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A COVARIANT FORMULATION OF A RELATIVISTIC HAMILTONIAN THEORY ON THE LIGHT CONE

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S U M M A R Y

In the covariant formulation of a relativistic Hamiltonian theory, given in refs./1-3/, the momenta of all physical particles belong to the mass-shell. In the vertices of diagrams the total 4-momentum of physical particles and the additional spurion line is conserved. The momentum of the spurion is proportional to the constant time-like vector on which the on-shell S-matrix does not depend.

In the present paper a version of a relativistic Hamiltonian theory is developed in which the 4-momentum of spurion lies on the light cone. The essentially new point in this scheme is a specific regularization, by which the singularities accompanying each field theory in the light-front variables are removed in all diagrams. The unitarity and causality conditions are analysed in detail. It is shown that the choice of regularization is defined by the causality condition.

I. INTRODUCTION

About 10 years ago in papers /1-3/ a covariant formulation of a relativistic Hamiltonian theory was proposed. The scheme is based on the Tomonaga-Schwinger equation for the scattering matrix $S(\sigma,-\infty)$, defined on the space-like surfaces of the type:

$$\lambda x = \lambda_0 x_0 - \vec{\lambda}\vec{x} = \sigma, \qquad (1.1)$$

where λ is the fixed time-like vector

$$\lambda^2 = 1, \quad \lambda_0 > 0. \qquad (1.2)$$

The Tomonaga-Schwinger equation has the form

$$i\frac{\partial S(\sigma,-\infty)}{\partial\sigma} = H(\sigma,\lambda)S(\sigma,-\infty), \qquad (1.3)$$

where the quantity $H(\sigma,\lambda)$ is connected with the interaction Hamiltonian $H(x)$ by the Radon transformation

$$H(\sigma,\lambda) = \int H(x)\delta(\sigma-\lambda x)d^4x. \qquad (1.4)$$

We can also express the operator $H(\sigma,\lambda)$ in terms of the Fourier-transform of the Hamiltonian

$$\widetilde{H}(p) = \int e^{-ipx}H(x)d^4x \qquad (1.5)$$

with the help of the relation

$$H(\sigma,\lambda) = \frac{1}{2\pi}\int e^{-i\sigma\varkappa}\widetilde{H}(\lambda\varkappa)d\varkappa, \qquad (1.6)$$

where $\varkappa$ is the one-dimensional invariant parameter.

In this paper we shall consider the Hamiltonian

$$H(x) = -g:\varphi^3(x): \quad , \qquad (1.7)$$

where $\varphi(x)$ is the operator of a scalar field in the interaction representation. The total scattering matrix $S(\infty,-\infty) = \lim_{\sigma\to\infty} S(\sigma,-\infty)$ has the form

$$S(\infty,-\infty)=T_\lambda \exp\left\{-i\int H(x)d^4x\right\}=$$

$$=1+\sum_{n=1}^{\infty}(-i)^n\int\theta(\lambda x_1-\lambda x_2)\cdots\theta(\lambda x_{n-1}-\lambda x_n)H(x_1)\cdots H(x_n)d^4x_1\cdots d^4x_n. \qquad (1.8)$$

In fact the S-matrix does not depend on λ, because in the time-like region we have always $\theta(\lambda x)=\theta(x^0)$, and in the space-like region the Hamiltonian ordering has no sense due to the locality condition

$$[H(x),H(y)]=0\,, \quad \text{when } (x-y)^2<0\,. \qquad (1.9)$$

The independence of the S-matrix of λ follows from the invariance of the sign of time in the time-like region. In fact, we can understand the substitution $x^0\longrightarrow\lambda x$, as the (proper) Lorentz transformation. The T_λ-ordering coincides with the usual T-ordering.

Let us put

$$S(\sigma,-\infty)=1+\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{R(\lambda æ)e^{iæ\sigma}}{æ-i\varepsilon}dæ, \qquad (1.10)$$
$$S(\infty,-\infty)=1+iR(0).$$

It is easily seen that the operator $R(\lambda æ)$ obeys the following integral equation

$$R(\lambda æ)=-\widetilde{H}(\lambda æ)-\frac{1}{2\pi}\int\widetilde{H}(\lambda æ-\lambda æ_1)\frac{dæ_1}{æ_1-i\varepsilon}R(\lambda æ_1). \qquad (1.11)$$

From this field-theoretic scheme there follows the specific diagram technique in which to φ-particles there corresponds the "propagator" - $D^{(+)}$-function, i.e.,these particles are on the mass shell. Besides, in each vertex there enter the lines, carrying the momentum $\lambda æ$ with propagator $\frac{1}{æ-i\varepsilon}$. In the vertices of diagrams the total 4-momentum of the physical and quasi-particles is conserved. To the virtual states there corresponds the non-zero quasi-particle momentum ($æ\neq0$). In this point the scheme considered differs essentially from the Feynman-Dyson formalism,where the total 4-momentum of the physical particles in vertices of diagrams is conserved, but the fields in the virtual states are not on the mass shell.

