ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E2 - 10105

D.I.Blokhintsev

DYNAMICS OF QUARKS

1976

E2 - 10105

D.I.Blokhintsev

DYNAMICS OF QUARKS

Submitted to ТМФ

The problem of quark confinement inside hadrons is an essential challenge to the modern quantum field theory. The known quark models of "bags" intended for the quark confinement inside hadrons prove to be rather complicated and are not adequate to the simplicity of the quark model.

Being the structural elements of hadrons the quarks should not follow the prescriptions of quantum field theory adjusted for the description of motion of point particles. For that reason it may be expected that the quarks are subjected to the dynamics essentially different from that dictated by the laws of quantum field theory. This note suggests a possibility of this type based on the stochastic Γ-geometry described in papers [1] and [2].

For the present study the author takes, from the above papers, the main assumption by which the coordinates of a point in such a geometry are operators

$$\hat{X}_\mu = x_\mu + \hat{\xi}_\mu , \tag{1}$$

where x_μ is the average coordinate and $\hat{\xi}_\mu$ is the stochastic fluctuation of the coordinate of that point. The latter is the operator given, in the Γ-geometry, by the formula

$$\hat{\xi}_\mu = a\gamma_\mu , \tag{2}$$

where a is the fluctuation scale and γ_μ are the standard Dirac matrices. Relation (2) results in the following commutation rule

$$[\hat{\xi}_\mu, \hat{\xi}_\nu] = 2ia^2 \Sigma_{\mu\nu}, \tag{3}$$

where $\Sigma_{\mu\nu}$ is the matrix $\gamma_\mu \gamma_\nu$, $\mu \neq \nu$.

Proceeding from the representation (1) we analyse properties of the stochastic vicinity of a point x. Rewriting (1) in the form $\hat{\xi}_\mu = \hat{X}_\mu - x_\mu$ results in the stochastic interval $\hat{S}^2$ which in the Γ-geometry equals

$$\hat{s}^2 = \sum_{\mu=1}^{4} (\hat{X}_\mu - x_\mu)^2 = \hat{\xi}_4^2 + \hat{\xi}_3^2 + \hat{\xi}_2^2 + \hat{\xi}_1^2 = 4a^2 \tag{4}$$

and is a constant, positive-definite quantity and indicates to the Euclidean nature of the considered Γ-geometry. On account of (4) we have

$$[\hat{s}^2, \hat{\xi}_\mu] = 0. \tag{5}$$

From (3) and (5) it is clear that the point of Γ-space in the vicinity of x can be specified by $\hat{s}^2$ and by the eigenvalue of one of the coordinates $\mathrm{Eigv}\,\hat{\xi}_\mu = \pm a$; the remaining three coordinates being undetermined. The eigenvalues of the coordinates $\hat{\xi}_\mu$ are determined from the equations

$$\hat{\xi}_\mu \Phi = \lambda \Phi, \tag{6}$$

where Φ is the bispinor which makes $\hat{\xi}_\mu$ diagonal.

It can be shown that the eigenvalues of the coordinate operators $\hat{\xi}_\mu$ equal $\pm a$.

From relations (3) and equations (6) it follows that in the vicinity of the point x one can place six objects having one of the spatial coordinates with definite value $+a$ or $-a$. These points are presented in the following table:

$$\begin{array}{ccc} \pm a & \hat{\xi}_2' & \hat{\xi}_3' \\ \hat{\xi}_1'' & \pm a & \hat{\xi}_3'' \\ \hat{\xi}_1''' & \hat{\xi}_2''' & \pm a \end{array} \tag{7}$$

If any of these points is occupied with the real particle with momentum p and mass M, then the operator of proper time $\hat{\tau}$:

$$\hat{\tau} = \frac{1}{i}(N\hat{\xi}) \tag{8}$$

with $N = \frac{p}{M}$, $N^2 = 1$ can be assigned to it in the Γ-space. This operator commutes with $\hat{s}^2$ but not with coordinates:

$$[\hat{\tau}, \hat{\xi}_\mu] = 2a^2(N_\alpha \Sigma_{\alpha\mu}), \quad \alpha \neq \mu. \tag{9}$$

