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DYNAMICAL BREAKDOWN OF CHIRAL SYMMETRY AND ABNORMAL PERTURBATION EXPANSIONS

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DYNAMICAL BREAKDOWN OF CHIRAL SYMMETRY AND ABNORMAL PERTURBATION EXPANSIONS

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1. Introduction

A great deal of current efforts in field theory is centered about the idea of a spontaneous breakdown of symmetries in gauge theories with massless fermions and vector mesons. In these approaches the particles acquire usually their masses spontaneously via the Higgs mechanism leaving us with renormalizable theories of massive fermions and vector mesons. This is at present a widely accepted method to construct unified theories of weak and electromagnetic interactions. It is, however, also desirable to investigate the possibility of a dynamical spontaneous breakdown of symmetry without introducing elementary Higgs fields having a vacuum expectation value. Thus, one has to seek for symmetry-violating solutions of the dynamical equations of the theory. This point of view has been formulated already a long time ago by Nambu and Jona-Lasinio[1] in their attempt to describe the spontaneous breakdown of chiral symmetry dynamically using the close analogy to superconductivity. They have stimulated a large amount of work in this kind of field[2,3].

In this paper we examine an Abelian gauge model of interacting quarks and vector gluons where the radiative corrections turn out to be the origin of the spontaneous symmetry breaking. Throughout the work functional methods are employed which have been recongnized to be a very effective tool for treating dynamical symmetry breaking. As will be shown in Sect. 2, the generation of a symmetry-violating scalar mass term in the fermion propagator may conveniently be studied by introducing a bilocal dynamical variable in the path-integral representation of the generating functional. The action principle applied to the effective action of the bilocal field yields then as the "classical" field equation the differential version of the Schwinger-Dyson equation in the lowest nontrivial approximation of perturbation theory. In Sect. 3 we shortly discuss the solutions of this equation that was years ago the starting point of the finite quantum electrodynamics of Baker, Johnson and Willey[4]. In Sect. 4 a systematic abnormal perturbation expansion of the path-integral has been worked out that uses the symmetry-breaking solution as the lowest order term. It takes into account higher order effects of quantum fluctuations around the stationary point of the effective bilocal action. Finally, the relation of the resulting abnormal perturbation diagrams to usual Feynman diagrams is demonstrated for the simplest Green's functions of the model.

2. Generating Functional and New Dynamical Variables

Let us consider a theory of massless fermions ("quarks") interacting with a massless neutral vector meson ("gluon") field (for simplicity internal degrees of freedom are not taken into account). The generating functional of all Green's functions, including the disconnected ones, is given in the path-integral formulation by*)

$$Z(J,\bar\eta,\eta) = N\int \mathcal{D}A_\mu\, \mathcal{D}\psi\, \mathcal{D}\bar\psi\, \delta[\partial_\mu A_\mu]\exp\Big\{i\int d^4x\Big[\bar\psi i\hat\partial\psi - \frac{1}{4}F_{\mu\nu}^2 + g\bar\psi\hat A\psi + J_\mu A_\mu + \bar\psi\eta + \bar\eta\psi\Big]\Big\} \tag{1}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad ; \quad \hat A = \gamma_\mu A_\mu$$

Here $J_\mu, \bar\eta, \eta$ are the external sources of the fields $A_\mu, \psi, \bar\psi$ and N is a normalization factor chosen such that $Z(0,0,0) = 1$. We remark that the Lagrangian appearing in eq. (1) is invariant with respect to (i) chiral and (ii) scale transformations, (i) $\psi \to \exp(i\gamma_5\lambda)\psi$, (ii) $\psi(x) \to \lambda^{3/2}\psi(\lambda x)$, $A_\mu(x) \to \lambda A_\mu(\lambda x)$.
Let us now show that the expression (1) may develop a scalar mass-like term for the fermion field leading to a dynamical

*) As we are not concerned with renormalization questions, necessary counterterms have been omitted in eq. (1).

breakdown of the γ_5-invariance of the theory. For this purpose we first integrate over the vector field A_μ and then introduce a new bilocal field containing a scalar component. The A_μ-integration yields

$$Z(J,\bar\eta,\eta) = N\int \mathcal{D}\psi\,\mathcal{D}\bar\psi \exp\Big\{i\int d^4x\Big[\bar\psi(x)\Big(i\hat\partial - g\gamma_\mu\int d^4y\, D_{\mu\nu}(x-y)J_\nu(y)\Big)\cdot\psi(x) + \bar\eta\psi + \bar\psi\eta\Big] - \frac{i}{2}\int d^4x\, d^4y\Big[J_\mu(x)D_{\mu\nu}(x-y)J_\nu(y) + g^2\,\bar\psi(x)\gamma_\mu\psi(x)D_{\mu\nu}(x-y)\bar\psi(y)\gamma_\nu\psi(y)\Big]\Big\} \tag{2}$$

where $D_{\mu\nu}(x)$ is the gluon propagator in the Landau gauge

$$D_{\mu\nu}(x) = (g_{\mu\nu} - \partial_\mu\partial_\nu/\Box)D^c(x)$$
$$D^c(x) = -\int\frac{d^4q}{(2\pi)^4}\,\frac{e^{-iqx}}{q^2+i\varepsilon} \tag{3}$$