The diagrams with the on-shell external lines reduce to the Feynman diagrams. The proof of this fact is trivial and is based on the well-known relation, the validity of last being directly connected with the causal properties of the theory

$$\mathcal{D}^c_\lambda(x)=\theta(\lambda x)\mathcal{D}^{(-)}(x)-\theta(-\lambda x)\mathcal{D}^{(+)}(x)\equiv\mathcal{D}^c(x), \qquad (1.12)$$

or, in the momentum representation,

$$\mathcal{D}^c_\lambda(p)=\int\frac{dæ}{æ-i\varepsilon}d^4k\left[\delta(p+k+\lambda æ)\mathcal{D}^{(-)}(k)+\delta(p+k-\lambda æ)\mathcal{D}^{(+)}(k)\right]= \qquad (1.13)$$
$$=\mathcal{D}^c(p)=\frac{1}{m^2-p^2-i\varepsilon}\,,$$

where

$$\mathcal{D}^{(\pm)}(k)=\theta(\pm k^0)\delta(k^2-m^2)\equiv\theta(\pm\lambda k)\delta(k^2-m^2). \qquad (1.14)$$

The covariant Hamiltonian formulation was applied to solve a number of problems. The ultraviolet divergences are concentrated here in one-dimensional dispersion-type integrals over $æ$. Exploiting this property of the theory it is possible to "curve" the one-dimensional momentum $æ$-space /4/ which makes the theory convergent.

When solving the two-body problem, the quasi-potential type equations /5/ naturally arise. The procedure analogous to that which in the usual theory leads to the Bethe-Salpeter equation gives in the new diagram technique three-dimensional equations /6/. The three-dimensional character of equations is evident from the very beginning. Due to the δ-function, entering into the "propagator" $D^{(+)}$, all integrals in momentum space are taken over the hyperboloid

$$p^2=m^2, \qquad (1.15)$$

on which the three-dimensional Lobachevsky geometry is realized. The expansion in terms of the matrix elements of the unitary irreducible representations of the Lobachevsky space (the Lorentz group) allows one to introduce the adequate configurational $\vec{r}$-representation /7/. The resulting scheme is a completely relativistic and possesses at the same time all the most important features of quantum mechanics. The difference is that the free energy operator here is the finite-difference one with step equal to the Compton wave length of the particle $\hbar/mc$.

The aim of this paper is to develop the version of a covariant Hamiltonian field theory in which the planes of the form

$$\mu x \equiv \mu_0 x_0 - \vec{\mu}\vec{x} = \sigma \tag{1.16}$$

are used instead of planes (1.1). Here μ is the fixed light-like vector

$$\mu^2 = 0\,, \quad \mu_0 > 0\,. \tag{1.17}$$

This formulation is an alternative to the Hamiltonian field theory with the time-like λ-vector. The transition from the description of the time dependence of the events to the description in terms of σ is equivalent to the transition to the light-front variables.

The original idea of the formulation of a field theory on hyperplanes tangent to the light cone goes back to Dirac /8/. In hadron physics the precursor of the light-front formulation was the infinite-momentum limit technique /9/. The kinematical properties of the generators of the Poincare group in this approach were made clear and their usefulness in the description of the properties of hadrons /10/ was established. There are important papers /11/ in which, on the basis of the quasipotential equation in terms of the light-front variables the form-factors, wave functions, and other features of the relativistic composite particles are investigated.

Let us note especially papers /12/, where the spurion technique was exploited for analysis of wave functions of the relativistic bound systems. We suppose that the elegant formalism of Refs /1-3/, being the covariant form of the old-fashioned perturbation theory, is the most appropriate basis for the construction of quantum field theory on the light front. In this scheme it is possible to formulate explicitly the causality condition in terms of the "proper time" μx. The additional singularities on the light cone which accompany each quantum field theory in the light-front variables are regularized by using the limiting procedure (see Sec.2) which is necessary for the formulation of the causality condition. The projecting properties of the wave functions and other quantities follow directly from the formulation itself and do not require any special proof (cf./9,11/).

