X and Y.

The coordinates of points D, E and F equal

$$T_{D,E} = t_{D,E} = \frac{1}{2W^2}[(W^2 - M^2)(S_X \pm \sqrt{\lambda_Y}) + 2M^2 Y]; \; T_F = S_X, t_F = Y.$$

For convenience, further we shall replace the set of invariants T and t by $\xi = t/T$ and t with the physical region restricted by lines

$$t_{max,min} = [V(S_X \pm \sqrt{\lambda_Y}) + 2M^2 Y]/2(\frac{V}{\xi} + M^2) \text{ and } \xi = 1, \qquad (13)$$

where $V = S_X - Y/\xi$ (Fig. 2). The abscissa of point f is $\xi_f = Y/S_X$.

Note that discriminant D_z is zero at $t_{max,min}$. This permits the procedure of analytic integration over t to be made essentially simpler.

Coefficients C_z and B_z in variables ξ and t can be written in the form

$$-C_z = At^2 + 2Bt + C, \; B_z = Et + F, \qquad (14)$$

where

$$A = (X + \frac{Y}{\xi})^2 - 4m^2(\frac{V}{\xi} + M^2),$$

$$B = VB_0 - Y\lambda_S, \quad C = Y^2\lambda_S,$$

$$E = XV - Y(\frac{S}{\xi} + 2M^2), F = YF_0, B_0 = SY + 2m^2 S_X,$$

$$F_0 = SS_X + 2M^2 Y.$$

3. Inclusive Cross Section

The differential cross section of the nonpolarized particle scattering (1) can be written as follows

$$d\sigma_R = \frac{\alpha^3 M}{\pi \lambda_S} S_{\mu\nu} W_{\mu\nu}\, dX dY \frac{dT dt dz}{t^2 \sqrt{R_z}} . \tag{15}$$

Here $S_{\mu\nu}$ is the lepton tensor /10/, $W_{\mu\nu}$ the hadron tensor:

$$W_{\mu\nu} = (2\pi)^6 \frac{p_{10}}{M} \frac{1}{2} \Sigma \int \langle p_1 | J_\mu | p_2 \rangle \langle p_2 | J_\nu | p_1 \rangle d\Gamma_h =$$

$$= W_1 (\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + \frac{1}{M^2} W_2 (p_{1\mu} - \frac{p_1 q}{q^2} q_\mu)(p_{1\nu} - \frac{p_1 q}{q^2} q_\nu), \tag{16}$$

and W_1 and W_2 are functions of invariants which can be composed of p_1 and q. As the latter, we take T and t.

The structure functions W_1 and W_2 in formula (16) are so normalised that the inclusive cross section of reaction (2) has the form

$$\frac{d^2\sigma_0}{dx dy} = \frac{2\pi\alpha^2 S}{\lambda_S\, xy^2} [2MW_1 \cdot y^2 (1 - \frac{2m^2}{Sxy}) + \nu_\ell W_2 \cdot \frac{2}{x} (1 - y(1 + \frac{M^2 x}{S}))], \tag{17}$$

where x and y are scaling variables: $x = Y/S_X$, $y = S_X/S$ and $\nu_\ell = S_X/2M$ is the energy loss by lepton in the lab. system.

In the contraction

$$\mathcal{S} = \frac{1}{2} M S_{\mu\nu} W_{\mu\nu} = 2 M W_1(T,t)\,\mathcal{S}_1 + \nu_h W_2(T,t)\frac{1}{T}\mathcal{S}_2, \qquad (18)$$

where $\nu_h = T/2M$ is the lab. energy transferred to the hadron system, we have

$$\mathcal{S}_1 = \frac{1}{2}\left(\frac{z_1}{z} + \frac{z}{z_1}\right) + \frac{1}{zz_1}(t^2 - 4m^4) - (t - 2m^2)\left(\frac{1}{z} - \frac{1}{z_1}\right) -$$

$$- m^2(t - 2m^2)\left(\frac{1}{z^2} + \frac{1}{z_1^2}\right);$$

$$\mathcal{S}_2 = -M^2\left(\frac{z_1}{z} + \frac{z}{z_1}\right) + \frac{1}{zz_1}[t(S(S-T) + X(X+T) - 2M^2(t+2m^2)) +$$

$$+ 2m^2((S - T)(X + T) + SX)] - \qquad (19)$$

$$- [\frac{1}{z}(XT - 2M^2 t) + \frac{1}{z_1}(ST + 2M^2 t)] -$$

$$- 2m^2[\frac{1}{z^2}(S(S - T) - M^2 t) + \frac{1}{z_1^2}(X(X + T) - M^2 t)],$$

where $z_1 = -2k_1 k = z + t - Y$.