(Throughout we use the notations and conventions of ref.[5].)
Let us next introduce the abbreviation

$$K_{(\alpha_1\beta_1;\alpha_2\beta_2)}(x_1y_1,x_2y_2) = (\gamma_\mu)_{\alpha_1\beta_2}D_{\mu\nu}(x_1-x_2)(\gamma_\nu)_{\alpha_2\beta_1}\delta(x_1-y_2)\delta(y_1-x_2) \tag{4}$$

and rewrite the four-fermion term in the exponent of eq. (2) as

$$\int d^4x\,d^4y\;\bar\psi(x)\gamma_\mu\psi(x)D_{\mu\nu}(x-y)\bar\psi(y)\gamma_\nu\psi(y) =$$
$$= -\int d^4x_1\,d^4y_1\,d^4x_2\,d^4y_2\,[\bar\psi_{\alpha_1}(x_1)\psi_{\beta_1}(y_1)]\,K_{(\alpha_1\beta_1,\alpha_2\beta_2)}(x_1y_1,x_2y_2)\,[\bar\psi_{\alpha_2}(x_2)\psi_{\beta_2}(y_2)] \equiv$$
$$\equiv -\bar\psi_{a_1}\psi_{b_1}K_{(a_1b_1;a_2b_2)}\bar\psi_{a_2}\psi_{b_2} = -\big([\bar\psi\times\psi],K[\bar\psi\times\psi]\big) \tag{5}$$

The sign minus in front of the integral in eq. (5) arises from the interchange of the $\psi,\bar\psi$ fields that have to be considered as anticommuting Grassman variables. Furthermore, for notational simplicity, a pair of discrete and continuous variables (α_i, x_i) is abbreviated by a latin index a_i (summation over equal latin indices includes an integration over the continuous variable) Eq. (5) is then linearized in the expression $\bar\psi_\alpha(x)\psi_\beta(y)$ by performing a (bilocal) Gauss integration*)

$$\exp\left\{\frac{i}{2}g^2\left([\bar\psi\times\psi],K[\bar\psi\times\psi]\right)\right\} =$$
$$= \left[\mathrm{Det}\,K^{-1}\right]^{1/2}\int\mathcal{D}\chi\exp\left\{-\frac{i}{2}(\chi,K^{-1}\chi)-i\bar\psi_a\psi_b\,g\,\chi_{ab}\right\} \qquad (6)$$
$$K^{-1}_{(ab,cd)}K_{(cd;ef)} = \delta_{(ab,ef)} = \delta_{ae}\delta_{bf}$$

where the bilocal field $\chi_{\alpha\beta}(x,y)$ is treated as a c -number field. It can be decomposed in scalar, pseudoscalar, vector, axial vector and tensor components

$$\chi(x,y) = S(x,y)+\gamma_5P(x,y)+\gamma_\mu V_\mu(x,y)+i\gamma_\mu\gamma_5A_\mu(x,y)+\frac{i}{2}[\gamma_\mu,\gamma_\nu]T_{\mu\nu}(x,y) \qquad (7)$$

With eq. (7) the integration measure reads $\mathcal{D}\chi = \mathcal{D}S\,\mathcal{D}P\,\mathcal{D}V\,\mathcal{D}A\,\mathcal{D}T$. Inserting eq. (6) in eq. (2) and performing the integration over the quark field the generating functional (1) can be rewritten in the final form

$$Z(J,\bar\eta,\eta) = N'\int\mathcal{D}\chi\, e^{iS(\chi)}Z(J,\bar\eta,\eta|\chi) \qquad (8)$$

with

$$S(\chi) = \left[-\frac{1}{2}(\chi,K^{-1}\chi) - i\,\mathrm{tr}\ln(1+gG_0\chi)\right] \qquad (9)$$

$$Z(J,\bar\eta,\eta|\chi) = \exp\Big[-\frac{i}{2}J_\mu\cdot D_{\mu\nu}\cdot J_\nu + i\bar\eta\,G(g\chi-g\hat A_{ext})\eta +$$
$$+\,\mathrm{tr}\ln\left(1-gG(g\chi)\hat A_{ext}\right)\Big] \qquad (10)$$

