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THE MOMENTUM SPECTRA OF LIGHT NUCLEI IN REACTIONS Ad → AX AT LARGE MOMENTUM TRANSFERS

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1. In refs. [1-3] a theoretical analysis of reactions $pd \to pX$ and $dd \to dX$ was performed within the multiple scattering model. It has been shown that the two-peak structure observed experimentally at $\vec{q}^{\,2} \geq 0.5 \div 1$ $(GeV/c)^2$ in the momentum spectra of scattered protons and deuterons is a direct consequence of the target composite structure. The composite structure of a scattered object in reaction $dd \to dX$ starts to be observed at momentum transfers $\vec{q}^{\,2} \geq 1.5$ $(GeV/c)^2$ and gives rise to the third peak in its momentum spectrum [4].

It is natural to trace what is the influence of further complication of the structure of interacting nuclei on the characteristics of reactions $A_1 A_2 \to A_1 X$ at large momentum transfers in the nonrelativistic approximations. The effect of relativistic contractions will be considered in attempts to generalize the obtained results to hadron-hadron interactions.

Theoretical results could be useful for planning experiments on the relativistic-ion beams which have recently formed an important part of the scientific program of many physical laboratories of the world.

2. In the present paper the reactions $^3He\,d \to {}^3He\,X$ and $^4He\,d \to {}^4He\,X$ are studied. Let us first trace, on a qualitative level, the dynamics of appearance of a possible structure in momentum spectra of nuclei, quasielastically scattered by deutron nucleons. As can be seen from calculational results presented below, the structure emerges at rather large momentum transfers, of an order of 0.5-0.7 GeV/c per nucleon of a scattered nucleus. It is obvious that the scattering of a nucleus with such a large transfer momentum (as compared with the Fermi-momentum) is possible provided that all its nucleons suffer at least one interaction with target nucleons. Amplitudes for the multiple scattering mechanisms, in which at least one nucleon of a scattered nucleus does not participate in the interaction, turn out to be proportional to some quantities like nuclear form factors and are negligible as compared to the amplitudes of interaction of all nucleus nucleons, which are proportional to combinations of NN-scattering amplitudes. Interaction mechanisms of a scattered nucleus A with a deuteron, which are significant at large transfer mo-

menta, are easily classified since each nucleon of the nucleus A can interact either with a proton or a neutron (single scattering) or with both of them successively (double scattering). Let m nucleons of the scattered nucleus undergo single interaction with a proton only, n nucleons with a neutron only, and the remaining $(A-m-n)$ nucleons with both the deuteron nucleons. Clearly, the condition of "survival" of nucleus A in the large-transfer-$\vec{q}$ momentum scattering is that each of its nucleons transfers the momentum $\vec{q}/A$ to deuteron nucleons which it interacts with. If a certain nucleon of the nucleus A interacts with both deuteron nucleons, this momentum is distributed almost equally between them. Thus, as a result of this mechanism, the deuteron proton acquires the momentum $\vec{q}(A+m-n)/2A$ and the neutron $\vec{q}(A-m+n)/2A$. If the deuteron nucleons were at rest and the described procedure of the momentum transfer were unambiguous, the energy loss by nucleus A required to transfer such momenta to deuteron nucleons would be

$$\Delta E_{mn} = \frac{\vec{q}^{\,2}}{8m_N A^2}[(A+m-n)^2+(A+n-m)^2] = \frac{\vec{q}^{\,2}}{4m_N} + \frac{1}{m_N}\left[\frac{\vec{q}(m-n)}{2A}\right]^2 \quad (1)$$

