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# TWO-LOOP RENORMALIZATION OF THE YANG-MILLS THEORY IN AN ARBITRARY GAUGE

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## 1. Introduction

Various kinds of asymptotically free theories were suggested as possible candidates for the strong interaction model. It is well-known that non-abelian gauge fields are the necessary ingredients of the asymptotically free theory. This stimulates the intensive investigation of non-abelian theories, particularly, with the use of renormalization group technique.

In the present note Yang-Mills theory renormalization group functions (the Gell-Mann-Low function and the anomalous dimensions of propagators) are calculated in the two-loop order. In contrast to earlier calculations[1,2,3] we do not fix the gauge parameter. This allows us to study the gauge dependence of the renormalization group equations and opens the possibility of using the most appropriate gauge in applications.

The self-interacting Yang-Mills Lagrangian is as a rule only a part of the total Lagrangian of the model but this part causes the main calculation difficulties. Therefore we present our results in the form which enables one to use them directly for studying more complicated models including matter fields.

## 2. The Method of Computations

The Lagrangian of the pure Yang-Mills field looks like

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}\left(\partial_\mu B_\nu^a - \partial_\nu B_\mu^a\right)^2 - \frac{1}{2\alpha}\left(\partial_\mu B_\mu^a\right)^2 - \partial_\mu \bar{\eta}^a \partial_\mu \eta^a + \\ & + g f^{abc} \bar{\eta}^a B_\mu^b \partial_\mu \eta^c - \frac{g}{2} f^{abc}\left(\partial_\mu B_\nu^a - \partial_\nu B_\mu^a\right) B_\mu^b B_\nu^c - \\ & - \frac{g^2}{4} f^{abc} f^{ade} B_\mu^b B_\nu^c B_\mu^d B_\nu^e ,\end{aligned} \tag{1}$$

where $f^{abc}$ are the antisymmetric group structure constants. We need only three renormalization constants, namely, the ones of the vector propagator $(Z_3^{-1})$, $\eta$ - propagator $(\tilde{Z}_3^{-1})$ and $\bar{\eta} B \partial \eta$ - vertex $(\tilde{Z}_1)$, to determine all renormalization group functions. From these three constants one can find the charge renormalization and the Gell-Mann-Low function;other vertex renormalizations are to be found from the Ward identities.

In the present note 't Hooft's scheme of renormalization[4] is used. This formalism is described in detail in the excellent work of Collins and Macfarlane[5] whose notations we adopt (note, however, our definition of space-time dimension: $n = 4-2\varepsilon$ ). The 't Hooft scheme exhibits some remarkable features, and among them explicit gauge invariance and mass- and momentum-independence of the renormalization constants. The latter property renders the renormalization group equations applicable for all momenta and enables us to simplify the computations of the diagrams putting several external momenta equal to zero.

It should be noted that from the subtraction point of view 't Hooft's scheme is the standard type $R$ -operation where only the pole parts of divergent diagrams are subtracted. This version of the 't Hooft scheme was suggested by Speer[6] who also proved the equivalence of this scheme to usual Bogolubov's approach.

The detailed description of computation of the relevant diagrams in the two-loop Yang-Mills theory can be found in ref.[2], where the Feynman gauge $\alpha = 1$ was used. The most unpleasant aspect of the calculations in an arbitrary gauge is the tremendous growth of the number of terms in algebraic expressions.



To overcome this difficulty we were forced to use a computer. Namely we used the symbolic calculation program SCHOONSCHIP[7] for the following operations: Lorentz-index summation, reduction of the numerator of the integrand and the evaluation of the standard two-loop momentum integrals. The total time required for calculating all the relevant diagrams with CDC-6400 computer proved to be 84 minutes.

### 3. Results

We introduce the notation $A = \frac{g^2 C_2(G)}{(4\pi)^2}$, where $C_2(G)\delta^{ab} = f^{acd} f^{bcd}$. The renormalization constants in the two-loop approximation look like

$$Z_3^{-1} = 1 + \frac{A}{\varepsilon}\left(\frac{\alpha}{2} - \frac{13}{6}\right) + \frac{A^2}{\varepsilon^2}\left(-\frac{35\alpha}{24} + \frac{455}{72}\right) + \frac{A^2}{\varepsilon}\left(\frac{\alpha^2}{8} + \frac{11\alpha}{16} - \frac{59}{16}\right),$$

$$\tilde{Z}_3^{-1} = 1 + \frac{A}{\varepsilon}\left(\frac{\alpha}{4} - \frac{3}{4}\right) + \frac{A^2}{\varepsilon^2}\left(-\frac{\alpha^2}{32} - \frac{3\alpha}{8} + \frac{53}{32}\right) + \frac{A^2}{\varepsilon}\left(-\frac{\alpha}{32} - \frac{95}{96}\right), \quad (2)$$

$$\tilde{Z}_1 = 1 - \frac{A}{\varepsilon}\frac{\alpha}{2} + \frac{A^2}{\varepsilon^2}\left(\frac{\alpha^2}{4} + \frac{3\alpha}{8}\right) + \frac{A^2}{\varepsilon}\left(-\frac{\alpha^2}{16} - \frac{5\alpha}{16}\right).$$

It is easy to verify that these results satisfy the renormalization group construints[4,5] on the $\frac{1}{\varepsilon^2}$ -terms. From (2) the renormalization group functions can be obtained immediately. To recall the definition of these functions let us write down the renormalization group equation for the propagator:

$$\left[\mu^2\frac{\partial}{\partial\mu^2} + \beta(g^2)\frac{\partial}{\partial g^2} + \delta(g^2,\alpha)\alpha\frac{\partial}{\partial\alpha} - \gamma(g^2,\alpha)\right]\mathcal{D}\left(\frac{K^2}{\mu^2}, g^2, \alpha\right) = 0, \quad (3)$$

where $K$ is an external momentum, $\mu$ is a new dimensional parameter ("unit of mass"[4]), an analog of the subtraction point. The Gell-Mann-Low function is

$$\beta(g^2) = -\frac{11}{3}\frac{g^4 C_2(G)}{(4\pi)^2} - \frac{34}{3}\frac{g^6 C_2^2(G)}{(4\pi)^4}. \tag{4}$$

The anomalous dimensions of the vector propagator ( $\gamma$ ) and $\eta$ -propagator ( $\tilde{\gamma}$ ) are

$$\gamma(g^2,\alpha) = \delta(g^2,\alpha) = A\left(-\frac{\alpha}{2}+\frac{13}{6}\right) + A^2\left(-\frac{\alpha^2}{4} - \frac{11}{8}\alpha + \frac{59}{8}\right),$$
$$\tilde{\gamma}(g^2,\alpha) = A\left(-\frac{\alpha}{4}+\frac{3}{4}\right) + A^2\left(\frac{\alpha}{16} + \frac{95}{48}\right). \tag{5}$$

In accordance with ref. [8] $\beta$ -function is gauge independent. However, the gauge dependence of the anomalous dimensions is non-trivial, unlike abelian theories, where only the one-loop contribution to the anomalous dimensions of propagators is gauge-dependent, while in the multi-loop contributions the gauge parameter is cancelled [9, eq.(40.24)].

In conclusion let us compare our results with the previous ones [1,2,3]. The expression (4) of $\beta$ -function agrees with that from ref. [2,3] and differs from [1]; in the latter paper the Landau gauge was used. If we put $\alpha = 1$ in (2) our results for each Green function coincide with Jones' ones [2].

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