The Green's functions $G(g\chi-g\hat A_{ext})$ and G_0 are defined as follows

$$i\hat\partial_x G_0(x,y) = -\delta(x-y)$$
$$(i\hat\partial_x+g\hat A_{ext})G(x,y|g\chi-g\hat A_{ext}) - g\int d^4z\,\chi(x,z)G(z,y|g\chi-g\hat A_{ext}) = -\delta(x-y) \qquad (11)$$

*) In ref. /6/ an analogous technique has been used for a nonrelativistic system of interacting fermions and bosons to obtain the gap equation of superconductivity.

where

$$\hat A_{ext}(x) = -\gamma_\mu\int d^4y\,D_{\mu\nu}(x-y)J_\nu(y) \qquad (12)$$

and we have used the abbreviation $J_\mu\cdot D_{\mu\nu}\cdot J_\nu = \int d^4x\,d^4y\,J_\mu(x)D_{\mu\nu}(x-y)J_\nu(y)$ Finally, to arrive at eq. (8), we have employed the identities

$$\mathrm{Det}\,A = \exp\mathrm{tr}\ln A \quad \text{and}$$
$$\mathrm{tr}\ln\left[G_0^{-1}+g(\chi-\hat A_{ext})\right] = \mathrm{tr}\ln G_0^{-1}[1+gG_0\chi] + \mathrm{tr}\ln\left[1-gG(g\chi)\hat A_{ext}\right]$$

where tr denotes the trace of the operator in square-brackets considered as an integral operator in the functional space as well as the usual trace over matrix indices.

Let us consider for a moment the fermion sector only by setting $J=0$. The expression $Z(0,\bar\eta,\eta|\chi) = \exp i\bar\eta G(g\chi)\eta$ may be understood as the generating functional of a fermion moving in an external bilocal random field $\chi(x,y)$ of probablity distribution exp $iS(\chi)$.

3. "Classical" Bilocal Field Equations and Discussion of Solutions

The functional $S(\chi)$ that appears as a weight factor in the functional integral (8) may be naturally interpreted as the effective action of the bilocal field $\chi(x,y)$. The "classical" field $M(x,y)$ is then determined by the solutions of the equation

$$g\frac{\delta S(\chi)}{\delta\chi}\Big|_{g\chi=M} = -K^{-1}M - ig^2G(M) = 0 \qquad (13)$$

following from the action principle. Multiplying eq. (13) from the left by the operator K yields also*)

$$M_{\alpha\beta}(x,y) = -ig^2D_{\mu\nu}(x-y)\left[\gamma_\mu G(x,y|M)\gamma_\nu\right]_{\alpha\beta}$$

where

$$i\hat\partial_x G(x,y|M) - \int d^4z\,M(x,z)G(z,y|M) = -\delta(x-y) \qquad (14)$$

As the equations (14) do not involve the external sources it is natural to assume translational invariance of the solutions, i.e.

*) Such a manipulation may in principle change the class of solutions.

$$M(x,y) = M(x-y) \quad , \quad G(x,y|M) = G(x-y|M)$$

Furthermore, going to the momentum space, eqs. (14) read

$$\begin{aligned} M(p) &= i \frac{g^2}{(2\pi)^4} \int d^4q \, D_{\mu\nu}(q-p) \, \gamma_\mu \frac{1}{M(q)-\hat{q}} \gamma_\nu \\ G(p|M) &= 1/M(p)-\hat{p} \end{aligned} \qquad (15)$$

Because of PT and T invariance the possible solutions contain only scalar and vector components. Moreover, the vector part can be shown to vanish in the transverse Landau gauge (3). Thus, we finally arrive at

$$M(p) = -i \frac{\lambda}{\pi^2} \int \frac{d^4q}{(q-p)^2} \frac{M(q)}{q^2 - M^2(q)} \quad , \lambda = \frac{3g^2}{(4\pi)^2} \qquad (16)$$

It is worth remarking that eqs. (15), (16) are just the Schwinger-Dyson equations for the quark propagator in the lowest nontrivial approximation of perturbation theory/4,5/. Moreover, eq. (13) is the differential form of this equation

$$\left(\frac{\partial}{\partial p_\mu}\right)^2 M(p) = -4\lambda \frac{M(p)}{p^2 - M^2(p)} \qquad (17)$$

It is equivalent to the integral equation (16) if one takes into account the boundary conditions

$$\left[p^2 \frac{dM(p)}{dp^2} + M(p)\right]_{p^2 \to -\infty} = 0 \quad ; \left[p^4 \frac{dM(p)}{dp^2}\right]_{p^2 \to 0} = 0 \qquad (18)$$

that can be read off from eq. (16) after performing the angular integrations.*) Eq. (16) admits, in particular, a trivial symmetric solution $M(p) = 0$. The nontrivial solutions of a linearized version of eq. (16) have been studied years ago in the finite quantum electrodynamics of Baker, Johnson and Willey/4,7/ for large values of space-like momenta. Recently, the solutions of the nonlinear equation (17) have been studied, too, and extended to the timelike region/8/.

