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OSCILLATIONS IN NEUTRINO BEAMS: STATUS AND POSSIBILITIES OF OBSERVATION

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SUMMARY

This review paper deals with the problem of possible oscillations in vacuum between different neutrino states. The ultimate sensitivity of oscillation experiments at various neutrino facilities (Sun, reactors, accelerators) is given in terms of a parameter, which is a function of the neutrino masses. Possible experiments are of the relative type, when one would look for a cosinusoidal term in the intensity of detected neutrinos,and of the absolute type, when the cosinusoidal term is vanishing as a result of averaging, and one would compare the absolute intensity of neutrinos of a given type with that expected in absence of oscillations. Various oscillation schemes are discussed, in which the two neutrinos with definite masses are described either by Majorana or Dirac fields. Schemes with N types of neutrinos (N > 2) are presented.

1. Introduction

The possibility that neutrinos in vacuum "oscillate" between different states was discussed a long time ago /1/ , when there appeared reports, later not confirmed, of lepton non-conservation. Since only one type of neutrinos was known at the time, possible lepton charge violations could have meant only oscillations of the type neutrino$\rightleftarrows$ antineutrino. It was natural (see the case i in table II) to describe possible oscillations $\nu \rightleftarrows \bar{\nu}_L$ in term of Majorana neutrinos ν_1 and ν_2 with different masses m_1 and m_2 (four states in all): coherent superpositions of the two Majorana fields would describe the four particles* ν , $\bar{\nu}$, ν_R, $\bar{\nu}_L$ of which

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[*] Usual neutrinos and antineutrinos are denoted $\nu, \bar{\nu}$. "Unusual" particles are denoted in a self-explanatory way, for example ν_R, etc. Particles with definite masses (Majorana as well as Dirac particles) will be denoted ν_1 , ν_2 ...

the first two were familiar objects, the last two are "sterile"*. In the case just mentioned, which is not realistic (because in nature there are at least two types of neutrinos), as well as in realistic cases which will be discussed below, there is a close analogy between neutrino oscillations and oscillations $K^\circ \rightleftarrows \bar{K}^\circ$: ν_1 and ν_2 are analogous to K°_S and K°_L (they have definite masses); the particles participating in the weak interaction $\nu_{e,\mu}$ and $\bar{\nu}_{e,\mu}$ are analogous to K° and $\bar{K}^\circ$ (they are not described by stationary states); the weak interaction of neutrinos is analogous to the strong interaction of kaons; some kind of superweak interaction of neutrinos, namely the interaction which, in a general case, would violate lepton charge conservation (what distinguishes ν_e from $\bar{\nu}_e$, ν_μ from $\bar{\nu}_\mu$) and/or mix the various neutrinos ν_e, ν_μ ... between themselves, is analogous to the weak interactions of kaons. The main differences between kaon and neutrino oscillations are:

i) While $|m_S - m_L| \ll m_K$, $|m_1 - m_2|$ may well be of the order of the neutrino masses.

ii) The kaons are bosons, while neutrinos are fermions, a fact which leads to kaon oscillations with only two states, while neutrino oscillations may exist only if there are at least four states.

iii) Kaons are unstable, while neutrinos are (as far as we know) practically stable (the electromagnetic decay of the "heavy" neutrinos into light ones can be neglected).

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[*] The notion of sterility of neutrinos plays a very important role and will be discussed later.

iv) Kaon oscillations have the maximum amplitude, while nautrino oscillations not necessarily so.

It was recognised that an experimental search for neutrino oscillations would be essential in the investigations of the problem of possible lepton charge violations (amplitudes are measured, instead of squared amplitudes, which are relevant in conventional methods of searching for lepton charge violations). It was also pointed out that the neutrino oscillation problem should have a bearing on the astrophysical interpretation of the first solar neutrino experiments, which had not yet been initiated. Reactor, accelerator and solar neutrino experiments were discussed in terms of neutrino oscillations, and it was shown that the sensitivity of oscillation experiments is indeed very high /2/ . In the simplest case the oscillation length is

$$L = \frac{2\pi p}{|m_1 - m_2| \frac{1}{2}(m_1 + m_2)} = \frac{4\pi p}{|m_1^2 - m_2^2|}, \qquad (1)$$

where p is the neutrino momentum. For the case of maximum oscillation amplitude and when the transition from a given neutrino state (say ν_e) is possible only into another single state (ν_μ), the intensity is given by the expression

$$I_{\nu_e}(R,p) = \frac{1}{2} I_{\nu_e}(R,p)(1 + \cos 2\pi \frac{R}{L}), \qquad (2)$$

where $I_{\nu_e}(R,p)$ is the intensity of ν_e with momentum p at a distance R from a source of ν_e and $I^{\circ}_{\nu_e}(R,p)$ is the intensity of ν_e, which would be expected if oscillations were absent. A quantitative theory of neutrino oscillations was first given in ref. /3/ .