Therefore, in the presence of a particle there is one more operator commuting with $\hat{s}^2$. The eigenvalues of the operator $\hat{\tau}$ are defined from the equation

$$\hat{\tau}\Psi = \tau\Psi \tag{10}$$

and equal

$$\tau = \pm \frac{1}{i} a \sqrt{N^2} = \pm a. \tag{11}$$

Equation (10) is compatible with the Dirac equation for a particle described by the spinor Ψ:

$$\Psi = \Psi(x_\mu + \hat{\xi}_\mu). \tag{12}$$

Note that the dependence of Ψ on argument $\hat{\xi}_\mu$ resembles that in the theory of superfield.

The Dirac equation

$$\gamma_\mu \frac{\partial \Psi}{\partial x_\mu} + M\Psi = 0 \tag{13}$$

for a "free" particle with momentum p reduces to the form

$$i(\gamma_\mu p^\mu)u(p) + Mu(p) = 0 \tag{14}$$

with the solution

$$\psi = U(p)e^{ip\hat{X}} = U(p)e^{ipx - M\tau}, \qquad (15)$$

where $U(p)$ is the usual bispinor for a free particle with momentum p.

From comparison of (11) and (14) it follows that the spinor ψ coincides with that in eq. (10) so that τ in (15) equals $\pm a$, depending on the sign of N_0 in (8) that corresponds to the sign of energy $E = \frac{1}{i} p_4$ in eq. (14).

If the eigenvalue of the proper time $\hat{\tau}$ of a particle is given, then its coordinates, according to (9), are undetermined: the definiteness of its location in the vicinity of x is incompatible with the information about its motion in the Γ-space. To obtain this information it is necessary to diagonalize the operators ξ_κ, $\kappa = 1,2,3$. The spinor Φ in (6) does not coincide with ψ in (13) and (14). The spinor Φ describes the tachyon states of a particle, possible of them are collected in (7). If we accept that two particles cannot be on the same spatial straight line in the Γ-space, then we conclude that in this space only three particles may be disposed which possess definite but different positions in the space Γ_3 and indefinite time $\hat{\xi}_4$. These particles are necessarily spinors.

In view of all said above it can be concluded that those particles are reasonable to identify with quarks: three possible positions of quarks hint at their correspondence with the quark colours. Placing the three quarks around point x we will consider their coordinates $\hat{\xi}_k$ as the relative ones and x_k as the coordinate of their center of masses. This is possible because the sum of any three coordinates, averaged for the state with definite $\hat{\tau}$ equals zero:

$$\langle \hat{\xi}_k' + \hat{\xi}_k'' + \hat{\xi}_k''' \rangle = 0. \qquad (16)$$

The three-quark system $q'q''q'''$ will be described by a three-index spinors, the "quark" mass of the baryon being equal to

$$m = M' + M'' + M'''. \qquad (17)$$

All the three quarks are in the state $\tau' = \tau'' = \tau''' = +a$, their coordinates $\hat{\xi}_\mu$ are indefinite but stay on the sphere (4) and cannot go beyond its limits. For the system "quark-antiquark" $q\bar{q}$ the position it possible with $\tau' = a$ and $\tau'' = -a$, respectively.

A specific feature of the presented quark dynamics consists in the following: despite the existence of finite dimensions of the quarks region of hadrons $\hat{s}^2 = 4a^2$ which cannot be left by quarks, the model possesses the scaling property and makes the presentation of confined quarks consistent with their description as free particles.

The dynamics is still undeveloped for hadrons with higher spins lying on the Regge trajectories.

In this case at least two variants are possible among which one cannot yet make a choice. Therefore the above presented idea of the "geometrical jail" for quarks should still be considered as an interesting abstract possibility.

The author is thankful to I.Todorov for useful discussions.

REFERENCES

1. *D.I.Blokhintsev. Teor.Mat.Fiz., 17, 153 /1973/.*
2. *D.I.Blokhintsev. Particles and Nucleus, 5, 606 /1974/.*

Received by Publishing Department on September 9, 1976.