Indeed, one easily finds nontrivial asymptotic solutions of eqs. (17), (18) by inserting the asymptotic expressions

*) In the functional approach used here there follow boundary conditions from the requirement that the action integral converges.

$M(p) \propto (-p^2)^{-c}, \sqrt{p^2}$ for $p^2 \to -\infty, +\infty$. This yields

$$\begin{aligned} M(p) &\propto M_0(-p^2)^{-\frac{1-\sqrt{1-4\lambda}}{2}} \quad , \quad -p^2 \to \infty \\ M^2(p) &\propto p^2(1 + \tfrac{4}{3}\lambda) \quad , \quad p^2 \to \infty \end{aligned} \qquad (19)$$

Furthermore, the numerical investigations performed in ref./8/ show that $M^2/p^2 < 1$ for $p^2 > 0$. Thus, the quark propagator $G(p|M) = \frac{M(p)+\hat{p}}{M^2(p)-p^2}$ does not develop a pole, the quarks being confined. As the authors of ref./8/ emphasize, the situation may change drastically if the vector gluon becomes massive, too.

Nambu and Jona-Lasinio/1/ pointed out first that eq. (16) is the relativistic analogue of the gap equation in superconductivity/9/. They suggested that a massive Dirac particle should be a mixture of bare fermions with opposite chiralities but with the same fermion number similarly as a quasi-particle in a superconductor is a mixture of bare electrons with opposite electric charges but with the same spin. It is convenient to decompose the Green's function (15) into its "normal" and "abnormal" parts

$$G(M) = \left[G_{RR}(M) + G_{LL}(M)\right] + \left[G_{RL}(M) + G_{LR}(M)\right] \qquad (20)$$

where

$$\begin{aligned} G_{\substack{RR\\LL}}(M) &= \frac{1 \pm \gamma_5}{2} G(M) \frac{1 \mp \gamma_5}{2} \\ G_{\substack{RL\\LR}}(M) &= \frac{1 \pm \gamma_5}{2} G(M) \frac{1 \pm \gamma_5}{2} \end{aligned} \qquad (21)$$

belong to the combinations $\psi_R \bar{\psi}_R, \psi_L \bar{\psi}_L$ etc. of quarks of right and left chiralities, $\psi_R = \frac{1+\gamma_5}{2}\psi$, $\psi_L = \frac{1-\gamma_5}{2}\psi$. We obtain

$$\begin{aligned} G_{\substack{RR\\LL}}(p|M) &= \frac{1 \pm \gamma_5}{2} \hat{p} \frac{1}{p^2 - M^2(p)} \\ G_{\substack{RL\\LR}}(p|M) &= \frac{1 \pm \gamma_5}{2} M(p) \end{aligned} \qquad (22)$$

Thus, the abnormal quark Green's functions vanish identically in the symmetric case of no mass gap. Assuming the invariance of the vacuum under chiral transformations yields the transformation law

$$e^{i\gamma_5\lambda} G \, e^{i\gamma_5\lambda} = G \qquad (23)$$

as can easily be seen by expressing G as a vacuum expectation value of the T-product of field operators. A nontrivial mass gap $M(p) \neq 0$ obviously violates eq. (23) being associated to a non-invariant vacuum. It is just the existence of such symmetry breaking solutions to the equations of the theory that has led to the concept of a dynamical spontaneous breakdown of a symmetry.

Concluding this section, we mention that eq. (16) when rewritten in terms of the function $M(p)/p^2 - M^2(p)$ is identical with the pseudoscalar sector of the homogeneous fermion-antifermion Bethe-Salpeter equation at total four-momentum $P^2 = 0$ with the nontrivial propagator (15) used/10/. Thus, for $M(p) \neq 0$ there exists also a nontrivial solution $\chi(p,0) \sim \gamma_5 \, M(p)/p^2 - M^2(p)$ of the homogeneous Bethe-Salpeter equation. As has been discussed at length in the literature (for a linearized propagator), these solutions belong, however, to a continuous spectrum and they are not normalizable/11/. An identification of these solutions with the usual Goldstone bosons arising if a symmetry is broken spontaneously seems thus to be excluded.