and in the momentum spectra of the scattered nucleus one would observe sharp (δ-shaped) peaks corresponding to energy losses ΔE_{mn}. From (1) it can be seen that the number of such peaks equals $(A+1)$ and they are equidistant in the variable $k = \sqrt{m_N \Delta E - \frac{\vec{q}^{\,2}}{4}} = \frac{\sqrt{M_X^2 - M_d^2}}{2}$, the momentum of relative motion of the neutron and proton /5/. If these peaks were really δ-shaped, then, in principle, all of them would be observable irrespective of the strength of the corresponding "lines". However, there exist some factors (apart from apparatus) which give rise to an essential smearing out of this clear structure of studied spectra. One of them is common for all the mechanisms of interaction of A nucleus nucleons with deuteron nucleons, it is the intranuclear motion. This factor is unique only for the mechanism of quasielastic scattering of a nucleus on a deuteron nucleon ($m=A$, $n=0$, $m=0$, $n=A$). In this case the whole momentum $\vec{q}$ is transferred to one nucleon and the mechanism of sharing of this momentum among nucleons of the nucleus A is thus of no significance. Therefore, the quasi-elastic-scattering peak should be the most narrow and should be determined by the width of nucleon distribution over the Fermi-momenta. The shape of peaks corresponding to $m \neq 0$, $n \neq 0$, with $m+n=A$, depends also on the Fermi-motion of nucleus A. Evidently, the mentioned above condition of "survival" of the nucleus A is fulfilled up to an order of magnitude

of the Fermi-momenta of nucleons of the nucleus **A**. Consequently, the width of the corresponding peaks will be a sum of widths of Fermi-distributions in both the nuclei. The most smeared peaks are those corresponding to mechanisms with $m+n<A$. The reason is that the latter include the interaction at least of one nucleon of the nucleus A with both the deuteron nucleons in which the momentum $\vec{q}/A$ is equally distributed between the proton and neutron up to an order of magnitude of $1/\sqrt{B}$, where B is the slope parameter of the NN-scattering differential cross section ($B^{-1/2}$ is of an order of magnitude of the quark Fermi-momentum in a nucleon). Overlapping of the peaks of finite width may smear out, completely or partially, the structure of the discussed spectra. One of the condition of separation of peaks corresponding to various interaction mechanisms (or to set of different mechanisms with equal $(m-n)$) is that the interval between their centers exceed noticeably their characteristic widths. From (1) it follows that this condition can be fulfilled for the reactions with large momentum transfers (with small cross sections). The second condition is the comparability of strengths of mechanisms corresponding to different sets of m, n. To establish when this condition holds requires concrete calculations results which are presented below.

3. Since the principal schemes of theoretical analysis of the reaction $dd \to dX$ and of more general ones, $Ad \to AX$ in the framework of the multiple scattering model have no difference and the first of them is rather thouroughly studied, we recall only the main approximations used and present final results of calculations for cross sections of inclusive reactions $Ad \to AX$ (for details, see refs. /1-3/).

First of all, calculations do not include the contribution into the high momentum part ($\Delta E \lesssim \vec{q}^{\,2}/2m_N$) of the spectrum of processes with the pion production, i.e., it is assumed that a nondetected system X consists of products of the deuteron disintegration (proton, neutron). The complete system of wave functions of the target final state was chosen in the form of plane waves, i.e., effects of np-interaction in the final state were neglected. A detailed discussion of this question is given in ref. /1/. Wave functions of the ground state of the deuteron and nucleus A are taken in the form of Gaussian functions of relative coordinates. This conventional approximation, together with the conventional Gaussian parametrization for profile functions of NN-scattering allows us to perform all integrations in the problem. The analytic final form of the differential cross sections of reaction $Ad \to AX$ to be investigated is as follows

