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Векторно-подобная модель лептонов с суперсимметричным взаимодействием

Предложена суперсимметричная векторно-подобная модель взаимодействия лептонов, основанная на группе $SU_2 \times U_1$. Фермионный сектор модели состоит из e, μ и одного тяжелого заряженного лептона E, двух тяжелых нейтральных дираковских лептонов и трех двухкомпонентных нейтрино (ν_e, ν_μ, ν_x). Суперсимметрия спонтанно нарушена, но нейтрино не являются голдстоуновскими фермионами.

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Vectorlike Lepton Model with Supersymmetric Interaction

A supersymmetric vectorlike model of lepton interactions based on the group $SU_2 \times U_1$ is proposed. The fermion sector of the model consists of e, μ and one heavy charged lepton E, two heavy neutral Dirac leptons, and three two-component neutrinos (ν_e, ν_μ, ν_x). Supersymmetry is spontaneously broken, but the neutrinos are not Goldstone fermions.

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Vectorlike models, recently considered by several authors [1,2,3], seem to be rather a promising possibility to describe weak and electromagnetic interactions. In these models the interaction between leptons and quarks mediated by the Yang-Mills mesons is assumed purely vectorial in the absence of mass terms. Mass terms introduce effective axial vector interaction and lead to parity and (possibly) lepton number nonconservation. In the considered models it was supposed that weak and electromagnetic gauge group was $SU_2 \times U_1$ and the physical spectrum of fermions and vector mesons is due to some kind of Higgs mechanism specific form of which was not discussed.

In this note we propose a particular vectorlike model of lepton interaction which is not only gauge invariant but also supersymmetric. Supersymmetric models provide a natural framework for the Higgs mechanism because in these models fermions are necessarily accompanied by (pseudo)scalar particles. However supersymmetry imposes severe restrictions on the form of interaction and up to now nobody succeeded in constructing satisfactory realistic supersymmetric model. In particular it seems extremely difficult to incorporate electron and muon simultaneously.

It was shown also [4,5] that if the electron neutrino is a Goldstone particle then the predicted lepton spectrum of β-decay disagrees with experiment.

So we face the alternative: either supersymmetry must be broken explicitly, or symmetry breaking is spontaneous but Goldstone fermion is not coupled to electron and there are other massless fermions which can be associated with neutrinos. (It seems there are no objections against identification of the Goldstone fermion with the muonic neutrino [6]).

Below we describe a mechanism of spontaneous supersymmetry breaking which does not suffer this difficulty and allows one to construct realistic supersymmetric models.The proposed mechanism is equivalent to a "soft" breaking of supersymmetry by special mass terms. (The term "soft" breaking is applied to the symmetry breaking preserving the symmetric renormalization procedure). The simplest example of "soft" supersymmetry breaking was considered in an early work by Illiopoulos and Zumino [7]. More elaborated procedure specially suited for non-Abelian gauge theories was proposed in our paper [8] where an alternative formulation involving spontaneous breaking of supersymmetry was also given.

In this paper we show that <u>any</u> mass term for scalar components of supermultiplet can be obtained in the framework of spontaneously broken symmetric theory.

Let us consider supersymmetric expression

$$\frac{1}{2}(\bar{D}D)(\Phi_+^+\Phi_-R_-)+\frac{1}{8}(\bar{D}D)^2(R_+\tilde{R}_-)+\eta(\bar{D}D)\tilde{R}_- + \text{h.c.} \qquad (1)$$

Here $\Phi_\pm$ are "physical" chiral multiplets, and $R_+ = \bar{R}_-^+$, $\tilde{R}_+ = \tilde{R}_-^+$ are auxiliary chiral superfields. Components of chiral superfields are denoted as usual $A_\pm$, $\psi_\pm$, $F_\pm$, where $A_\pm$, $F_\pm$ are (pseudo)scalars and $\psi_\pm$ are two-component spinors, D is a covariant derivative[9].