2. THE CAUSALITY AND UNITARITY CONDITIONS

In the $\sigma=\lambda x$-representation (see /1/) the formulation of the causality condition is based on the group property of the S-matrix

$$S(\infty,\sigma)\, S(\sigma,-\infty) = S(\infty,-\infty). \tag{2.1}$$

It is important here that relation (2.1) reflects the causal properties of the theory due to the validity of the "strong" condition of the space-like character of the hyperplanes(1.1)

$$|x_0 - x_0'| < \omega\, |\vec{x} - \vec{x}'|$$
$$\lambda x = \lambda x' = \sigma\,, \quad \omega = \mathrm{const} < 1\,. \tag{2.2}$$

Due to the arbitrariness in choosing of λ this condition allows one always to separate, by the hyperplane of the type (1.1), two space-time regions of switching on the interaction $G_1 \lessapprox G_2$ (the sign $\lessapprox$ means that points $x_2 \in G_2$ are either space-like with respect to points $x_1 \in G_1$, or $x_1^0 < x_2^0$).

In the case of light-like μ -vector the condition (2.2) is not fulfilled. All surfaces of the type (1.16) are tangent to the light cone along the generatrix

$$x - x' = \rho\mu, \quad x\mu = x'\mu = \sigma, \qquad -\infty < \rho < \infty . \tag{2.3}$$

There exist consequently a very wide class of switching on regions, which are unseparable by the planes of the type (1.16). It is possible to connect such regions by the light signal propagating along the straight line (2.3).

To formulate the causality principle in terms of the hyperplanes (1.16), we consider the light-like 4-vector μ as a limit of the sequence of the time-like vectors λ_δ

$$\lim_{\delta\to 0} \lambda_\delta = \mu, \qquad \lambda_\delta^2 = \delta^2, \quad \lambda_\delta^0 > 0. \tag{2.4}$$

Supposing that the limit $R(\mu\varkappa) = \lim_{\delta\to 0} R(\lambda_\delta \varkappa)$ exists, we come to the following equation (cf./1/)

$$R(\mu\varkappa) - \tilde{R}^+(-\mu\varkappa) = \frac{1}{2\pi}\int \frac{d\varkappa'}{\varkappa' - i\varepsilon}\left[\tilde{R}^+(\mu\varkappa' - \mu\varkappa)R(\mu\varkappa') - \tilde{R}^+(\mu\varkappa')R(\mu\varkappa' + \mu\varkappa)\right]. \tag{2.5}$$

Here the operator $\tilde{R}$ is introduced, which obeys an equation of the form (1.11) with the propagator $\frac{1}{\varkappa + i\varepsilon}$.

Following to /1/ we shall call this equation in what follows the causality condition in the $\varkappa$ -representation. Another, equivalent equation describing the causality condition in the p-representation has the form

$$\frac{\delta j(p)}{\delta\varphi(-\kappa)} = \frac{1}{2\pi}\int\frac{d\varkappa}{\varkappa - i\varepsilon}\left[j(p-\mu\varkappa), j(\kappa+\mu\varkappa)\right] + \tilde{\Lambda}(p-\kappa), \tag{2.6}$$

where $j(p)$ is the current operator

$$j(p) = i\frac{\delta S}{\delta\varphi(-p)} S^+, \tag{2.7}$$

which obeys in the configurational representation the locality condition (cf.(1.9)); $\tilde{\Lambda}(p-\kappa)$ is a Fourier-transform of the so-called quasilocal terms and S is a scattering matrix extended off the energy-momentum shell.

Let us also present the relation which provides the equivalence of the μ -vector formulation with the Feynman-Dyson theory (cf.(1.12))

$$\mathcal{D}^c_\mu(x) = \Theta(\mu x)\mathcal{D}^{(-)}(x) - \Theta(-\mu x)\mathcal{D}^{(+)}(x) . \tag{2.8}$$

This relation is also established by taking the limit (2.4).