4. Abnormal Perturbation Expansion

The nontrivial solution of the nonlinear eqs. (17), (18) defines a quark propagator that includes already a definite class of radiation corrections of ordinary perturbation theory. Let us now show how further corrections to this symmetry breaking solution, as e.g. vertex and gluon propagator corrections, may be taken into account in a systematic way by expanding the integrand of eq. (8) around the "classical" solution.
Taking the nontrivial confinement solution /8/ of the Schwinger-Dyson gap equation as a lowest order term, this approximation scheme has the advantage of preserving automatically some of the nonlinear features of field theory. For this purpose, we shift the field variable χ around the solution $M(x,y)$ of eqs. (17), (18)

$$g\chi(x,y) = M(x-y) + g\phi(x,y) \tag{24}$$

and expand the integrand of all the relevant path-integrals in the bilocal quantity $g\phi(x,y)$ that characterizes the strength of the field fluctuations around M. The generating functional (8) is then rewritten as

$$Z(J,\bar\eta,\eta) = \bar N \int \mathcal{D}\phi \, e^{-\frac{i}{2}(\phi, S^{(2)}(M)\phi)} \left[e^{iS_{int}(\phi)} Z(J,\bar\eta,\eta | M + g\phi) \right] \tag{25}$$

where

$$S_{int}(\phi) = i \sum_{n=3}^{\infty} \frac{(-g)^n}{n} \, tr\left[G(M)\phi \right]^n \tag{26}$$

$$S^{(2)}(M)_{(\alpha_1\beta_1;\alpha_2\beta_2)} = K^{-1}_{(\alpha_1\beta_1;\alpha_3\beta_3)} \left[\delta_{\alpha_3\beta_1}\delta_{\alpha_2\beta_2} - ig^2 K_{(\alpha_3\beta_1,\kappa\ell)} G_{\ell\alpha_2}(M) \times G_{\beta_2\kappa}(M) \right] \tag{27}$$

In eq. (25) we have absorbed the term $\exp iS(M)$ in the new normalization constant $\bar N$ and used the fact that the linear term in the functional Taylor expansion of $S(\chi)$ is absent due to eq. (13). In the following the path-integral averaging, e.g. of eq. (25), will be written in short as

$$Z(J,\bar\eta,\eta) = \frac{\langle e^{iS_{int}(\phi)} Z(J,\bar\eta,\eta | M + g\phi) \rangle_\phi}{\langle e^{iS_{int}(\phi)} \rangle_\phi} \tag{28}$$

Let us now define the propagator $\mathcal{D}^\phi_{(\alpha\beta;\gamma\delta)}(xy, x'y')$ of the bilocal field $\phi_{\alpha\beta}(x,y)$ by*)

$$\mathcal{D}^\phi_{(\alpha\beta;\gamma\delta)}(xy;x'y') = i \langle \phi_{\alpha\beta}(xy)\phi_{\gamma\delta}(x'y') \rangle_\phi = \left[S^{(2)-1}(M) \right]_{(\alpha\beta,\gamma\delta)}(xy,x'y') \tag{29}$$

Note that $\mathcal{D}^\phi$ acts as a two-particle propagation kernel that satisfies the following integral equation of the Bethe-Salpeter type

$$\mathcal{D}^\phi_{(\alpha\beta;\gamma\delta)} = K_{(\alpha\beta,\gamma\delta)} + ig^2 K_{(\alpha\beta,\kappa\ell)} G_{\ell i}(M) G_{j\kappa}(M) \mathcal{D}^\phi_{(ij,\gamma\delta)} \tag{30}$$

From eqs. (29), (27) we obtain the perturbation expansion

$$\begin{aligned}
\mathcal{D}^\phi_{(\alpha\beta;\gamma\delta)}(xy;x'y') &= (\gamma_\mu)_{\alpha\delta}(\gamma_\nu)_{\gamma\beta} D_{\mu\nu}(x-y)\,\delta(x-y')\,\delta(y-x') \\
&+ ig^2 \left[\gamma_\mu G(x,y'|M)\gamma_5\right]_{\alpha\delta} D_{\mu\nu}(x-y) \left[\gamma_\sigma G(x',y|M)\gamma_\nu\right]_{\gamma\beta} D_{\varsigma\sigma}(y'-x') \\
&+ (ig^2)^2 \int d^4z\, d^4w \left[\gamma_\mu G(x,w|M)\gamma_\lambda G(w,y'|M)\gamma_5\right]_{\alpha\delta} \times \\
&\times \left[\gamma_\tau G(x',z|M)\gamma_\sigma G(z,y|M)\gamma_\nu\right]_{\gamma\beta} D_{\mu\nu}(x-y) D_{\tau\varsigma}(x'-y') D_{\sigma\lambda}(z-w) + \dots
\end{aligned} \tag{31}$$

*) These definitions always include a normalization factor $\langle 1 \rangle_\phi$ which is suppressed for brevity.