The last term is nothing but $\eta(\tilde{F}_-^R + \tilde{F}_+^R)$. Therefore F_+^R acquires nonzero vacuum expectation value, and one should perform a shift $F_{R\pm} \to F_{R\pm} + \eta$ which eliminates the linear term, and produces the mass term

$$\frac{\eta}{2}(\bar{D}D)(\Phi_+^+\Phi_- + \Phi_-^+\Phi_+)_A = \eta(A_+^+A_- + A_-^+A_+). \qquad (2)$$

Variation over $\tilde{R}$ leads to the free field equations for R and therefore auxiliary fields R and $\tilde{R}$ completely decouple from the "physical" fields $\Phi_\pm$. The only observable effect of these fields is the appearance of the mass term (2). Goldstone neutrino is a component of R and is not coupled to the physical fields. The analogous mechanism described in our paper [8] allows one to obtain mass terms of the form

$$\xi_+A_+^+A_+ + \xi_-A_-^+A_-. \qquad (3)$$

These arguments show that "soft" breaking of supersymmetry by arbitrary mass terms for the scalar fields can be realized as spontaneous symmetry breaking and consequently preserves supersymmetric renormalization procedure developed in our papers [10,11,12] (There exists a general theorem that mass terms change logarithmic counterterms, i.e., the wave function and charge renormalization only by finite numbers [13,14]. Our statement is stronger: soft supersymmetry breaking does not produce also independent mass renormalization and preserves generalized Ward identities).

The proposed model is based on the gauge group $SU_2 \times U_1$. The gauge fields are described by pseudoscalar superfields $\Psi^a = \{ A^a, \chi^a, M^a, N^a, A^a_\mu, \lambda^a, D^a \}$ and $\Psi = \{ A, \chi, M, N, A_\mu, \lambda, D \}$. $A^{(a)}$, $M^{(a)}$, $N^{(a)}$, $D^{(a)}$ are (pseudo) scalars, $\chi^{(a)}$ and $\lambda^{(a)}$ are Majorana spinors, $A^{(a)}_\mu$ are vector fields $(a = 1,2,3)$. Ψ^a and Ψ^μ transform as SU_2 triplet and singlet, correspondingly. The gauge fields interact with complex chiral superfields $\Phi_{\pm 1,2} = \{ A^a_{\pm 1,2}, \psi^a_{\pm 1,2}, F^a_{\pm 1,2} \}$ which are isodoublets with respect to SU_2 (a is an isotopic index). The most general supersymmetric and gauge invariant Lagrangian is [15,16]

$$\mathcal{L} = \frac{1}{8} (\bar{D}D)^2 \{ \Phi^+_{+i} e^{g\Psi + g_1 \Psi_o} \Phi_{+i} + \Phi^+_{-i} e^{-g\Psi_1 - g_1 \Psi_o} \Phi_- \} -$$

$$- \frac{1}{2} (\bar{D}D) \{ M_{ij} \Phi^+_{-i} \Phi_{+j} \} + \frac{1}{128} (\bar{D}D)^2 \, \mathrm{Tr} \{ V^{(0)}_\mu V^{(0)}_\mu + V^{(1)}_\mu V^{(1)}_\mu \} + \mathrm{h.c.} \tag{4}$$

$$V^{(0,1)}_\mu = - \frac{1}{g} [C^{-1} \gamma^\mu \frac{1 + i\gamma_5}{2}]^{\alpha\beta} D_\alpha [e^{-g\Psi(0,1)} D e^{g\Psi(0,1)}],$$

$$\Psi_1 = \Psi^a \tau^a, \quad \Psi^o \equiv \Psi .$$

According to the previous discussions, spontaneous supersymmetry breaking allows one to introduce mass terms

$$\xi_{ij+} A^{+}_{i+} A_{j+} + \xi_{ij-} A^{+}_{i-} A_{j-} + \eta_{ij} (A^{+}_{i+} A_{j-} + A^{+}_{j-} A_{i+}). \quad (5)$$

Let us assume that a stable minimum is achieved for some values $\langle A_{\pm i} \rangle = \alpha_{\pm i}$. Then vector mesons acquire the following masses:
Charged components

$$W_{\pm} = \frac{A^1_\mu \pm A^2_\mu}{\sqrt{2}}, \quad M^2_w = \frac{g^2}{2} \Sigma \alpha^2_{\pm i} .$$