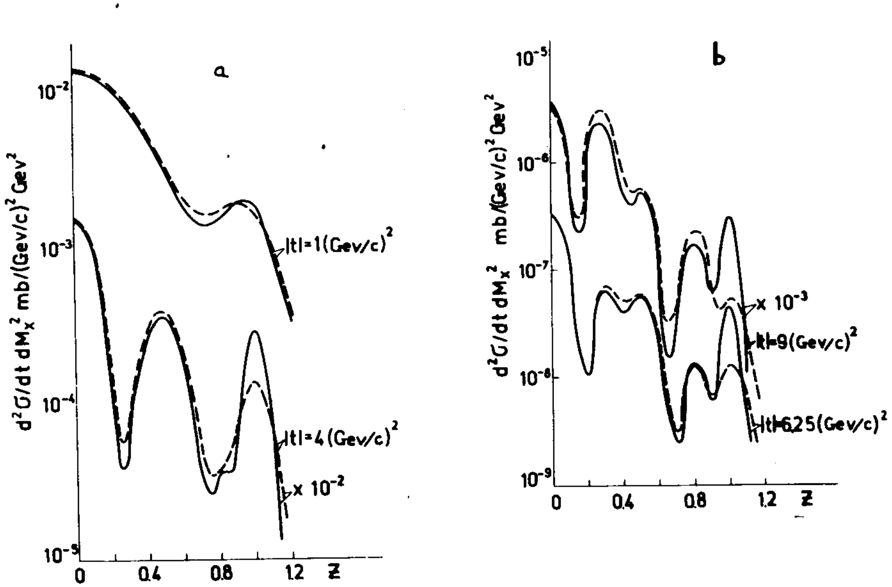

Fig.1. Helium spectra with respect to the missing mass in reactions ^{4}Hed $\to$ ^{4}HeX. Solid lines are the nonrelativistic approximation, dashed lines with the relativistic contraction of nuclei taken into account.

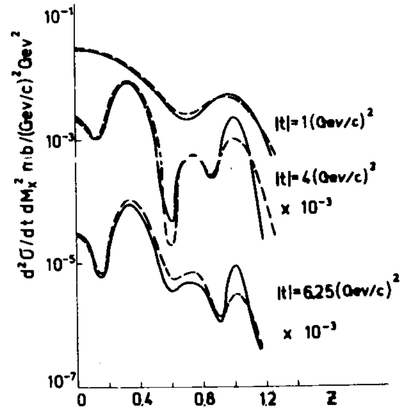

Fig.2. The same as Fig.1 but for reaction ^{3}Hed $\to$ ^{3}HeX.

$$\frac{d^2\sigma}{dt\,dM_x^2} = \frac{\pi[\Gamma(A+1)]^2(R^2+2B)^2(R^2+B)^2}{4R_d^2}\times \qquad (2)$$

$$\times\sum_{k=0}^{A}\sum_{\ell=0}^{A-k}\sum_{m=0}^{A}\sum_{n=0}^{A-m}(-\lambda_1)^{k+\ell+m+n}(\lambda_2)^{2A-k-\ell-m-n}\beta_{k\ell}\beta_{mn}\times$$

$$\times\frac{\exp[-a_{k\ell}-a_{mn}-(\beta_{k\ell}+\beta_{mn})p^2-\beta_{k\ell}q_{k\ell}^2-\beta_{mn}q_{mn}^2]I_0[2p(\beta_{k\ell}q_{k\ell}+\beta_{mn}q_{mn})]}{\Gamma(k)\Gamma(\ell)\Gamma(m)\Gamma(n)\Gamma(A-k-\ell+1)\Gamma(A-m-n+1)[AR^2+B(2A-k-\ell)][AR^2+B(2A-m-n)]},$$

$$a_{k\ell}=\frac{Bq^2[2AB+R^2(A+k+\ell)]}{4A[AR^2+B(2A-k-\ell)]},\quad a_{mn}=\frac{Bq^2[2AB+R^2(A+m+n)]}{4A[AR^2+B(2A-m-n)]},$$

$$q_{k\ell}=\frac{q(k-\ell)(R^2+B)}{2[AR^2+B(2A-k-\ell)]},\quad q_{mn}=\frac{q(m-n)(R^2+B)}{2[AR^2+B(2A-m-n)]},$$

$$\beta_{k\ell}=\left\{\frac{2}{R_d^2}+\frac{A-k-\ell}{B}+\frac{4k\ell}{(k+\ell)(R^2+2B)}+\frac{(k-\ell)^2(A-k-\ell)}{(k+\ell)[AR^2+B(2A-k-\ell)]}\right\}^{-1}, \qquad (2)$$

$$\beta_{mn}=\left\{\frac{2}{R_d^2}+\frac{A-m-n}{B}+\frac{4mn}{(m+n)(R^2+2B)}+\frac{(m-n)^2(A-m-n)}{(m+n)[AR^2+B(2A-m-n)]}\right\}^{-1},$$

$$\lambda_1=\frac{\sigma}{2\pi(R^2+2B)},\quad \lambda_2=\frac{\sigma^2}{16\pi^2B(R^2+B)},$$