2. Sensitivity of Oscillation Experiments at Various Neutrino Facilities

The physical effects of oscillations may be observable only if the oscillation length is comparable or smaller than the distance from the source to the detector. Thus, as it is suggested by expressions (1), (2), it is advantageous to perform experiments with relatively low energy neutrinos and at sufficiently large distances R. Experiments with reactors imply $p \gtrsim 1$ MeV, $R \lesssim 100$ m, experiments at meson factories imply $p \gtrsim 10$ MeV, $R \lesssim 100$ m, experiments at high energy accelerators imply $p \gtrsim 1$ GeV, $R \lesssim 10$ km, solar neutrino experiments imply $p \gtrsim 200$ KeV, $R \simeq 150 \cdot 10^6$ km (the Sun-Earth distance). The ultimate sensitivity of the oscillation method (that is the sensitivity when the maximum oscillation amplitude is assumed) is obtained from the condition $L \lesssim R$ for various experimental facilities in terms of the parameter

$$M^2_{min} = 4\pi \frac{p_{min}}{R_{max}}$$

(see Table I, which of course has only value of illustration)*.

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* This Table is quite conservative; for example, at the Tbilisi Conference A.K.Mann reported on a proposal to detect neutrino oscillations from Fermilab accelerator with $R \simeq 1000$ km (A.K.Mann, H.Primakoff, preprint, Univ. of Penn., 1976).

For values of $M^2 \geq M^2_{min}$, where $M^2 = |m_1^2 - m_2^2|$, oscillation effects might be observable.

Table I

Neutrino source	p_{min} (mev)	R_{max} (km)	M^2_{min} (ev)2
Reactor	1	0.1	0.03
Meson factory	10	0.1	0.3
High energy accelerator	1000	10	0.3
Sun	0.2	$150 \cdot 10^6$	$4 \cdot 10^{-12}$

Already now from a reactor experiment /4/, showing that the $\bar{\nu}_e + p \rightarrow e^+ + n$ cross section is about equal to the value expected in absence of oscillations, and from an accelerator experiment /5/, showing that the $\nu_\mu + n \rightarrow e^- + p$ cross section is at most 2% of the $\nu_\mu + n \rightarrow \mu^- + p$ cross section, it is known that M^2 cannot be much larger than 1 eV^2. An upper limit of ~1 eV is obtained for $|m_1 - m_2|$. An idea of the sensitivity of oscillation methods can be obtained by compairing the values of M_{min} with the best direct measurement /6/ of the neutrino mass ($m_{\nu_e} \leq 35$ eV); one may see, that if solar neutrino experiments will be performed in a way which is relevant from the point of view expressed here, one may hope for an improvement by many orders of magnitude in measuring upper limits of neutrino (differences of) masses.

3. Relative and Absolute Measurements

In principle there are two ways of looking experimentally for oscillation effects.

a) One is to observe the oscillating term of equation (2) in relative measurements. Relative measurements might be performed only if M^2 is relatively large, more than say 0.3 eV^2 by using neutrino detectors of good energy resolution: at meson factories, where the spatial smearing of the neutrino source is less severe than at other facilities, or at high energy accelerators, if the decay path of the neutrino generating mesons is much less than the oscillation length.

A good way of revealing oscillations would be to measure at a given distance R from the effective source of neutrinos the ratio r between the intensities of electron and muon neutrinos as a function of their momentum p. If the dimensions of the neutrino detector may be neglected and if the intensity of ν_e at the neutrino source location is negligible r will vary with momentum as follows

$$r(p) = \frac{I_{\nu_{e/\mu}}}{I_{\nu_e}} = \frac{1 - \cos a/p}{1 + \cos a/p},$$

where a is a constant equal to $\frac{1}{2}R|m_1^2 - m_2^2|$ (this expression of r is given for maximum oscillation amplitude).