Neutral components

$$Z_\mu = (g^2 + g^2_1)^{-\frac{1}{2}} (g_1 A_\mu - g A^3_\mu), \; M^2_Z = 2^{-1} (g^2 + g^2_1) \Sigma \alpha^2_i ,$$

$$a_\mu = (g^2 + g^2_1)^{-\frac{1}{2}} (g A_\mu + g_1 A^3_\mu), \; M_a = 0.$$

An additional mass term for the fermions also arises

$$\frac{ig}{\sqrt{2}} \{ \alpha_{+i} (\bar{\lambda}_1 + i\bar{\lambda}_2) \psi^1_{+i} + \alpha_{-i} (\bar{\lambda}_1 + i\bar{\lambda}_2) \psi^1_{-i} \} + \quad (6)$$
$$+ \frac{i}{\sqrt{2}} \{ \alpha_{+i} (g_1 \bar{\lambda} - g\bar{\lambda}_3) \psi^2_{+i} + \alpha_{-i} (g_1 \bar{\lambda} - g\bar{\lambda}_3) \psi^2_{-i} \} + \text{h.c.}$$

The component

$$i (g^2 + g^2_1)^{-\frac{1}{2}} (g\bar{\lambda} + g_1 \bar{\lambda}_3) = \bar{\nu} \quad (7)$$

remains massless and may be identified with one of neutrinos, for example, muonic one.

Now it is clear why at least two chiral multiplets are necessary: the neutrino enters the Yang-Mills interaction

$$g\epsilon^{k\ell m}\bar{\lambda}^{k}A_{\mu}^{\ell}\lambda^{m} .$$

Therefore the components of the charged fermion $(\bar{\lambda}_1 + i\bar{\lambda}_2)_{\pm}$ cannot both belong to electron and μ-meson. A heavy charged lepton is unavoidable.

If the model is assumed to conserve lepton number, the values $a_{\pm i}$ cannot be all different from zero. Indeed, if all masses M_{ij} are nonzero, the lepton conservation is associated with the transformations/17/

$$\begin{bmatrix} A_{+i} \\ \psi_{+i} \\ F_{+i} \end{bmatrix} \to \begin{bmatrix} A_{+i} \\ \psi_{+i}e^{i\alpha} \\ F_{+i}e^{2i\alpha} \end{bmatrix}, \quad \begin{bmatrix} A_{-i} \\ \psi_{-i} \\ F_{-i} \end{bmatrix} \to \begin{bmatrix} A_{-i}e^{2i\alpha} \\ \psi_{-i}e^{i\alpha} \\ F_{-i} \end{bmatrix}, \quad \begin{bmatrix} A_{\nu} \\ \lambda \\ D \end{bmatrix} \to \begin{bmatrix} A_{\nu} \\ e^{\alpha\gamma_5}\lambda \\ D \end{bmatrix} \qquad (8)$$

$\langle A_{-i}\rangle_0 = a_{-i} \neq 0$ means spontaneous breakdown of lepton number conservation. On the other hand the formula (6) shows that if all $a_{-i} = 0$ then the massless charged fermion $(\bar{\lambda}_1 + i\bar{\lambda}_2)_-$ is inevitable. To avoid the appearance of such an undesirable particle one of the a_{-i} for example a_{-2}, must be $\neq 0$. It can be achieved without violation of lepton number conservation, only if $M_{22}=M_{12}=\eta_{11}=\eta_{21}=0$. In this case the transformation properties of Φ_{-2} can be chosen independently on Φ_{+i}, and one can put

$$\begin{vmatrix} A_{2-} \\ \psi_{2-} \\ F_{2-} \end{vmatrix} \rightarrow \begin{vmatrix} A_{2-} \\ \psi_{2-} \; e^{-i\alpha} \\ F_{2-} \; e^{-2i\alpha} \end{vmatrix} . \tag{9}$$

With this assignment the charged components $2^{-1/2}(\lambda_2+i\lambda_1)_+$ and ψ_{2-} are antileptons, and other charged components are leptons.