The graphical representation of eq. (29) and (31) is given in Fig. 1.

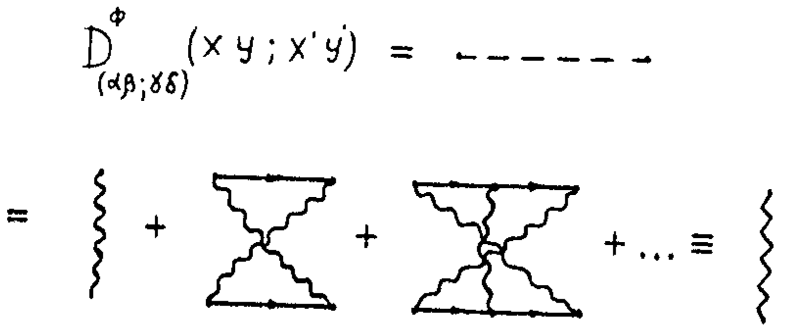

Fig. 1.

Here the dashed line symbolizes the propagator of the original bilocal field ϕ which will be also sometimes written in the form of a contraction $\overline{\phi_{\alpha\beta}(x,y)\,\phi_{\gamma\delta}(x',y')}$. In the second row of Fig. 1 we give the expansion of the propagator in terms of Feynman graphs where the solid lines represent the quark Green's functions $G(x,y|M)$ that are solutions of eq. (14). This class of graphs will also be represented by a zigzag line symbolizing the many-gluon exchange. The expectation value of a product of n bilocal fields may now easily be evaluated by means of a Wick theorem. We have, for example,

$$\langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle_\phi = \overline{\phi_1 \phi_2}\;\overline{\phi_3 \phi_4} + \overline{\phi_1 \overline{\phi_2 \phi_3} \phi_4} + \overline{\phi_1 \overline{\phi_2 \phi_3 \phi_4}} \tag{32}$$

$$\langle \phi_1 \phi_2 \dots \phi_{2n+1} \rangle_\phi = 0$$

In the following we use the Wick theorem also in the compact form

$$\langle f(\phi) \rangle_\phi = e^{-\frac{i}{2}\left(\frac{\delta}{\delta B},\, D^\phi \frac{\delta}{\delta B}\right)} f(B)\Big|_{B=0} \tag{33}$$

Let us now compute as examples the complete two-particle Green's functions G, Δ of the quarks and vector gluons as well as the quark-gluon vertex Γ. We get*)

*) One may also obtain a slightly modified expansion of the quark Green's function by seeking for the stationary point of the whole integrand of eq. (34). Instead of the field equation (13) we have then $\delta/\delta M\,[e^{iS(M)} G(x,y|M)] = 0$. It can easily be shown that the resulting Schwinger-Dyson gap equation includes now the averaged field $a_\mu(x)$ of eq. (37). A similar result has been obtained for a system of interacting Fermi and Bose particles in ref./12/.

$$G_{\alpha\beta}(x,y) = \frac{1}{iZ} \frac{\delta}{\delta \eta_\beta(y)} \frac{\delta Z}{\delta \bar{\eta}_\alpha(x)}\Big|_{\eta=\bar{\eta}=0}$$

$$= \langle e^{iS_{int}(\phi)} Z(J,0,0|M+g\phi)\, G_{\alpha\beta}(x,y|M+g\phi-g\hat{A}_{ext}) \rangle_\phi \tag{34}$$

or, using eq. (33),

$$G_{\alpha\beta}(x,y) = e^{-\frac{i}{2}\left(\frac{\delta}{\delta B},\, D^\phi \frac{\delta}{\delta B}\right)} \Big[e^{iS_{int}(B)} Z(J,0,0|M+gB) \times G_{\alpha\beta}(x,y|M+gB-\hat{A}_{ext}g) \Big]_{B=0} \tag{35}$$

and, similarly,

$$\Delta_{\mu\nu}(x,y) = i \frac{\delta}{\delta J_\nu(y)} \frac{\delta \ln Z}{\delta J_\mu(x)}\Big|_{\eta=\bar{\eta}=0}$$

$$= D_{\mu\nu}(x-y) - g^2 \int d^4z\, d^4u\, d^4v\, \langle e^{iS_{int}(\phi)} D_{\mu\nu}(x-z) \times$$

$$\times\, \mathrm{tr}\left[\gamma_\alpha G(z,u|M+g\phi-g\hat{A}_{ext})\, \gamma_\tau G(u,v|M+g\phi-g\hat{A}_{ext}) \right] D_{\tau\alpha}(v-y) \rangle_\phi$$

$$+ g^2 \left[\langle e^{iS_{int}(\phi)} a_\mu(x) a_\nu(y) \rangle_\phi - \langle e^{iS_{int}(\phi)} a_\mu(x) \rangle_\phi \langle e^{iS_{int}(\phi)} a_\nu(y) \rangle_\phi \right] \tag{36}$$