At reactor facilities one might observe relative effects of oscillations by measuring in some way the antineutrino spectrum. This spectrum at the place where $\bar{\nu}_e$ are emitted is more or less known and could be measured more accurately by determining the spectrum of electrons in fission (at saturation). According to (1) and (2) the spectrum of $\bar{\nu}_e$ far away from the source is distorted. Thus a measurement of the spectrum of electrons emitted in the reaction $\bar{\nu}_e + p \to e^+ + n$ in principle might permit to see the spectrum distortion.

As far as neutrino astronomy is concerned, the possibilities of doing relative measurements have been discussed in some detail early [3,7,8]. Briefly, relative experiments are to remote at the present time and would require neutrino detectors of an incredibly high energy resolution.

b) In the second way of looking for oscillation effects, relative measurements connected with the oscillation term of equation (2) (which on averaging is equal to zero) are not made; one is to compare the absolute intensity of a given type of neutrinos, say ν_e, measured at a given distance from a source of neutrinos (say ν_e or ν_μ) with that which is expected at the same distance in absence of oscillations. This method can be used at all the facilities listed in table I (in particular, at high energy accelerators one would measure the presence of ν_e at large distances from a source of ν_μ).).

If $M^2 \ll 0.01\ (\mathrm{eV})^2$, the only hope of revealing oscillation effects is based on cosmic ray neutrino, especially on solar neutrino experiments. Absolute solar neutrino experiments will be considered below.

According to any real situation, one must average the expression (2) over the neutrino spectrum, the neutrino active region of the Sun, etc.; thus the expression (2), if $L \leq R$, becomes simple $\overline{I_{\nu_e}}=1/2\overline{I^{\circ}_{\nu_e}}$. The value $1/2 I^{\circ}_{\nu_e}$ is the minimum intensity of ν_e (maximum loss of intensity) which corresponds to the maximum oscillation amplitude (assumed in expression (2)). For various types of oscillations which are considered further, we may write for maximum mixing

$$\overline{I_{\nu_e}} = \delta_{min} \overline{I^{\circ}_{\nu_e}} , \qquad (3)$$

where δ_{min} is the minimum value of the decrease coefficient given in Table II. Of course together with electron neutrinos, the intensity of which is $\delta_{min} \overline{I^{\circ}_{\nu_e}}$ at the place of measurement, there are other types of "sterile" neutrinos with an intensity $(1-\delta_{min})\overline{I^{\circ}_{\nu_e}}$, which are called sterile, because they are undetectable (at least by the detector which is being used). These sterile neutrinos may be of different types, depending on the type of oscillations which are considered, that is depending upon the interaction which is responsible for lepton charge violation and for mixing. Their sterility may be connected with the energy (for example ν_μ cannot be detected through the charged current channel if their energy is not larger than the muon mass), with the character of the usual weak interaction ("right" neutrinos, even if they exist, would be practically undetectable, etc.). The sterility, however, is a relative notion: low energy particles (like ν_μ, as well as any new type of neutrinos) could be scattered because of possible neutral currents and in principle might be detectable in the future; "right" neutrinos might be active together with new heavy charged leptons, etc.

It is clear from the above discussion that the "theoretical" intensity $I^0_{\nu_e}$ must be estimated reliably[7,8,10]. This requirement implies that one should preferably detect solar neutrinos from the main thermonuclear reaction in the Sun $p+p \to d+e^+ + \nu_e$ (and $e^- + p + p \to d + \nu_e$). Fortunately the possibility of fulfilling such a program arose recently with the development of the Ga-Ge radiachemical method of detecting neutrinos[11,12] *.

In cosmic ray neutrino physics the appearence of sterile neutrinos (active or not) through oscillations might occur in a number of situations, which will not be discussed here. Let us mention only that in ref.[14] it was shown that the presence of ("sterile-active") neutrinos of a new type, interacting together with heavy charged leptons in the weak interaction, could explain the apparent discrepancy between the Kolar goldmine events[15] and the accelerator evidence, if the oscillation length is ~100 km (distances R ≃ 100 km are quite typical for high energy neutrinos generated

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* For the state of the well known Cl-Ar solar neutrino experiment see ref.[13].

in the atmosphere and detected under the earth surface at a depth of few km of water equivalent.