The unitarity condition of the S-matrix, formulated in paper /1/, is transferred to the given scheme without change. We have only to remember the necessity to take the limit (2.4). This condition has the form

$$R(\mu\varkappa) - R^+(-\mu\varkappa) = \frac{1}{2\pi}\int\frac{d\varkappa'}{\varkappa' - i\varepsilon}\left[R^+(-\mu\varkappa')R(\mu\varkappa - \mu\varkappa') + R^+(\mu\varkappa' - \mu\varkappa)R(\mu\varkappa')\right] \tag{2.9}$$

On the energy-momentum shell $\varkappa = 0$ and (2.9) coincides with the unitarity condition

$$R(0) - R^+(0) = i\,R(0)R^+(0), \tag{2.10}$$

or $SS^+ = 1$.

The formal solution of the Tomonaga-Schwinger equation (1.3) in the case of surfaces given by relation (1.16) has the form

$$S = T_\mu \exp\{-\int H(x)d^4x\} = 1 + \sum_{n=1}^{\infty}(-i)^n\int\Theta(\mu x_1 - \mu x_2)\cdots\Theta(\mu x_{n-1} - \mu x_n)H(x_1)\cdots H(x_n)d^4x_1\cdots d^4x_n . \tag{2.11}$$

In calculating the matrix elements of this operator there will arise expressions containing the products of singular functions with singularities, coinciding not only at the top of the light cone, but along the generatrix (2.3). It turns out that the limiting procedure (2.4), which allows one to formulate the causality condition, is at the same time a regularizing procedure. These singularities do not contribute to the matrix elements of S-matrix.

3. THE DIAGRAM TECHNIQUE

In paper /2/ it was shown that iterations of the equation (1.11) in the λ-formulation lead to a peculiar diagram technique which differs from the Feynman one. In the case of the μ-formulation we get the appropriate diagram technique by only slightly modifying the diagram technique of the λ approach. Let us formulate these rules (cf./2,9,12/).

1. In the Feynman diagram corresponding to a given process in the usual approach, number all vertices in an arbitrary way. To the physical φ-particles there correspond solid lines, which are oriented along the direction of decreasing vertex number. Omit all diagrams (except vacuum ones), which have at least one vertex with all lines incoming (or outgoing). Such diagrams we shall call in what follows the diagrams with vacuum transitions, DVT.

2. Connect the first vertex with the second, the second with the third and so on, by dotted lines. Orient these dotted lines along the direction of increasing vertex number and ascribe to each of them the 4-momenta $\mu\varkappa_j$ ($j = 1,2....n-1$), where n is the order of the diagram. Lead into the first vertex and lead out from the last n-th vertex dotted lines with a free end, ascribing to them the 4-momenta $\mu\varkappa$ and $\mu\varkappa'$, respectively.

3. Assign to each internal solid line with the 4-momentum k

the function $D^{(+)}(k)$, to each internal dotted line with the 4-momentum $\mu\varkappa_j$ - the "propagator" $\frac{1}{2\pi}\frac{1}{\varkappa_j - i\varepsilon}$.

4. Assign to the i-th vertex the factor $\frac{g}{\sqrt{2\pi}}\delta(\mu\varkappa_{i-1}-\mu\varkappa_i-\sum_{s=1}^{3}k_s)$, where the sum $\sum k_s$ is the total momentum of the incoming and outgoing solid lines in given vertex. Assign to each external solid line the factor $(2\pi)^{3/2}(2p_0)^{-1/2}$, where p is the 4-momentum of the given solid line.

5. Integrate over all independent 4-momentum k and all $\varkappa_j$ in the infinite limits.

6. Sum the coefficient functions, which result from all n! numbering of vertices of the given diagram. Symmetrize the resulting total coefficient function in those momenta k, in which it remains unsymmetrical after summation. Multiply the result by the factor 1/r, where r is a number of permutations of the external lines which enter into the diagram symmetrically.

Let us show that the DVT in fact give no contribution. Let us introduce three 4-vectors $\mu^*, e^{(1)}, e^{(2)}$, which satisfy the conditions

$$\mu^{*2} = \mu e^{(1,2)} = \mu^* e^{(1,2)} = e^{(1)} e^{(2)} = 0, \quad e^{(1)2} = e^{(2)2} = -1, \quad \tfrac{1}{2}\mu\mu^* = 1, \tag{3.1}$$

and corresponding projections of 4-momentum k on these directions

$$k^+ = \mu k, \quad k^- = \mu^* k, \quad \tilde{k} = ((k e^{(1)}),(k e^{(2)})), \quad k^2 = k^+ k^- - \tilde{k}^2. \tag{3.2}$$

It is easy to see that in each vertex the k^+-projection of the momentum of physical particles is conserved (the projection of the momentum of a quasi-particle on the 4-direction μ equals zero). In the case of a DVT it follows from the existence of the factor $\Theta(k^0)$ in the propagator that the k^+-projection of each individual line equals zero. Such diagrams, as is easily seen, equal zero.