Integrating (18), (19) over z in limits (9) gives the following expression for the invariant lepton spectrum of process (1)

$$\frac{d^2\sigma_R}{dxdy} = \frac{2\alpha^3}{\lambda_S} S^2 y \int \frac{dTdt}{t^2}\,\mathcal{S}(T,t), \qquad (20)$$

where

$$\delta(T,t)=\frac{1}{\pi}\int\frac{dz}{\sqrt{R_z}}\, S = 2MW_1(T,t)\,\delta_1(T,t)+\nu_h W_2(T,t)\frac{1}{T}\delta_2(T,t);$$

$$\delta_1(T,t)=\Big\{\frac{1}{\sqrt{-C_z}}\Big[\frac{t-Y}{2}+\frac{(t-2m^2)(Y+2m^2)}{t-Y}\Big]-m^2(t-2m^2)\frac{B_z}{(-C_z)^{3/2}}\Big\}-$$

$$-\{S \leftrightarrow -X\}+\frac{1}{\sqrt{\lambda_Y}},$$

$$\delta_2(T,t)=\Big\{\frac{1}{\sqrt{-C_z}}[M^2(t+Y)-XT]+$$

$$+\frac{1}{(t-Y)\sqrt{-C_z}}[t(S(S-T)+X(X+T)-2M^2(t+2m^2))+ \qquad (21)$$

$$+2m^2((S-T)(X+T)+SX)]-2m^2\frac{B_z}{(-C_z)^{3/2}}[S(S-T)-M^2t]\Big\}-$$

$$-\{S \leftrightarrow -X\}-\frac{2M^2}{\sqrt{\lambda_Y}}.$$

The obtained expression (20) is consistent with analogous formula (38) of paper [1]. Really, the structure functions $W_1(T,t)$ and $W_2(T,t)$ are related to the charge $G_E(t)$ and magnetic $G_M(t)$ nucleon form factors as follows

$$2MW_1(T,t) = tG_M^2(t)\,\delta(T-t),$$

$$\frac{W_2(T,t)}{2M} = \frac{G_E^2(t) + \tau G_M^2(t)}{1+\tau}\,\delta(T-t), \qquad (22)$$

where $\tau = t/4M^2$. The latter and relations (20), (21) produce formula (38) of /1/.

4. Removal of Infrared Divergence

Formula (20) defines the contribution of the radiative tail from continuum to the observed cross section of deep inelastic ℓN -scattering. It is a sum of all radiative tails of finite hadron states with $M_f^2 \le W^2$. At $M_f^2 = W^2$ $T = S_X$ and $t = Y$ (this corresponds to point F in Fig. 2), i.e., the momentum of the real photon $\sqrt{\lambda_k}/2M = 0$ and $\delta(T,t)$ becomes infinite (see formulae (21) and (7)). Thus, integral (20) diverges at point F (the well known infrared divergence). To remove the latter, it is necessary to consider the contribution to the inclusive cross section from diagrams with the virtual photon exchange, as well (formula (3)). The sum of R- and V-contributions is free of that divergence.

Now let us separate from $d^2\sigma_R/dxdy$ the infrared divergent part /7/. Consider the identity

$$\frac{d^2\sigma_R}{dxdy} = \frac{d^2\sigma_R}{dxdy} - \frac{d^2\sigma_R^{IR}}{dxdy} + \frac{d^2\sigma_R^{IR}}{dxdy} \equiv \frac{d^2\sigma_R^{F}}{dxdy} + \frac{d^2\sigma_R^{IR}}{dxdy}, \qquad (23)$$

where $d^2\sigma_R^{IR}/dxdy$ is the infrared-divergent part and $d^2\sigma_R^F/dxdy$ is the convergent part (i.e., the photon mass λ may be neglected in the latter part). On passing in Eq. (20) to the variable ξ and using a method [7] for constructing $d^2\sigma^{IR}/dxdy$ we obtain