4. Various Oscillation Schemes

When it was found that there exist (at least) two types of neutrinos, the theoretical description of hypothetical oscillations became a more subtle problem, and the question arose about the various types of oscillations which might exist in nature. There are several schemes of lepton charge violation and mixing which lead to different oscillations, tabulated in Table II, but below attention is paid only to the more attractive schemes.

Scheme 1 /3/, which was the first scheme with a sound theoretical basis, moves from the Gribov interaction Hamiltonian, in which phenomenologically all the possible lepton charge violations and mixing are taken into account

$$M = m_{ee}\bar{\nu}_e^c\nu_e + m_{\mu\mu}\bar{\nu}_\mu^c\nu_\mu + m_{e\mu}(\bar{\nu}_e^c\nu_\mu + \bar{\nu}_\mu^c\nu_e) + \text{h.c.}, \quad (4)$$

where $\nu^c = C\bar{\nu}$ is the charge conjugated spinor. The expression (4) is written under the assumption that there are four states in all, since the two types of bare particles are massless and described by two-component spinors. The interaction (4) is easily diagonalized. The particles with definite masses are two Majorana neutrinos ν_1, ν_2 with different masses m_1 and m_2

$$m_{1,2}=\frac{1}{2}(m_{ee}+m_{\mu\mu}\pm\sqrt{(m_{ee}-m_{\mu\mu})^2+4m_{e\mu}^2}) .$$

The neutrino fields ν_e, ν_μ are described by orthogonal superpositions

$$\begin{aligned}\nu_e &= \nu_1\cos\alpha+\nu_2\sin\alpha,\\ \nu_\mu &= -\nu_1\sin\alpha+\nu_2\cos\alpha,\end{aligned} \qquad (5)$$

(ν_1, ν_2 -Majorana fields),
where α is the mixing angle related to the parameters m_{ee}, etc., by the expression:

$$\sin^2 2\alpha = \frac{4m_{e\mu}^2}{(m_{ee}-m_{\mu\mu})^2+4m_{e\mu}^2} . \qquad (6)$$

The particles ν_e and ν_μ are no longer described by stationary states and there arise oscillations $\nu_e \to \nu_\mu$, $\bar{\nu}_e \to \bar{\nu}_\mu$. In this scheme it is attractive that only usual particles participate in the oscillations. The intensity of ν_e with momentum p at a distance R from a source of ν_e is

$$I_{\nu_e}(R,p)=I^{\circ}_{\nu_e}(R,p)(1-\frac{1}{2}\sin^2 2\alpha+\frac{1}{2}\sin^2 2\alpha\cos 2\pi\frac{R}{L}) \qquad (7)$$

where $I^{\circ}_{\nu_e}(R,p)$ is the intensity which is expected in the absence of oscillations, the oscillation length is

$$L=\frac{4\pi p}{|m_1^2-m_2^2|}=\frac{4\pi p}{(m_{ee}+m_{\mu\mu})\sqrt{(m_{ee}-m_{\mu\mu})^2+4m_{e\mu}^2}}$$