Let us consider, as an example, the diagram of self-energy in the lowest order

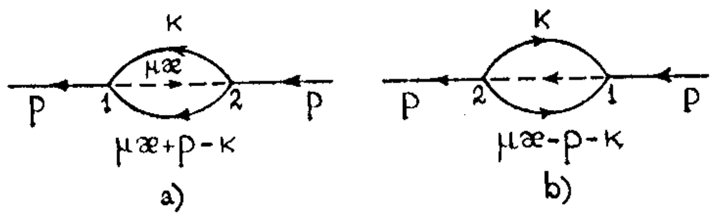

Fig. 1.

The diagram b) is a DVT and does not contribute. The matrix element, corresponding to the diagram a), has the form:

$$F = g^2 \int \frac{d\text{æ}}{\text{æ}-i\varepsilon} \mathcal{D}^{(+)}(k)\mathcal{D}^{(+)}(\mu\text{æ}+p-k)\, d^4k , \qquad (3.3)$$

or, after integrating over æ ,

$$F = g^2 \int \mathcal{D}^{(+)}(k) \frac{\theta(\mu p - \mu k)}{m^2-(p-k)^2-i\varepsilon} d^4k \, . \qquad (3.4)$$

The presence of the θ-function in the integrand reflects the so-called projecting properties of this formalism (cf./11/).

It is convenient to introduce the variables (3.2), then after integration over k^- we get

$$F = g^2 \int \frac{dk^+}{2k^+} d\tilde{k} \frac{\theta(k^+)\,\theta(p^+-k^+)}{m^2-(p^+-k^+)\left(p^- - \frac{m^2+\tilde{k}^2}{k^+}\right)+(\tilde{p}-\tilde{k})^2-i\varepsilon} \, . \qquad (3.5)$$

After putting $k^+=xp^+$ and $\tilde{k}=\tilde{k}'+x\tilde{p}$, we come to the usual (Feynman's) expression for F

$$F = \frac{g^2}{2} \int_0^1 dx \int \frac{d\tilde{k}'}{m^2+\tilde{k}'^2 - x(1-x)p^2-i\varepsilon} \qquad (3.6)$$

4. THE QUASI-POTENTIAL TYPE EQUATION

As it was shown in Ref./6/, the Bethe-Salpeter type equation

for the two-particle scattering amplitude in a field theory with a λ-vector turns out to be a three-dimensional one. Let us demonstrate that in the μ-technique a three-dimensional equation of the quasi-potential type also arises. In the light-front variables this equation coincides with that obtained in the noncovariant old-fashioned perturbation theory /9/ (in the infinite-momentum frame). Another derivation was given in /11/. Let us emphasize that the projecting properties of the amplitudes and wave functions in the scheme with a μ-vector follow immediately from the diagrammatic rules, formulated in the preceding section and do not need any proof.

Consider two particle scattering. We denote the initial and final four-momenta of particles by q_1, q_2 and p_1, p_2 , correspondingly.

The definitions of Ref./6/ are naturally transferred to the case of light-like μ-vector under consideration. Let us recall them. Each connected diagram is called irreducible, if it cannot be divided into two connected diagrams which are linked by two solid lines oriented from right to left, and by one dotted line oriented in the opposite direction. It is supposed that the time axis on the graphs is directed from right to left and external ends are oriented in the proper way. If such a division is possible, the diagram is called reducible. The procedure analogous to that which leads in the usual technique to the Bethe-Salpeter equation gives here

$$T(0,p_1,p_2;q_1,q_2,\mu\text{æ}) = V(p_1,p_2;q_1,q_2,\mu\text{æ}) + \int \frac{d^4k_1\, d^4k_2}{(2\pi)^3} \mathcal{D}^{(+)}(k_1)\mathcal{D}^{(+)}(k_2) \cdot \qquad (4.1)$$

$$\cdot \frac{d\text{æ}'}{\text{æ}'-i\varepsilon} V(p_1,p_2;k_1,k_2,\mu\text{æ}')\, T(\mu\text{æ}',k_1,k_2;q_1,q_2,\mu\text{æ})\, \delta(k_1+k_2-p_1-p_2-\mu\text{æ}'),$$

where V is the "potential", corresponding to the sum of all irreducible diagrams, T is two-particle amplitude.