The conditions formulated above allow one to identify

$$\begin{aligned} 2^{-1/2}(\lambda_2+i\lambda_1)_+ + \psi_{2-} &= \mu \\ \psi_{2+} + 2^{-1/2}(\lambda_2+i\lambda_1)_- - \epsilon\psi_{1-} &= E, \\ \psi_1 + 2^{-1/2}\epsilon(\lambda_2+i\lambda_1)_- &= e \ , \end{aligned} \tag{10}$$

where ϵ is a small parameter that would be equal to zero if the electron weak interaction were exactly (V-A). One finds

$$\begin{aligned} &M_{11}=m_e, \ \ M_{21}=-M_E\epsilon, \ \ g\alpha_{1+}=\epsilon m_e, \ \ g\alpha_{2+}=M_E, \\ &g\alpha_{2-}=m_\mu, \ \ \alpha_{1-}=M_{12}=M_{22}=0. \end{aligned} \tag{11}$$

It can be easily verified that the parameters ξ, η may be chosen in such a way to provide for $\alpha_{\pm i}$ the values (11).

A sum rule follows from eq. (11),

$$M_w^2 = M_E^2 + m_\mu^2 + \epsilon m_e^2 : \qquad (12)$$

Eqs. (6) (12) determine uniquely also the neutral lepton spectrum. This spectrum includes the second "neutrino" and massive leptons which unfortunately cannot be identified without contradiction to experiment. The massive neutral lepton with the ,mass $\sim m_\mu$ arises. Its appearance cannot be avoided even if one does not demand lepton conservation. The minimal model (4) is not compatible with the experiment and the number of leptons should be increased. The closest possibility is a model with the same number of charged leptons and one new neutral lepton. Such a model is described by the Lagrangian (4) plus additional term $\Delta\mathcal{L}$, responsible for the interaction of multiplets $\Phi_{\pm 1,2}$ with a neutral chiral superfield $S_+ = S_-^+$.

$$\Delta\mathcal{L} = -\frac{1}{2}(\bar{D}D)a_{ij}\, S_-\Phi_{+i}^+\Phi_{-j} + \frac{1}{16}(\bar{D}D)^2(S_+S_-) + \text{h.c.} \qquad (13)$$

We assume that S transforms under (8) as follows:

$$\begin{bmatrix} A_{S-} \\ \psi_{S-} \\ F_{S-} \end{bmatrix} \to \begin{bmatrix} A_{S-}\, e^{2i\alpha} \\ \psi_{S-}\, e^{i\alpha} \\ F_{S-} \end{bmatrix}$$

Then lepton number is conserved if $a_{11} = a_{21} = 0$.
Now the spectrum of neutral leptons is defined by the quadratic form

$$-m_e \bar{\psi}_1^2 \psi_1^2 + M\epsilon \bar{\psi}_{2+}^2 \psi_{1-}^2 - \gamma \bar{N}(m_e \epsilon \psi_{1+}^2 + M\psi_{2+}^2 + m_\mu \psi_{2-}^2) -$$

$$- \{ C\bar{\psi}_S \psi_{2+}^2 + D\bar{\psi}_S \psi_{2-}^2 + A\bar{\psi}_S \psi_{1+} \} + \text{h.c.} \tag{14}$$

$$A = \frac{a_{12}}{g} m_\mu , C = \frac{a_{22} m_\mu}{g} , D = \frac{a_{22} M + a_{12} \epsilon m_e}{g} , \gamma = \frac{(g^2 + g_1^2)^{1/2}}{g}.$$