$B=7\ (\mathrm{GeV}/c)^{-2}$, $\sigma=39$ mb.

where q^2 is the momentum transfer squared, p the momentum of a scattered nucleus, σ the total cross section of NN-interaction, B the slope parameter of the NN-scattering differential cross section, A the number of nucleons of nucleus A, I_0 the Bessel function of imaginary argument, R and R_d are oscillator parameters of Gaussian distributions of the nucleon density in nuclei A and d, respectively. The latter were taken to equal

$R_d=2.28$ fm, $R_{^3He}=1.7$ fm, $R_{^4He}=1.37$ fm.

Note that the destructive interference of amplitudes with both $(k-\ell)$ and $(m-n)$ equal to unity, i.e., of those giving the most significant contribution into neighbouring peaks, like in the case of reactions $pd\to pX$ and $dd\to dX$ (see, e.g., refs. /1-3/) provides a more clear separation of peaks in momentum spectra. The dynamics of evolution of the structure in momentum spectra of nuclei ^{3}He and ^{4}He with growing transfer momenta is shown in Fig.1,2.

Instead of the traditional variable p, the scattered-nucleus momentum, in terms of which the picture is strongly deformed, we use the dimentionless variable

$$z=\sqrt{(4m_N\Delta E-\vec{q}^{\,2})/\vec{q}^{\,2}}=2\sqrt{(M_x^2-M_d^2)/|t|}\,.$$

In terms of this variable the positions of peaks are almost equidistant and do not depend on the invariable transfer t. From Figs. 1,2 it is seen that with increasing transfer momenta the structure of spectra becomes more clear, however, the cross sections of processes sharply fall in magnitude.

4. Interest to the problem of a possible generalization of the results obtained to the processes of hadron-hadron interactions (e.g., $N\pi \to NX$) arises for two reasons: first, based on the model of hadrons as composite systems within the quark hypothesis, the analogy of structures of hadrons and light atomic nuclei, and second, the assumption of the same interaction mechanism. At first sight it seems natural to conclude that a similarity should exist in the behaviour of characteristics of nculeus-nuclear and hadron-hadron processes. The discovery of the diffraction structure in the differential cross sections of elastic pp-scattering confurms to an extent this conclusion. However, a detailed comparison of calculations, within the multiple-scattering model performed by the traditional scheme of the theory of nucleus-nucleus scattering revealed a noticeable discrepancy between the theory and experiment. The theory predicted two diffraction minima in the differential cross sections of pp -scattering at transfer momenta $-t \simeq 0.7 \div 0.8$ (GeV/c)^2; $-t \simeq 3.5 \div 4.5$ (GeV/c)^2 while experimentally only one minimum was registered at $-t \simeq 1.3 \div 1.4$ (GeV/c)^2. This discrepancy has led authors of ref. /6/ to the premature conclusion about the invalidity of the quark model and necessary inclusion into the hadron structure of a considerable gluon component. This difficulty was resolved in refs. /7/ the authors of which stressed for the first time the importance of the consideration of a number of relativistic effects and, on the basis of the relativization scheme they proposed for the multiple scattering model, described the data on high-energy pp-scattering without going beyond the scope of the quark model.

Relativistic effects should be taken into account also in attempting to generalize the results we have obtained for the cross sections of inclusive reactions $^3He\,d \to {}^3He\,X$ to the case of reactions $N\pi \to NX$.