Here

$$a_\mu(x) = -\int d^4y\, D_{\mu\nu}(x-y) \left[J_\nu(y) + ig\, \mathrm{tr}\, \gamma_\nu G(y,y|M+g\phi-g\hat{A}_{ext}) \right] \tag{37}$$

is a superposition of the external vector potential $\hat{A}_{ext}(x)$ and of the effective averaged potential induced by the external current in the vacuum in the presence of the external bilocal field χ. The three-point function is given by the following expression (including radiation corrections on the external lines)

$$\tilde{\Gamma}_\mu(x,y,z) = \frac{1}{iZ} \frac{\delta}{\delta \bar{\eta}(y)} \frac{\delta}{\delta \eta(x)} \frac{\delta Z}{\delta J_\mu(z)}\Big|_{\eta=\bar{\eta}=0} =$$

$$= \langle e^{iS_{int}(\phi)} Z(J,0,0|M+g\phi) \times$$

$$\times \Big\{ \int d^4x' G(y,x'|M+g\phi-g\hat{A}_{ext}) D_{\mu\nu}(x'-z)\gamma_\nu G(x',x|M+g\phi-g\hat{A}_{ext})$$

$$+ g^2 \int d^4x' \Big(J_\nu(x') + i\,\mathrm{tr}\Big[\frac{1}{1-g^2 G(M+g\phi)\hat{A}_{ext}} G\gamma_\nu\Big]_{(x',x')}\Big) \times$$

$$\times D_{\nu\mu}(x'-z) G(y,x|M+g\phi-g\hat{A}_{ext})\Big\} \rangle_\phi \qquad (38)$$

The lowest order term of the three-point function (for $J=0$)

$$\int d^4x' G(y-x'|M)[\gamma_\nu] G(x'-x|M) D_{\nu\mu}(x'-z) \qquad (39)$$

just leads to the bare coupling of eq. (14) as it should be. Finally, Fig. 2 shows, for illustration, some graphical examples of the abnormal perturbation expansion of the Green's functions (34), (36), (38). The graphs on the left-hand side represent contributions obtained by expanding the integrand of the path-integrals in the bilocal field ϕ (closed quark loops arise from $S_{int}(\phi)$), whereas on the right-hand side the corresponding classes of Feynman graphs are drawn. We recall that a solid line denotes the nonperturbative quark propagator $G(p|M)$ of eq. (14).

5. Discussions and Concluding Remarks

It has been shown that the dynamical breakdown of chiral symmetry may conveniently be studied by using bilocal integration variables in the path-integral representation of the generating functional. It turns out that the "classical" equation of motion of the bilocal field is identical with the differential version of the Schwinger-Dyson equation for the quark propagator in the lowest nontrivial approximation of perturbation theory. The existence of nontrivial solutions to this differential equation with the boundary conditions (18) has been shown in ref. /8/. There it has been found that the quark propagator