$$\frac{d^2\sigma_R^{IR}}{dxdy} = \frac{2\alpha^3}{\lambda_S} S^2 y \int_{\xi_{min}}^{1} \frac{d\xi}{x^2} J^\lambda \left[\frac{\mathcal{S}^{IR}}{Y} \right], \tag{24}$$

where

$$\xi_{min} = x\left(1 - \frac{2\lambda M}{S_X}\right); \quad J^\lambda[A] \equiv \frac{1}{\pi} \int \frac{dtdz}{\sqrt{R_z}} A; \tag{25}$$

$$\mathcal{S}^{IR} = 2MW_1(x,Y)\mathcal{S}_1^{IR} + \nu_\ell W_2(x,Y)\frac{x}{Y}\mathcal{S}_2^{IR};$$

$$\mathcal{S}_1^{IR} = (Y - 2m^2)F^{IR}, \quad \mathcal{S}_2^{IR} = 2(SX - M^2 Y)F^{IR}; \tag{26}$$

$$F^{IR} = \frac{Y + 2m^2}{zz_1} - m^2\left(\frac{1}{z^2} + \frac{1}{z_1^2}\right).$$

By applying (26) and (17) formula (24) may be written as

$$\frac{d^2\sigma_R^{IR}}{dxdy} = \frac{d^2\sigma_0}{dxdy} \cdot \frac{\alpha}{\pi} \frac{Y}{x^2} \int_{\xi_{min}}^{1} d\xi J^\lambda [F^{IR}]. \tag{27}$$

Thus in the cross section $d^2\sigma_R^{IR}/dxdy$ we factorize the cross section of nonradiative process (2). Calculating integrals $J^\lambda[\frac{1}{zz_1}]$, $J^\lambda[\frac{1}{z^2}]$ and $J^\lambda[\frac{1}{z_1^2}]$ allowing for the infinitesinal photon mass λ we get

$$J^\lambda[\frac{1}{zz_1}] = \frac{\xi}{\sqrt{C_m^\lambda}}\ln\frac{v^2(Y+2m^2)+v_{min}^2 a+\sqrt{C_m^\lambda}\sqrt{v^2-v_{min}^2}}{v^2(Y+2m^2)+v_{min}^2 a-\sqrt{C_m^\lambda}\sqrt{v^2-v_{min}^2}}, \quad (28)$$

$$J^\lambda[\frac{1}{z^2}] = \frac{\xi\sqrt{v^2-v_{min}^2}}{m^2v^2+\lambda^2\lambda_S x^2}, \quad J^\lambda[\frac{1}{z_1^2}] = \frac{\xi\sqrt{v^2-v_{min}^2}}{m^2v^2+\lambda^2\lambda_X x^2}. \quad (29)$$

Here

$$v = \xi V = \xi S_X - Y, \quad v_{min} = \xi_{min} S_X - Y = 2\lambda M x;$$

$$C_m^\lambda = v^2\lambda_m + v_{min}^2 b, \quad \lambda_m \equiv \lambda(-(k_1+k_2)^2, -k_1^2, -k_2^2) = Y(Y+4m^2),$$

$$a = \frac{SX}{2M^2} - (Y+2m^2), \quad b = \frac{Y}{M^2}[SX - M^2Y - m^2(\frac{Y}{x^2} + 4M^2)]. \quad (30)$$

Changing to variable v in (27) gives

$$\frac{d^2\sigma_R^{IR}}{dxdy} = \frac{d^2\sigma_0}{dxdy}\cdot\frac{\alpha}{\pi}\delta_R^{IR}, \quad (31)$$

where

$$\delta_R^{IR} = \int_{v_{min}}^{v_{max}} dv\,(1 + \frac{v}{Y}) I^{\lambda}(X,Y,v) \tag{32}$$

and $v_{max} = S_x - Y$, , $I^{\lambda}(X,Y,v) = J^{\lambda}[F^{IR}]/\xi$.