In the above presented results of calculations for reaction $Ad \to AX$ the effect of relativistic contraction of interacting nuclei was not taken into account. Being taken into account this effect leads to the dependence of oscillator parameters R_d and R of Gaussian distributions of nucleon density in the deuteron and nuclei on the momentum transfer:

$$R_d^2(q) = \frac{R_d^2}{1 + \frac{q^2(z^2+1)^2}{4M_d^2}}, \qquad R_A^2(q) = \frac{R_A^2}{1 + \frac{q^2}{2Am_N^2}}.$$

This effect, though somewhat deforming the picture (see dashed lines in Figs.1,2), does not change the main conclusion on a possible observation of the (A+1)-peak structure of momentum spectra of scattered nuclei.

A completely different picture is observed for reaction $N\pi \to NX$. If the cross sections of this reaction are calculated within the quark model without the effects of relativistic contraction of hadrons, then the multiple scattering model predicts for nucleon momentum spectra almost the same properties as for spectra of nuclei ^3He in reaction $^3He\,d \to {}^3He\,X$, with the only difference that this structure should completely reveal itself at values of the transfer momentum squared $\vec{q}^{\,2}$ almost $\langle r^2\rangle_A / \langle r^2\rangle_h \simeq 6$ times as large as those in the reaction $Ad \to AX$. For reaction $N\pi \to NX$ these momenta are of an order of $15 \div 20$ (GeV/c)^2. However, the inclusion of the effect of relativistic contraction in reaction $N\pi \to NX$ results in full disappearance of the diffraction picture (see Fig.3). In this case we used the following values of parameters $R_\pi \simeq 0.59$ fm, $R_N \simeq 0.84$ fm, $B = 0.5$ (GeV/c)$^{-2}$, $\sigma_{qq} = 5.38$ mb. Absence of the structure does not contradict the multiple scattering model, because in hadron-hadron interactions the very relativistic nature of hadrons is significant, which we have not yet taken into account. Therefore, conclusions about invalidity of the model or interaction mechanisms would be premature, like for pp-scattering. However, this example clearly shows how dangerous are predictions on the basis of the analogy of

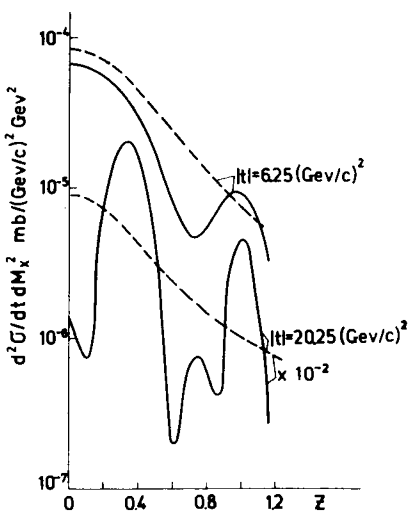

Fig.3. Proton spectra with respect to the missing mass in reaction $N\pi \to NX$. Solid lines are the nonrelativistic approximation, dashed lines with the relativistic contraction of nuclei taken into account.

structures of hadrons and light atomic nuclei without allowing for specific features of hadron-hadron interactions.

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Саакян С.Б.Импульсные спектры легких ядер E2-82-409
в реакциях Ad→AX при больших передачах импульса

В рамках модели многократного рассеяния рассчитаны сечения инклюзивных реакций ^{3}Hed → ^{3}HeX и ^{4}Hed → ^{4}HeX. Показано, что при значениях переданного импульса на один нуклон рассеиваемого ядра больше чем 0,5 ГэВ/с в импульсных спектрах последнего возникает сложная структура, обусловленная разделением вкладов различных механизмов многократного рассеяния. В спектрах нуклонов в реакции Nπ→NX подобная структура замазывается релятивистскими эффектами сжатия.

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Saakian S.B. The Momentum Spectra of Light Nuclei E2-82-409
in Reactions Ad → AX at Large Momentum Transfers

Within the multiple scattering model the inclusive cross sections ^{3}Hed → ^{3}HeX and ^{4}Hed → ^{4}HeX are calculated. At momentum transfers per nucleon of a scattered nucleus of 0.5 GeV/c in momentum spectra a fine structure arises because of the separation of contributions from different mechanisms of multiple scattering. In nucleon spectra in the reaction Nπ → NX such a structure is suppressed by relativistic contraction.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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