Table II. Schemes of Neutrino Oscillations

No	Number of neutrino types	Lepton charge violation	Mixing	Number of neutrino states	Neutrino bare masses	Particles with definite masses	Typical oscillations in available ν_e beams	δ_{min}	Refer.
1	one	yes		4	$\neq 0$	2 Majorana neutrinos	$\nu \rightleftarrows \bar{\nu}_L$	1/2	/1/
1	two (electron and muon)	yes	yes	4	0	2 Majorana neutrinos	$\nu_e \rightleftarrows \nu_\mu$	1/2	/3/
2	"_"	yes	no	8	$\neq 0$	4 Majorana neutrinos	$\nu_e \rightleftarrows \bar{\nu}_{eL}$	1/2	/2/
3	"_"	no	yes	8	$\neq 0$	2 four-component neutrinos	$\nu_e \rightleftarrows \nu_\mu$	1/2	/17,18/
4	"_"	yes	yes	8	$\neq 0$	4 Majorana neutrinos	$\nu_e \rightleftarrows \nu_\mu$ $\nu_e \rightleftarrows \nu_{eL}$ $\nu_e \rightleftarrows \nu_{\mu L}$	1/4	/24/
1a	N >2(electron, muon, M...)	yes	yes	2N	0	N Majorana neutrinos	$\nu_e \rightleftarrows \nu_\mu$ $\nu_e \rightleftarrows \nu_M$	1/N	/16/
2a	"_"	yes	no	4N	$\neq 0$	2N Majorana neutrinos	$\nu_e \rightleftarrows \bar{\nu}_{eL}$	1/2	
3a	"_"	no	yes	4N	$\neq 0$	N four-component neutrinos	$\nu_e \rightleftarrows \nu_\mu$ $\nu_e \rightleftarrows \nu_M$	1/N	/16,21/
4a	"_"	yes	yes	4N	$\neq 0$	2N Majorana neutrinos	$\nu_e \rightleftarrows \nu_\mu$, $\nu_e \rightleftarrows \nu_M$ $\nu_e \rightleftarrows \bar{\nu}_{eL}$, $\nu_e \rightleftarrows \bar{\nu}_{\mu L}$ $\nu_e \rightleftarrows \bar{\nu}_{ML}$, ...	1/2N	/24/

and $\sin^2 2\alpha$ is given by expression (6). The expression (7) reduces to expression (2) in the case of maximum amplitude of oscillations ($\alpha = \frac{\pi}{4}$) which is obtained either when $m_{ee} = m_{\mu\mu}$, $m_{e\mu} \neq 0$ or when $m_{ee}, m_{\mu\mu} \ll m_{e\mu}$.

The scheme 1^a is an extension of scheme 1 to the case of N types of neutrinos, δ_{min} being equal to 1/N [16].

Scheme 3 [17,18] leads to consequences, similar the those of scheme 1 although the starting points are different. It is based on a theory of the weak interaction of the four leptons which is analogous to the theory of the weak interaction of four quarks [19]. The two neutrinos ν_1 and ν_2 with definite masses m_1 and m_2 are Dirac particles, participating in the interaction through two orthogonal combinations [17,18,20]

$$\nu_e = \nu_1 \cos\theta + \nu_2 \sin\theta,$$

$$\nu_\mu = -\nu_1 \sin\theta + \nu_2 \cos\theta,$$

(ν_1, ν_2-four-component fields),

where θ is a mixing angle which a priori has nothing to do with the Cabibbo hadron angle. Clearly ν_e and ν_μ are not described by stationary states and there appear oscillations $\nu_e \to \nu_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\mu$, the origin of which can be traced to the mass difference $|m_1 - m_2|$. In scheme 3 there are 8 neutrino states in all.

Note, that in spite of neutrino mixing, such processes like $\mu \to e\gamma$, etc., have an extremely small (though finite) probability [20,22], for the same reason which makes very unlikely such processes like $K \to \mu\bar{\mu}$, etc. This incidentally is true also for scheme 1 [23]. From (7) it is seen that in a solar exp. it is possible to measure, directly and irrespactive of the value of M, the mixing angle by measuring the ν_e intensity (if the last turns out to be less than $I^{\circ}_{\nu_e}$).

The intensity I_{ν_e}, already defined, (with the obvious replacements of a by θ) is given by the expression (7), which reduces to the expression (2) for maximum mixing $\theta = \frac{\pi}{4}$. This corresponds to $\delta_{min} = \frac{1}{2}$.

Scheme 3a /11,21/ is an extension of the scheme 3 to the case of N types of neutrinos, δ_{min} being $\frac{1}{N}$.

Schemes 4 and 4a extend schemes 1 and 1a to the case where the bare particles are described by four component spinors, and consequently allow oscillations $\nu_e \rightleftarrows \bar{\nu}_{eL}$, etc. /24/ . The attraction of this scheme would increase if "right" currents existed, that is if the right components of the neutrino field would participate in the weak interaction together with new charged leptons, as required, for example, by vector-like theories /25, 26/. In the case of two types of neutrinos a value $\delta_{min} = 1/4$ is obtained and in the general case of N types of neutrinos $\delta_{min} = 1/2N$.

Conclusion

The main points related to oscillation phenomena are: finite neutrino masses, neutrino mixing, lepton charge violation, number of neutrino types. Thus some questions which might be answered in experiments based on the neutrino oscillation ideology directly concern the very nature of neutrinos.

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