The procedure [7] for calculating the integral of type (32) gives

$$\delta_R^{IR} = \delta_R^{\lambda} + \delta_R^{S} + \delta_R^{H} , \tag{33}$$

where

$$\delta_R^{\lambda} = J_0 \ln \frac{v_{max}}{v_{min}}, \tag{34}$$

$$\delta_R^{S} = J_0 \ln 2 + \frac{1}{2}(SL_S + XL_X) + S_{\phi}(Y + 2m^2, \lambda_m, a, b), \tag{35}$$

$$\delta_R^{H} = J_0 \frac{v_{max}}{Y} . \tag{36}$$

Here

$$J_0 = 2[(Y + 2m^2)L_m - 1], \; L_m = \frac{1}{\sqrt{\lambda_m}} \ln \frac{\sqrt{\lambda_m} + Y}{\sqrt{\lambda_m} - Y},$$

$$L_S = \frac{1}{\sqrt{\lambda_S}} \ln \frac{S + \sqrt{\lambda_S}}{S - \sqrt{\lambda_S}}, \; L_X = \frac{1}{\sqrt{\lambda_X}} \ln \frac{X + \sqrt{\lambda_X}}{X - \sqrt{\lambda_X}}, \tag{37}$$

$$S_{\phi}(s,\lambda,a,b) = \frac{s}{2\sqrt{\lambda}} \sum_{i=1}^{2} (-1)^i \sum_{j=1}^{4} \delta_j \sum_{k=1}^{2} [\Phi(\frac{\gamma_i - \gamma}{\gamma_i - \gamma_k^j}) + \Phi(\frac{\gamma + (-1)^i}{\gamma_k^j + (-1)^i})] \Big|_{\gamma_1}^{\gamma_u} , \tag{38}$$

where $\Phi(x)$ is the Spence function and parameters of function (38) are expressed as follows

$$\delta_j = \{1,1,-1,-1\};\ \gamma_{1,2} = \mp\frac{(\sqrt{b}\mp\sqrt{\lambda})^2}{b-\lambda},$$

$$\gamma^j_{1,2} = -(\alpha_j\sqrt{b}\pm\sqrt{b\alpha_j^2+\tau_j^2})/\tau_j,\ \gamma_u = (\sqrt{b+\lambda}-\sqrt{b})/\sqrt{\lambda};$$

$$\alpha_1=\alpha_2=s-\sqrt{\lambda},\ \alpha_3=\alpha_4=s+\sqrt{\lambda}, \tag{39}$$

$$\tau_{1,2} = -a\sqrt{\lambda}+\frac{1}{2}(b-\lambda)\mp\sqrt{D},\ \tau_{3,4} = -a\sqrt{\lambda}-\frac{1}{2}(b-\lambda)\mp\sqrt{D};$$

$$D = (s+a)(\lambda a - sb) + \frac{1}{4}(\lambda+b)^2.$$

The infrared divergence is fully concentrated in δ^λ_R, and δ^S_R and δ^H_R are covariant analogs of a part of contributions of soft and hard photons.

Let us now show the vanishing of the photon mass λ under summing $d^2\sigma^{IR}_R/dxdy$ with the contribution from virtual photon exchange diagrams $d^2\sigma_V/dxdy$. The latter can be represented as a sum of the infrared free part $d^2\sigma^F_V/dxdy$ and the one infrared divergent $d^2\sigma^{IR}_V/dxdy$:

$$\frac{d^2\sigma_V}{dxdy} = \frac{d^2\sigma^F_V}{dxdy} + \frac{d^2\sigma^{IR}_V}{dxdy}.$$

The part $d^2\sigma_V^{IR}/dxdy$ can be easily represented in the form /7,11/

$$\frac{d^2\sigma_V^{IR}}{dxdy} = \frac{d^2\sigma_0}{dxdy} \cdot \frac{\alpha}{\pi}\delta_V^{IR}, \tag{40}$$

where

$$\delta_V^{IR} = \delta_V^{\lambda} + \delta_V^{L}, \tag{41}$$

$$\delta_V^{\lambda} = J_0 \ln\frac{\lambda}{m}, \tag{42}$$

$$\delta_V^{L} = -(Y + 2m^2)\{L_m \ln\frac{\lambda_m}{m^2 Y} + \frac{1}{\sqrt{\lambda_m}}[\Phi(\frac{Y+\sqrt{\lambda_m}}{Y-\sqrt{\lambda_m}}) - \Phi(\frac{Y-\sqrt{\lambda_m}}{Y+\sqrt{\lambda_m}})]\}. \tag{43}$$