On passing in (4.1) to the total P,Q,K and relative p,q,k momenta by means of formulae

$$p_{1,2} = \frac{P}{2} \pm p\,, \quad q_{1,2} = \frac{Q}{2} \pm q\,, \quad k_{1,2} = \frac{K}{2} \pm k \tag{4.2}$$

and performing the integration over the variable K, we get:

$$T(\tilde{p},\tilde{q}) = V(\tilde{p},\tilde{q};\tilde{P}) + \frac{1}{4(2\pi)^3}\int \frac{\theta(\beta^+)\theta(\Delta^+)}{\beta^+} \cdot \frac{V(\tilde{p},\tilde{k};\tilde{P})\,T(\tilde{k},\tilde{q})\,dk^+ d\tilde{k}}{m^2+\tilde{k}^2 - \frac{\Delta^+}{2}\left(P^- + \frac{m^2+\tilde{\beta}^2}{\beta^+} - \frac{m^2+\tilde{\Delta}^2}{\Delta^+}\right) - i\varepsilon}\,, \tag{4.3}$$

where

$$\beta = \frac{P}{2} + k\,, \qquad \Delta = \frac{P}{2} - k\,. \tag{4.4}$$

Here we have taken into account the following representation for $D^{(+)}$- function

$$\mathcal{D}^{(+)}(k) = \frac{\theta(k^+)}{k^+}\,\delta\left(k^- - \frac{m^2+\tilde{k}^2}{k^+}\right). \tag{4.5}$$

Let us pass on to the variables

$$x = \frac{1}{2} + \frac{p^+}{P^+}\,, \qquad y = \frac{1}{2} + \frac{q^+}{P^+}\,, \qquad z = \frac{1}{2} + \frac{k^+}{P^+}\,. \tag{4.6}$$

In terms of the Z-variable the factor $\theta(\beta^+)\theta(\Delta^+)$ is rewritten in the form $\theta(z)\theta(1-z)$ [*]. Taking into account (4.6), the equation (4.3) is represented in the form

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[*] This projecting factor, being typical of all field-theoretic schemes in the light-front variables (cf./9,11/), arises here automatically due to the transition to the propagator $D^{(+)}$. As a result, the projecting properties of this scheme do not require any additional verification.

$$T(x,\tilde{p};y,\tilde{q}) = V(x,\tilde{p};y,\tilde{q};\tilde{P}) + \int \frac{d\tilde{k}}{(4\pi)^3}\int_0^1 \frac{dz}{z(1-z)} \frac{V(x,\tilde{p};z,\tilde{k};\tilde{P})\,T(z,\tilde{k};y,\tilde{q})}{\frac{m^2+(\tilde{P}/2+\tilde{k})^2}{z} + \frac{m^2+(\tilde{P}/2-\tilde{k})^2}{1-z} - P^+P^- - i\varepsilon}\,. \tag{4.7}$$

The equation (4.7) is written in an arbitrary reference frame.

Consider also another form of the eq.(4.1). Performing the integration over $d\text{æ}, dk_2$, and dk_1^0, we get

$$T(\vec{p}_1,\vec{q}_1) = V(\vec{p}_1,\vec{q}_1;\vec{P},\mu) + \frac{1}{2(2\pi)^3}\int \frac{d\vec{k}_1}{k_1^0} \frac{\theta(\mu P - \mu k_1)}{2Pk_1 - P^2 - i\varepsilon} V(\vec{p}_1,\vec{k}_1;\vec{P},\mu)\,T(\vec{k}_1,\vec{q}_1) \tag{4.8}$$

Since for the external ends

$$\theta(\mu P - \mu q_1) \equiv 1\,, \tag{4.9}$$

we can rewrite the equation (4.8) in the form

$$T(\vec{p}_1,\vec{q}_1) = \tilde{V}(\vec{p}_1,\vec{q}_1;\vec{P},\mu) + \frac{1}{2(2\pi)^3}\int \frac{d\vec{k}_1}{k_1^0} \frac{\tilde{V}(\vec{p}_1,\vec{k}_1;\vec{P},\mu)\,T(\vec{k}_1,\vec{q}_1)}{2Pk_1 - P^2 - i\varepsilon} \tag{4.10}$$

In the c.m.s. ($\vec{P} = 0$) we get an equation with the simple Green function $E_q(E_k - E_q - i\varepsilon)^{-1}$, which coincides with the equation obtained earlier /13/ in another way.

* * *

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