Imposing the condition

$$m_e MD + m_\mu \epsilon MA - m_e m_\mu C = 0 \tag{15}$$

which means

$$a_{22} \approx - \frac{\epsilon m_\mu^2}{M m_e} a_{12} , \ D \approx - \frac{a}{g} \frac{\epsilon m_\mu^2}{m_e} , C \approx D \frac{m_\mu}{M} \ll D$$

one finds the following spectrum:
1) two massless "two-component neutrinos"

$$\nu_{1+} \approx \{ 1 + m_e^2 (\epsilon m_\mu)^{-2} \} \{ \psi_{1+}^2 - \frac{m_e}{M\epsilon} \psi_{2+}^2 + \frac{m_e}{\epsilon m_\mu} \psi_{2-}^2 \} ,$$

$$\nu_{2-} \approx \{ 1 + \gamma^2 \epsilon^2 + \gamma^2 m_e^2 (\epsilon m_\mu)^{-2} g^2 a_{12}^{-2} \}^{-1/2} \times \tag{16}$$

$$\times \{ N - \gamma \epsilon \psi_{1-}^2 + \frac{\gamma m_e}{\epsilon m_\mu} \frac{g}{a_{12}} \psi_{S-}^2 \} .$$

2) two massive Dirac fermions with the masses

$$M_1 \approx \gamma M, \; M_2 \approx \sqrt{D^2 + A^2} \approx \frac{a_{12}}{g} m_\mu (1 + \epsilon^2 m_\mu^2 m_e^{-2})^{1/2} \quad : (17)$$

if

$$1 \gg \epsilon \gg m_e m_\mu^{-1}, \; a_{12} \gtrsim g, \quad \epsilon^{-1} \gg \gamma$$

then

a) $\nu_{1+} \approx \psi^2_{1+}$ and may be identified with the electronic neutrino

b) $\nu_{2-} \approx N_-$.

Vector currents including this "neutrino" contain with respect to normal ones the factor $g g_1^{-1}$.

c) both massive fermions are heavy: $M_i \gg m_\mu > m_K$.

d) Due to lepton conservation the mass of muonic neutrino is automatically zero to all orders. The condition (15) guarantees masslessness of ν_1 and ν_2 only at lowest order. To provide ν_1 and ν_2 with zero masses to all orders one should take into account in eq. (15) radiative corrections. It can always be done, because the form of the mass matrix (14) is fixed by lepton conservation.

Besides fermions and vector mesons, the model includes five complex scalar mesons. Three Hermitian components of these mesons become Goldstone bosons and are eliminated by gauge transformation. One Hermitian scalar acquires a mass $\sim \gamma M$. Masses of remaining scalars can be done arbitrary by appropriate choice of parameters ξ, η.

Finally we have the model describing weak and electromagnetic interactions of electron, muon, one heavy charged lepton, two heavy neutral leptons, and three neutrinos. Explicit form of interaction is easily derived from (4). In Zumino-Wess gauge one has:

$$
\begin{aligned}
\mathcal{L}_{int} &= gg_1(g^2+g_1^2)^{-1/2} \, a_\mu(\bar{e}\gamma^\mu e + \bar{\mu}\gamma^\mu\mu) + \\
&+ gg_1 2^{-1/2}(g^2+g_1^2)^{-1/2} \, W_-^\mu\{\bar{\mu}\,\gamma^\mu(\nu_\mu - gg_1^{-1}\nu_2)\} + g2^{-1/2}\bar{e}W_-^\mu\gamma^\mu\nu^e + \\
&+ (g^2+g_1^2)^{-1/2} \, Z^\mu\{(g_1^2-g^2)\{\bar{e}\gamma^\mu e - \bar{\mu}_-\gamma^\mu\mu_-\} + \\
&+ (g^2+g_1^2)\,\bar{\nu}_e\gamma^\mu\nu_e - g^2\bar{\mu}_+\gamma^\mu\mu_+\} + \ldots.
\end{aligned} \tag{18}
$$

where ... denotes terms including heavy leptons. Contrary to the standard Weinberg model the mass of the charged intermediate boson is fixed unambiguously. $M_W = 37.3$ GeV. The sum rule (11) fixes also a mass of the charged heavy lepton. It is clear that $\mu - e$ universality demands $g = g_1$.

The model considered seems to be the minimal supersymmetric model of lepton interactions. There exists a number of models with more leptons. In particular the model with 3 chiral doublets and without chiral singlet may be of some interest, because it is characterized by only one dimensionless coupling constant.

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