Clearly, the sum $\delta_R^{\lambda} + \delta_V^{\lambda}$ does not contain λ and equals

$$\delta^{\lambda} \equiv \delta_R^{\lambda} + \delta_V^{\lambda} = J_0 \ln\frac{v_{max}}{2mMx}. \tag{44}$$

The convergent part is

$$\frac{d^2\sigma_V^F}{dxdy} = \frac{d^2\sigma_0}{dxdy} \cdot \frac{\alpha}{\pi}\delta_V^F + \frac{d^2\sigma^{AMM}}{dxdy}. \tag{45}$$

Here

$$\delta_V^F = [(\frac{3}{2}Y + 4m^2)L_m - 2] +$$

$$+\{[\frac{2}{3}(Y+2m^2)L_m - \frac{10}{9} + \frac{8m^2}{3Y}(1-2m^2L_m)] + [m \to \mu]\}, \quad (46)$$

where the expression in braces corresponds to the vacuum polarization by electron and muon, and

$$\frac{d^2\sigma^{AMM}}{dxdy} = \frac{2\alpha^3 S}{\lambda_S xy^2} m^2 L_m [2MW_1 \cdot 3y^2 + \nu_\ell W_2 \cdot \frac{2}{x}(1 - y(1 + \frac{2M^2x}{S} + \frac{y}{4}))] \quad (47)$$

corresponds to the contribution of the lepton anomalous magnetic moment. The latter must be taken into account at small x.

5. Infrared Free Part of Cross Section

The infrared free part of the cross section of (1) $d^2\sigma_R^F/dxdy$ can be written in the form

$$\frac{d^2\sigma_R^F}{dxdy} \equiv \frac{d^2\sigma_R}{dxdy} - \frac{d^2\sigma_R^{IR}}{dxdy} =$$

$$= \frac{2\alpha^3}{\lambda_S} S^2 y \int_x^1 d\xi (\frac{1}{\xi^2} J[\frac{\mathcal{S}}{t}] - \frac{1}{x^2} J[\frac{\mathcal{S}^{IR}}{Y}]), \quad (48)$$

where the integrand is finite. By using formulae (28) and (29) at $\lambda = 0$ and (26) we have

$$J[\frac{\delta^{IR}}{Y}] = 2MW_1(x,Y)J[\frac{\delta_1^{IR}}{Y}] + \nu_\ell W_2(x,Y)xJ[\frac{\delta_2^{IR}}{Y^2}], \quad (49)$$

where

$$J[\frac{\delta_1^{IR}}{Y}] = \frac{1}{Y}(Y-2m^2)\frac{J_0}{V}, \; J[\frac{\delta_2^{IR}}{Y^2}] = \frac{2}{Y^2}(SX-M^2Y)\frac{J_0}{V}. \quad (50)$$

To calculate $J[\frac{\delta}{t}]$ it is necessary to establish the t-dependence of the structure functions W_1 and W_2. However, it is possible, in principle, to calculate $d^2\sigma_R^F/dxdy$ at computer by using an experimental fit of the structure functions.

Here we obtain an exact expression for $J[\frac{\delta}{t}]$ under the assumption of the Bjorken scaling for $2MW_1$ and $\nu_h W_2$ /12/:

$$2MW_1(\xi,t) = f_1(\xi), \quad \nu_h W_2(\xi,t) = \xi f_2(\xi). \quad (51)$$

In this case

$$\frac{1}{\xi^2}J[\frac{\delta}{t}] - \frac{1}{x^2}J[\frac{\delta^{IR}}{Y}] = \quad (52)$$

$$= f_1(\xi)(\frac{1}{\xi^2}J[\frac{\delta_1}{t}] - \frac{1}{x^2}J[\frac{\delta_1^{IR}}{Y}]) + f_2(\xi)(J[\frac{\delta_2}{t^2}] - J[\frac{\delta_2^{IR}}{Y^2}]) +$$
$$+ (f_1(\xi) - f_1(x))\frac{1}{x^2}J[\frac{\delta_1^{IR}}{Y}] + (f_2(\xi) - f_2(x))J[\frac{\delta_2^{IR}}{Y^2}].$$