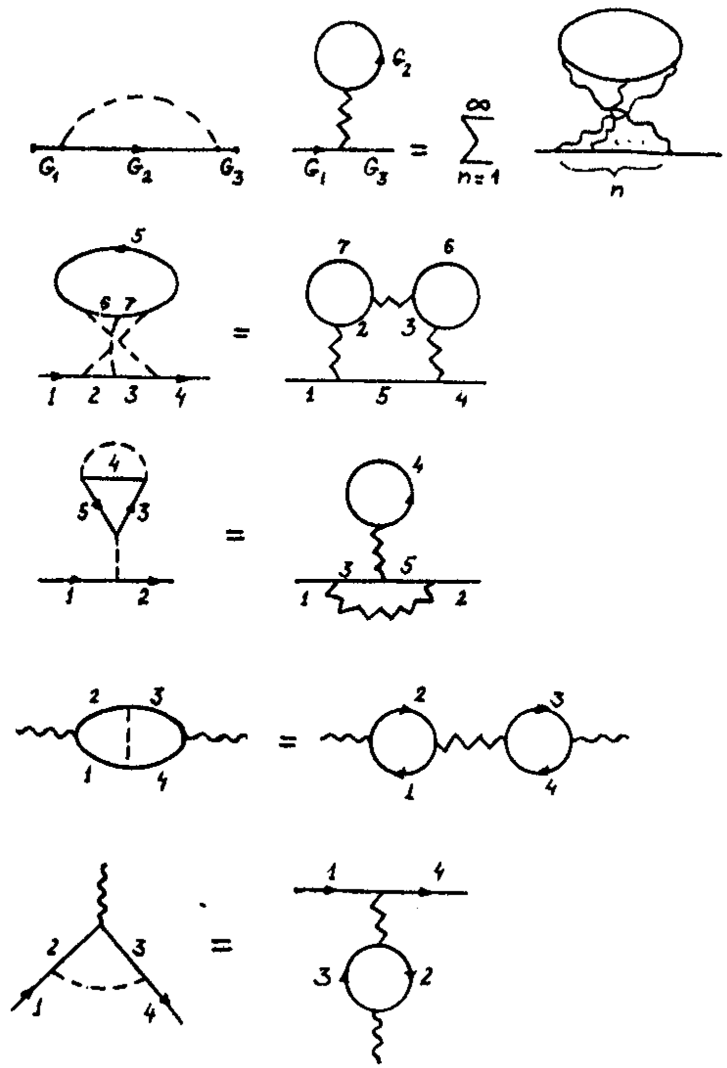

Fig. 2.

does not develop a particle pole in the timelike region, the quarks being confined. It should be remarked that this proof heavily relies on the boundary conditions (18) obtained from the integral form of the Schwinger-Dyson gap equation. It would be interesting whether quark confinement holds also if the additional radiative corrections are taken into account.

Finally, an abnormal perturbation expansion taking the nonperturbative solution of the Schwinger-Dyson equation as the lowest order term has been formulated and the correspondence of these diagrams to the Feynman diagrams has been established.

Concluding we mention that our approach bears a resemblence to the method of bilocal sources of Jackiw, Cornwall and Tomboulis/13/ which has been used to construct an effective potential for composite operators. These authors have obtained the Schwinger-Dyson gap equation from the minimum of the effective potential.

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Added note

After completing this work we have obtained a paper of H.Kleinert/14/ where the gap equation (13) has been derived, too, using bilocal techniques.

References

1) Y.Nambu and G.Jona-Lasinio, Phys.Rev., 122, 345 (1961); ibida 124, 246 (1961),
see also
V.G.Vaks and A.I.Larkin, Journal Theor. i Exp.Fis., 40, 282 (1961) (in Russian);
B.A.Arbuzov, A.N.Tavkhelidze and R.N.Faustov. Dokl. AN SSSR 139, 345 (1961) (in Russian).

2) R.Jackiw and K.Johnson, Phys.Rev., D8, 2386 (1973);
J.Cornwall and R.E.Norton, Phys.Rev., D8, 3338 (1973);
D.Gross and A.Neveu, Phys.Rev., D10, 2332 (1974).

3) A.T.Filippov, Proceedings of the IV International Conference on Nonlocal Quantum Field Theory (Alushta 1976) D-2-9788 Dubna 1976;
P.I.Fomin et al, Preprint ITP-75-93E Kiev (1975).

4) K.Johnson, M.Baker and R.Willey, Phys.Rev., 136, B1111 (1964);
M.Baker and K.Johnson, Phys.Rev., D8, 1110 (1973).

5) N.N.Bogolubov and D.V.Shirkov, Introduction in the Theory of Quantized Fields, Moscow 1974 (in Russian).

6) M.A.Braun, preprint FIAN SSSR, No.140 Moscow 1971.

7) R.Haag and A.J.Maris, Phys.Rev., 132, 2325 (1963);
P.Langacker and H.Pagels, Phys.Rev., D9, 3413 (1974).

8) R.Fukuda and T.Kugo, preprint KUNS 350, Kyoto (1976).

9) J.Bardeen, L.Cooper and J.Schrieffer, Phys.Rev., 108, 1175 (1957);
N.N.Bogolubov, Nuovo Cim., 7, 794 (1958).

10). N.Nakanishi, Progr. in Theor.Phys., Suppl. 43, 1 (1969).

11) J.S.Goldstein, Phys.Rev., 91, 1516 (1953).

12) E.S.Fradkin, O.K.Kalashnikov, Preprint FIAN SSSR, No. 140 Moscow (1971).

13) J.M.Cornwall, R.Jackiw and E.Tomboulis, Phys.Rev., D10, 2428 (1974).

14) H.Kleinert, Univ. de Genève prepr. (1976).

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