Calculating differences in the first two terms of (52) gives

$$\frac{1}{\xi^2} J\left[\frac{\delta_1}{t}\right] - \frac{1}{x^2} J\left[\frac{\delta_1^{IR}}{Y}\right] = \left\{\frac{1}{2\xi^2}(L_t + L_A) - \frac{1}{2Y^3}\left(S_X + \frac{Y}{\xi}\right)(Y - 2m^2)J_0 - \right. \tag{53}$$

$$\left. - \frac{2m^2 B_0}{\xi^2 Y^2 \lambda_S} + \frac{1}{\xi^2}\left[-\frac{1}{2} + \frac{2m^2}{Y} + \frac{2m^4}{Y^2}\left(2 + \frac{F_0}{\lambda_S}\right)\right] L_S \right\} + \{S \leftrightarrow -X\};$$

$$J\left[\frac{\delta_2}{t^2}\right] - J\left[\frac{\delta_2^{IR}}{Y^2}\right] - \left\{-\frac{V}{Y^2} + \frac{1}{Y}\left(S_X + \frac{2m^2}{\xi}\right) L_m + \right.$$

$$+ \frac{2}{Y^2 \lambda_S}\left[\frac{VF_0}{2}\left(1 - \frac{4m^2 SX}{M^2 Y^2}\right) - \frac{m^2 S^2 V}{Y^2}\left(\frac{\lambda_Y}{M^2} + \frac{3d}{\lambda_S}\right) + \right. \tag{54}$$

$$\left. + S(SX - M^2 Y) + 2m^2 (S + X)\left(\frac{S}{\xi} + M^2\right)\right] -$$

$$- \left[\frac{1}{Y^2 \lambda_S}\left(M^2 V B_0\left(1 - \frac{4m^2 SX}{M^2 Y^2}\right) + \frac{2m^2 S^2}{Y}\left(E - \frac{3BF_0}{Y\lambda_S}\right) - 2m^2 F_0\left(\frac{S}{\xi} + M^2\right)\right) + \right.$$

$$\left. \left. + \frac{1}{Y}\left(\frac{X}{\xi} - 2M^2\right) + \frac{1}{Y^2}\left(\lambda_S + X^2 + 2m^2 \frac{S_X}{\xi}\right) + \frac{8m^2 SX}{Y^3}\right] L_S \right\} + \{S \leftrightarrow -X\},$$

where

$$L_t = \frac{1}{\sqrt{\lambda_Y}} \ln \frac{V(S_X + \sqrt{\lambda_Y}) + 2M^2 Y}{V(S_X - \sqrt{\lambda_Y}) + 2M^2 Y},$$

$$L_A = \frac{1}{\sqrt{A}} \ln \frac{X + \frac{Y}{\xi} + \sqrt{A}}{X + \frac{Y}{\xi} - \sqrt{A}},$$

$$d = 4[m^2\Lambda_Y - Y(SX - M^2Y)] .$$

The sum of two last terms in (52) is

$$(f_1(\xi) - f_1(x))\frac{1}{x^2}J[\frac{\delta_1^{IR}}{Y}] + (f_2(\xi) - f_2(x))J[\frac{\delta_2^{IR}}{Y^2}] = \quad (55)$$

$$= \frac{\xi J_0}{Sx^2y^3}[\Lambda_1(x,\xi)\cdot y^2(1-\frac{2m^2}{Sxy}) + \Lambda_2(x,\xi)\cdot 2(1-y(1+\frac{M^2x}{S}))] ,$$

where

$$\Lambda_{1,2}(x,\xi) = \frac{f_{1,2}(\xi) - f_{1,2}(x)}{\xi - x} .$$

The contribution of (55) to integral (48) is quite large at small x. However, it is completely compensated by the contribution of δ_R^H. Therefore it is convenient to make integration over ξ just for the sum of these terms.

6. Discussion of Results

By formulae derived in sects. 4 and 5 we have calculated numerically EC in the region $W > 2$ GeV to the cross section of the process

$$\mu + p \to \mu + \text{hadrons} \quad (56)$$

at muon energy E = 150 and 250 GeV. The results are presented in Figs. 3 and 4 (solid lines), where

$$\delta(x,y) \equiv \left(\frac{d^2\sigma_R}{dxdy} + \frac{d^2\sigma_V}{dxdy}\right) / \frac{d^2\sigma_0}{dxdy}. \qquad (57)$$

For the structure function we use a fit [9] within a parton model, i.e.,

$$f_1(x) = f_2(x) = f(x), \qquad (58)$$

where $f(x)$ is a fitted function.

As is seen from the Figures, the EC varies from -0.2 at $x \sim 1$ up to a large positive value at $x \sim 0$. The EC grows also as $y \to 1$ because of the large probability of the photon emission at large lepton energy losses. This growth is most prominent at small x. Besides, with growing E the value of $|\delta(x,y)|$ increases at fixed x and y.

Our consideration of EC is restricted to the region of x and y where

$$|\delta(x,y)| < 0.3,$$

i.e., where the EC are not too large. Outside this region the accuracy of the performed calculations may be insufficient and the consideration of higher-order EC will be required.

Prior to the quantitative study of that problem no interpretation should be made for data outside the region of x and y indicated above.

The boundary of this region should be defined in each experiment by $\delta(x,y)$ calculat-

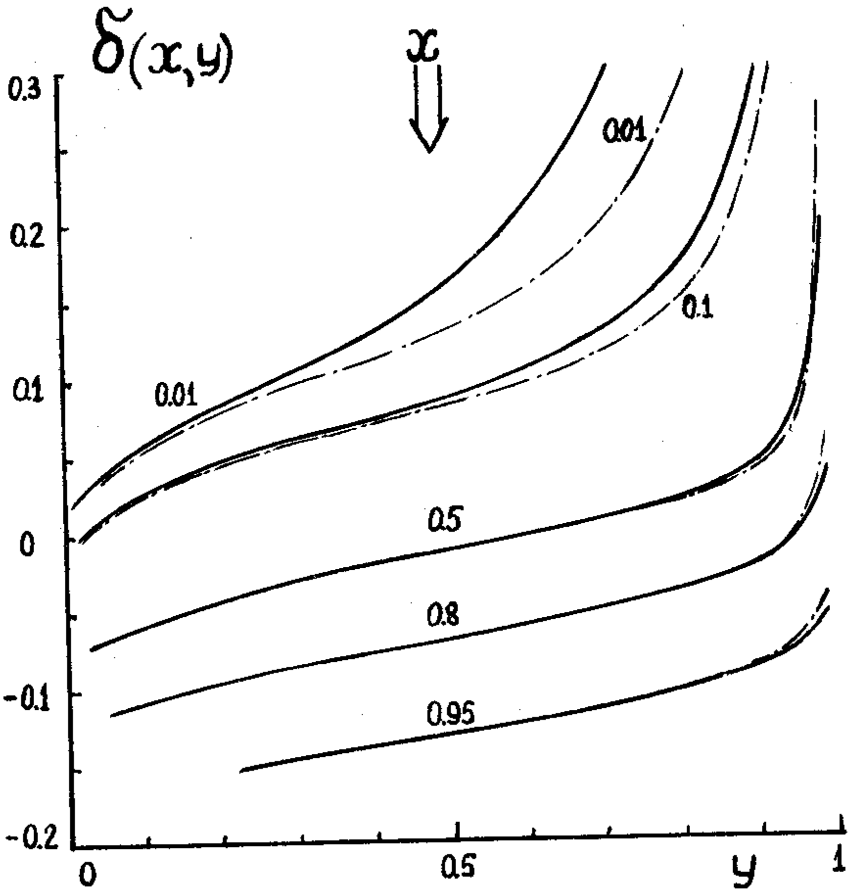


Fig. 3. Electromagnetic corrections to process $\mu + p \rightarrow \mu$ + hadrons at E = 150 GeV.

ed on the basis of measured functions $2MW_1$ and νW_2. The boundary in the Figures (the end of solid lines at $\delta(x,y) = 0.3$) may be considered as an illustration.

Now let us compare our calculations of EC to process (56) with the one by the often

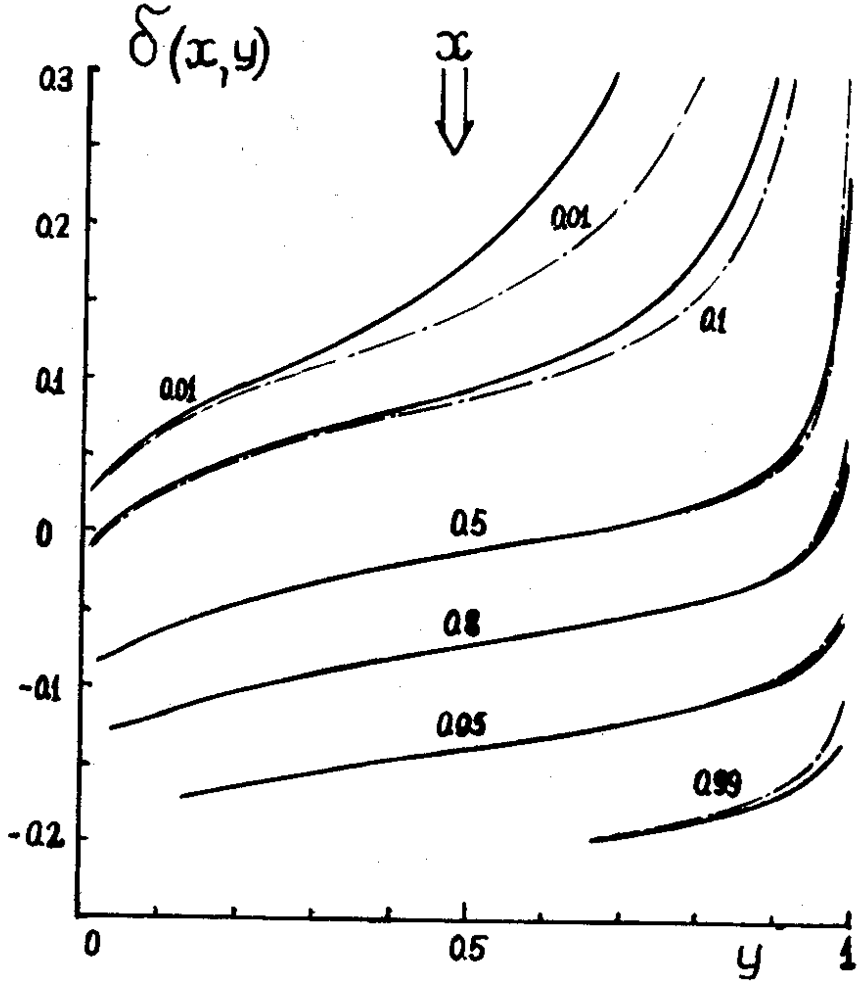


Fig. 4. Electromagnetic corrections to process $\mu + p \rightarrow \mu$ + hadrons at E = 250 GeV.

used approximate formula (C7) of paper /2/ derived in the Peaking Approximation (PA) (dash-dotted lines in Figs. 3 and 4). In the latter calculation the parameter Δ =15 MeV was taken.

As is clear from Figures, two calculations are consistent in a large region of x and y The PA produces incorrect results only for $x \leq 0.01$ and $y \geq 0.95$. Besides, at small x and y the PA does not work at all: formula (C7) gives negative values (here dash-dotted lines are cut off).

This, in general, quite good agreement, in contrast to the case of elastic radiative tail [1], is due to dominating the soft photon (for which the PA works well) contribution to the continuum radiative tail, while to the elastic radiative tail the main contribution comes from hard photons. Inapplicability of PA in the above regions is also the consequence of the large hard photon contribution.

In the (x,y) region, where the PA does not work, the standard procedure for infolding the experimental data [2] is inapplicable,as being based essentially on the PA formulae. To obtain information on the structure functions in this region requires some procedure based on exact formulae, i.e., an iterative one.

In conclusion we note that we have obtained the cross section $d^2\sigma_R^F/dxdy$ as a single integral over ξ in the approximation of the Bjorken scaling. Since the scaling breaking, probably, is not large [13], this approximation may turn out to be sufficient for calculations of the EC to continuous spectrum in the region $W > 2$ GeV. However, the formulae derived allow one, in principle, to calculate EC also for any known structure functions W_1 and W_2 by means of numerical integration of (48) over the two-dimensional (T,t) region (see Fig. 2). Such an integration is necessary, e.g., in exact calculating the contribution of resonance radiative tail to the measured cross section. These questions will be discussed in a subsequent paper.

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